A Novel Multi-Agent Decision Making Architecture Based on Dual's Dual Problem Formulation

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***Abstract*—In this paper, a novel multi-agent decision making architecture based on dual's dual problem formulation is designed for economic operation. The benefits of such architecture include its limited information exchange among autonomous entities and the ease of implementation. Existing AC optimal power flow (OPF) software packages can be easily incorporated. The proposed decision making is based on dual's dual problem formulation, which is the first time seen in the literature to tackle an OPF problem. Subgradient-based iterative solving procedure results in a multi-agent decision making architecture. Convergence property of the algorithm is analyzed. The application scope of the proposed method is also discussed to identify challenges for meshed networks. Case studies are given to demonstrate the feasibility of the proposed method in providing approximate solutions to IEEE 14-bus system AC OPF. Implementing the proposed decision making architecture in real world is also demonstrated in a dynamic simulation platform.**

***Index Terms*—Distributed optimization; Primal Dual Decomposition; Multi Agent Systems; frequency control; convergence property**

I. Introduction

In this paper, a novel multi-agent decision making architectures for microgrids and power system optimization and control will be investigated. Introduction of numerous smart buildings, distributed energy sources and energy storages poses challenges in operation and control. A centralized control center may over burden its SCADA system and computing machines to collect every piece of measurements and calibrate optimal operation schemes. On the other hand, due to privacy concerns, communities are not willing to share all information. Multi-agent decision making strategies become appealing for the above mentioned reasons. Information exchange topologies, convergence properties, and real-world implementation are all challenging issues to be tackled.

Multi-agent decision making strategies have been seen in power system applications in the literature, e.g., demand and utility interactions [1], [2], economic dispatch [3], distributed DC OPF [4]–[7], and distributed AC OPF [8]–[12]. There are various names referring to this type of decision making architecture, including distributed control, decentralized control and multi-agent systems. A variety of distributed optimization problem formulations and algorithms corresponds to a variety of information exchange topologies.

In terms of distributed problem formulation, the two main approaches are primal decomposition and dual decomposition [13]. Dual-based decomposition is suitable for optimization problems with decomposed cost functions and global constraints [14]. Power system economic operation falls into this category. Most of the earlier papers on OPF focus on dual decomposition only. For example, [4] is on dual decomposition method to solve DC OPF problems. Areas are separated at the boundary buses. These buses are treated as generators with constant prices. With given prices, each area carries out DC OPF and calibrates how much power to export or import. The prices are updated based on the power difference on the tie-lines. Dual decomposition has also been applied in voltage control [15], electric vehicle control [16]. In [8], instead of formulating a Lagrangian function using Lagrangian relaxation, an augmented Lagrangian function is formulated.

In addition to Lagrangian relaxation-based dual decomposition, in some cases, another layer of relaxation is imposed to have a primal-dual problem. This formulation has been seen in [17], [18] to explain that a generator's droop control is aligned with a primal-dual update process. The optimization problem formulated in [17], [18] considers a dc network without line limits. The primal-dual formulation is adopted by the authors in [7] and developed a new type of distributed DC OPF algorithm considering line limits for radial networks.

In this paper, inspired by the primal-dual approach, a novel problem formulation is proposed. The formulation is named as dual's dual problem. Instead of going through layers of Lagrangian relaxation, in this paper, only one layer of relaxation of the original OPF problem is examined. This approach proves that the Lagrangian multiplier of the dual problem in [17], [18] is indeed the tie-line flow. This formulation will be extended to AC OPF. In addition, challenges in meshed network application will be identified for DC OPF.

In terms of iterative procedures, popular approaches include subgradient-based update [19], alternating direction method of multipliers (ADMM) [20], saddle-point-flow method [21] and center free consensus-based update [22]. Subgradient-based strategies have many multi-agent applications, e.g. [4], [15], [23]. The authors have also adopted subgradient-based multi-agent control strategy in [12] for utility-community interaction. Center-free consensus-based update is also seen in power system applications. For example, in [3], an economic dispatch problem is first converted to a consensus problem: prices at every area should be the same. Then each area will update its price based on the price information from the neighboring areas. The price update uses a weighting matrix (Markov stochastic matrix) to take into account of the price information from other areas. With the converging property of a Markov matrix, the prices everywhere will be the same finally. To apply center free consensus-based update, the

