Interarea Oscillations Revisited

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***Abstract*—This letter revisits interarea oscillation analysis using networked control analysis techniques. The power system analysis problem is analyzed as a networked control problem, specifically consensus control of homogeneous systems with static output feedback. The power grid is represented by a graph Laplacian matrix. Stability of the entire system can be evaluated by individual system dynamics and graph Laplacian's eigenvalues. Through this technique, the classical large-scale power system analysis problem is decomposed into multiple small-scale system analysis problems. Analysis of the classical two-area four-machine system is conducted by the proposed approach and compared with the small-signal analysis results from Power System Toolbox. The interarea oscillation mode is found to be related to the second smallest eigenvalue of the graph Laplacian matrix while the local oscillation modes are related to the other eigenvalues of the graph Laplacian matrix.**

***Index Terms*—Interarea oscillations, consensus control, multi-agent systems**

I. Introduction

Small-signal stability analysis is a standard approach for power system oscillations, especially interarea oscillations. The state-space model of the entire system consisting multiple generators is first built. Eigenvalue and participation factor analysis are then carried out to identify related states to an oscillation mode [1]. The recent advance in consensus control of multi-agents over a network provides a new approach for stability analysis. In this letter, stability analysis technique for homogeneous system consensus control through static output feedback [2] is adopted for power system oscillation analysis. To apply the analysis technique, the power system is first converted to a networked control problem with homogenous systems and static output feedback. The power grid is represented by a graph Laplacian matrix. Through this technique, the classical large-scale power system analysis problem is decomposed into multiple small-scale system analysis problems. In the case study section, analysis of the classical two-area four-machine system is conducted by the proposed decomposed approach and compared with the small-signal analysis results from Power System Toolbox [3]. The interarea oscillation mode is found to be related to the second smallest eigenvalue of the graph Laplacian matrix while the local oscillation modes are related to the other eigenvalues of the graph Laplacian matrix.

II. Power system viewed as a networked control problem

Consider a system with n generator. For every generator in the power system, classic model is assumed. The dynamics of each subsystem is expressed in a state-space model, where δ notates rotor angle in rad/s, ω notates speed in pu, and P_D notates area load in pu.

$$\underbrace{\begin{bmatrix}\dot{\Delta\delta_i}\\ \dot{\Delta\omega_i}\end{bmatrix}}_{\dot{x}_i} = \underbrace{\begin{bmatrix}0 & \omega_0\\ 0 & -\frac{D_i}{H_i}\end{bmatrix}}_{A_i}\underbrace{\begin{bmatrix}\Delta\delta_i\\ \Delta\omega_i\end{bmatrix}}_{x_i} + \underbrace{\begin{bmatrix}0\\ \frac{-1}{2H_i}\end{bmatrix}}_{B_i}\underbrace{\Delta P_{tie,i}}_{u_i} + \begin{bmatrix}0\\ \frac{-1}{2H_i}\end{bmatrix}\underbrace{\Delta P_{D,i}}_{d_i}$$

$$\underbrace{\Delta\delta_i}_{y_i} = \underbrace{\begin{bmatrix}1 & 0\end{bmatrix}}_{C}\begin{bmatrix}\Delta\delta_i\\ \Delta\omega_i\end{bmatrix} \quad (1)$$

If we treat the total tie-line flow $P_{tie,i}$ as the system's input u_i, and the rotor angle $\Delta\delta_i$ as the output y_i, then the entire system can be viewed as a consensus or synchronizing control over a network. The input u_i has the following structure.

$$u_i = \sum_j \Delta P_{ij} = \sum_{j\neq i}^{n} T_{ij}(\Delta\delta_i - \Delta\delta_j) = \sum_{j\neq i}^{n} T_{ij}(y_i - y_j)$$

where P_{ij} is the tie-line flow on the line between bus i and bus j and $T_{ij} = \frac{\partial P_{ij}}{\partial \delta_{ij}}$.

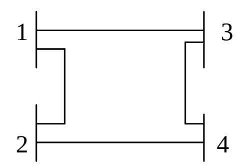

Fig. 1. A four-bus network.

For a network shown in Fig. 1, we will have the input vector u as the following.

$$\begin{bmatrix}u_1\\u_2\\u_3\\u_4\end{bmatrix} = \underbrace{\begin{bmatrix}T_{12}+T_{13} & -T_{12} & -T_{13} & 0\\ -T_{12} & T_{12}+T_{24} & 0 & -T_{24}\\ -T_{13} & 0 & T_{13}+T_{34} & -T_{34}\\ 0 & -T_{24} & -T_{34} & T_{24}+T_{34}\end{bmatrix}}_{L}\begin{bmatrix}y_1\\y_2\\y_3\\y_4\end{bmatrix} \quad (2)$$

Note that L is a weighted graph Laplacian matrix. A Laplacian matrix is defined as follows.

$$L_{ij} = \begin{cases}-1 & i,j \text{ are connected through a link}\\ 0 & i,j \text{ are connected through a link}\end{cases} \quad (3)$$

$$L_{ii} = \sum_j -L_{ij}, i \neq j \quad (4)$$

If every link is associated with a weight T_{ij}, then we have a weighted graph Laplacian matrix.