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network has to be assumed to be lossless and the line limits are ignored.ADMM is another way to carry out distributed AC OPF. ADMM comes from dual ascent subgradient method. To enhance convergence performance, the objective function is augmented with a quadratic function of the equality term. This method is called method of multiplier. ADMM is based on the method of multiplier and implemented in a distributed fashion. When separated into subproblems carried out by multiple agents, each agent is given information of the Lagrangian dual (price) and information from other agents. Prices will be updated based on the decision variables found by agents. In ADMM based AC OPF, information such as voltage phasors are exchanged among agents (see Fig. 1 in [24]) and each agent solves an augmented optimization problem.

In this paper, subgradient-based algorithm is developed, which results an easy to implement information exchange architecture.

While information level decision making can be tested by software packages such as MATPOWER [25], there is a need to test decision making strategies in dynamic simulation platforms to examine their effect on power system frequency responses and dynamics. Few research papers in the literature have examined distributed control and power system dynamic performance. Among them, the distributed optimization-based generator control was implemented into a real-time digital simulation platform in [3], where the generators' turbine-governor power references are updated every 0.1 seconds with additional frequency feedback. In our research, implementation of the proposed decision making architecture is also tested in a dynamic simulation platform.

Overall, the objective of this paper is to design a novel multi-agent decision making architecture based on dual's dual problem formulation. Convergence property and applicability scope are also investigated. How to implement the architecture in real-world is illustrated through a dynamic simulation platform. This paper is the first kind to apply dual's dual method in DC or AC OPF solving. The resulting architecture is easy to be implemented.

The rest of the paper is organized as follows. Section II presents the dual's dual problem formulation and the resulting multi-agent decision making architecture. Section III examines the convergence property. Section IV examines the challenges for the proposed method to be implemented in a meshed network. Section V presents case studies, one on distributed AC OPF solving and the other on implementation in a dynamic simulation platform. Section VI concludes the paper.

II. A Novel Multi-Agent Decision Making Architecture

A. Optimization Problem Formulation

Let us first look at a two-area system shown in Fig. 1. The original economic dispatch problem Prob_1 is as follows.

$$\begin{aligned}
\text{Prob}_1 \quad & \min_{P_1,P_2,P_{12}} && f_1(P_1)+f_2(P_2) && \text{(1a)}\\
& \text{subject to} && \lambda_1: P_1-P_{12}=D_1 && \text{(1b)}\\
& && \lambda_2: P_2+P_{12}=D_2 && \text{(1c)}\\
& && -d\le P_{12}\le d && \text{(1d)}
\end{aligned}$$

where P_i is the real power generated from Area i, D_i is the load value (real power consumption), $f_i(P_i)$ is the cost function of a generator, and P_{12} is the line flow from Area 1 to Area 2, and d is assumed to be the line limit. λ_1 and λ_2 are the Lagrangian multipliers corresponding to the two equality constraints. Their definitions are the same as the locational marginal price (LMP) at Bus 1 and 2. Here we assume that $\underline{P}_1 \le \pm d + D_1 <= \bar{P}_1$, and $\underline{P}_2 \le \pm d + D_2 <= \bar{P}_2$, *i.e.*, the sums of line limits and loads are within the area capacity limits, or the generators will not hit limits before the line hits limit.

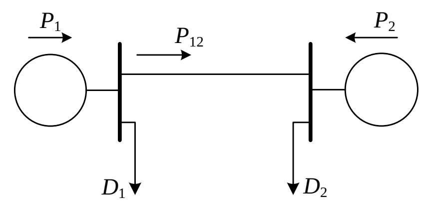

Fig. 1. A two-area system.