$$L_{ij} = \begin{cases}-T_{ij} & i,j \text{ are connected through a link}\\ 0 & i,j \text{ are connected through a link}\end{cases} \quad (5)$$

$$L_{ii} = \sum_j T_{ij}, i \neq j \quad (6)$$

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TABLE I
EIGENVALUES COMPUTED BY PST AND THE PROPOSED METHOD FOR TWO CASES

case 1			case 2		
PST	proposed	eig(L)	PST	proposed	eig(L)
$-0.0111+0.0000i$	$-0.0000+0.0000i$	0.0000	$0.0126+0.0000i$	$0.0000+0.0000i$	-0.0000
$0.0111+0.0000i$	$-0.0077+0.0000i$		$-0.0126+0.0000i$	$-0.0077+0.0000i$	
$-0.0000\pm 3.5319i$	$-0.0038\pm 3.6000i$	4.0221	$-0.0000\pm 2.0444i$	$-0.0038\pm 2.1495i$	1.4339
$0.0000\pm 7.5092i$	$-0.0038\pm 7.5047i$	17.4789	$0.0000\pm 7.4936i$	$-0.0038\pm 7.3983i$	16.9865
$-0.0000\pm 7.5746i$	$-0.0038\pm 8.5494i$	22.6838	$-0.0000\pm 7.5204i$	$-0.0038\pm 8.1449i$	20.5879

It can be seen that the matrix L in (2) is indeed a weighted graph Laplacian matrix. If every generator has the same parameters, then we have homogeneous systems and $A_i = A, B_i = B$. Therefore

$$\dot{x} = (I \otimes A + L \otimes BC)x + (I \otimes B)d \tag{7}$$

where $\otimes$ notates Kroneck product.

Research in [2] give the stability criterion for the above system. The system $I \otimes A + L \otimes BC$ is Hurwitz if and only if all the matrices $A + \lambda_i BC$ are Hurwitz, where λ_i are the eigenvalues of L.

III. CASE STUDY

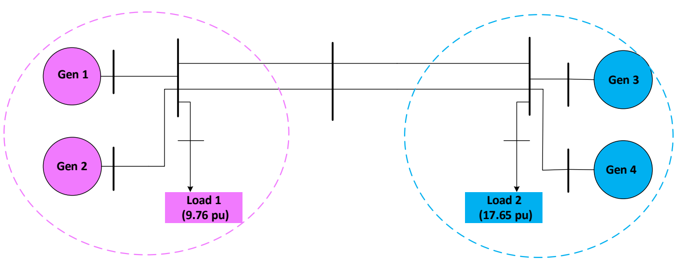

Fig. 2. Two-area four-machine test system.

The two-area four-machine power system used for interarea oscillation is shown in Fig. 2. The power grid connection of the system is equivalent to the connection in Fig. 1. Therefore, we can adopt the graph Laplacian matrix L in (2) for stability analysis.

If we assume that the system symmetric and $T_{13} = T_{24}$, $T_{12} = T_{34}$, then the eigenvalues of L are:

$$\begin{bmatrix}\lambda_1 & \lambda_2 & \lambda_3 & \lambda_4\end{bmatrix} = \begin{bmatrix}0 & 2T_{13} & 2T_{12} & 2(T_{13}+T_{12})\end{bmatrix} \tag{8}$$

Effect of power transfer level and tie-line length
It can be seen that λ_2, the second smallest eigenvalue of the Laplacian matrix is related to T_{13}. For simplicity of analysis, assume that the line is lossless. Then

$$P_{13} = \frac{E_1 E_3}{X_{13}} \sin(\delta_1 - \delta_3) \tag{9}$$

$$T_{13} = \frac{E_1 E_3}{X_{13}} \cos(\delta_1 - \delta_3) \tag{10}$$

A longer line corresponds to a greater line reactance X_{13}, in turn a smaller λ_2. A heavier power transfer corresponds to a greater angle difference $\delta_1 - \delta_3$ and also a smaller T_{13} or λ_2.

On the other hand, the other two eigenvalues λ_3 and λ_4 are dominant by T_{12} as T_{12} is much greater than T_{13} due to the close connection between Gen 1 and Gen 2.

The system eigenvalues are determined by the following matrices' eigenvalues:

$$A - \lambda_i BC = \begin{bmatrix} 0 & \omega_0 \\ \frac{-\lambda_i}{2H} & \frac{-D}{2H} \end{bmatrix} \tag{11}$$

In turn, the system eigenvalues are also determined by the following polynomials.

$$s^2 + \frac{D}{2H}s + \frac{\lambda_i \omega_0}{2H} = 0 \tag{12}$$

$$s \approx -\frac{\sqrt{2}}{4}\frac{1}{\sqrt{\lambda_i H \omega_0}} \pm j\sqrt{\frac{\lambda_i \omega_0}{2H}} \tag{13}$$

Observing (13), we can see that for a small λ_i, the oscillation mode's frequency is low, while for a large λ_i, the oscillation mode's frequency will be large. This indicates that the low frequency inter-area oscillation corresponds to the oscillation mode associated to λ_2, while the local oscillation modes are associated to the other Laplacian matrix eigenvalues.

Table I lists the eigenvalues computed by Power System Toolbox using the conventional overall system analysis approach versus the eigenvalues computed using small-scale matrices in (11) at a typical operation case (case 1) and the same operation case with one of the tie-lines tripped (case 2). The comparison shows that the proposed small-scale matrix eigenvalue calculation can capture the system dynamics adequately.

IV. CONCLUSION

This letter applies consensus control analysis technique for homogeneous systems into power system stability analysis. The new analysis approach improves understanding of inter-area oscillation modes and local oscillation modes from a network point of view. The technique has the potential of decomposing a large-scale system analysis problem into small-scale system analysis problems.

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