The dual problem Prob_2 after relaxing the two equality constraints is as follows.

$$\begin{aligned}
\text{Prob}_2 \quad \max_{\lambda_1,\lambda_2} \quad \min_{P_1,P_2,P_{12}} \quad & f_1(P_1)+\lambda_1(D_1-P_1+P_{12})\\
& + f_2(P_2)+\lambda_2(D_2-P_2-P_{12})\\
\text{subject to} \quad & -d\le P_{12}\le d
\end{aligned}\tag{2}$$

If we treat the line flow P_{12} separately from the generator power, then we have Prob_3.

$$\begin{aligned}
\text{Prob}_3 \quad \min_{P_{12}} \quad \max_{\lambda_1} \quad & \left(\min_{P_1}\left(f_1(P_1)-\lambda_1P_1\right)+\lambda_1\left(D_1+P_{12}\right)\right)\\
+\max_{\lambda_2} \quad & \left(\min_{P_2}\left(f_2(P_2)-\lambda_2P_2\right)+\lambda_2(D_2-P_{12})\right)\\
\text{subject to} \quad & -d\le P_{12}\le d
\end{aligned}\tag{3}$$

The above formulation Prob_3 is similar to the dual problem of the following optimization problem Prob_4.

$$\begin{aligned}
\text{Prob}_4 \quad \max_{\lambda_1,\lambda_2} \quad & q_1(\lambda_1)+q_2(\lambda_2)\\
\text{subject to} \quad & \lambda_1=\lambda_2
\end{aligned}\tag{4}$$

where $q_1(\lambda_1)=\min_{P_1}\left(f_1(P_1)-\lambda_1P_1\right)+\lambda_1D_1$ and $q_2(\lambda_2)=\min_{P_2}\left(f_2(P_2)-\lambda_2P_2\right)+\lambda_2D_2$.
The dual problem of Prob_4 is notated as Prob_5 and is described in the following.

$$\text{Prob}_5 \quad \min_{\pi} \quad \max_{\lambda_1,\lambda_2} \quad \left(q_1(\lambda_1)+q_2(\lambda_2)+\pi(\lambda_1-\lambda_2)\right)$$

Comparing Prob_3 and Prob_5, it is shown that the Lagrangian multiplier π associated with $\lambda_1=\lambda_2$ is indeed the tie-line flow P_{12}, if the line flow limit is not hit. Prob_3 is equivalent to the following optimization problem.

$$\begin{aligned}
\min_{P_{12}} \quad & \left(\begin{array}{l}\min f_1(P_1)\\ \text{s.t. } \ P_1-P_{12}=D_1\end{array}\right)+\left(\begin{array}{l}\min f_2(P_2)\\ \text{s.t. } \ P_2+P_{12}=D_2\end{array}\right)\\
\text{subject to} \quad & -d\le P_{12}\le d
\end{aligned}\tag{5}$$

Remarks:
This problem has a strong economic meaning and application value. We can decompose a system that is connected by a tie-line by assuming a tie-line power flow. Each area will consider the tie-line flow injection or exporting as a negative (or positive) load. Each area carries out optimization and finds the LMP for the interfacing bus. At next iteration, the tie-line flow is updated based on the price difference of the sending end and receiving end buses. If the price at the sending end is higher than that at the receiving end, then the tie-line flow should be reduced. Otherwise, the tie-line flow should be increased.

B. Example

For the two-area system in Fig. 1, assume that each area has two generators. Gen 1 and Gen 2 are in Area 1 while Gen 3 and Gen 4 are in Area 2. The cost functions of the four generators are $7P_{g1}$, $8P_{g2}$, $9P_{g3}$, and $10P_{g4}$. The power of each generator should be in the range of [0, 10] pu. The loads in two areas are 1 pu and 10 pu respectively. Line flow limit is ignored. For this example, both dual decomposition and dual's dual method are applied. For dual decomposition, each area solves a minimization problem for a given λ^k.

$$\min_{P_{g1},P_{g2}} \quad 7P_{g1} + 8P_{g2} + \lambda^k(D_1 - P_{g1} - P_{g2}) \\ s.t. 0 \le P_{g1} \le 10, 0 \le P_{g2} \le 10 \tag{6}$$

$$\min_{P_{g3},P_{g4}} \quad 9P_{g3} + 10P_{g4} + \lambda^k(D_2 - P_{g3} - P_{g4}) \\ s.t. 0 \le P_{g3} \le 10, 0 \le P_{g4} \le 10 \tag{7}$$

The dual variable updates based on the power imbalance:

$$\lambda^{k+1} = \lambda^k + \alpha_k\left(\sum D_i - \sum P_{gi}\right) \tag{8}$$

Fig. 2 gives the iterative results of dual decomposition method. Fig. 3 gives the iterative results of the dual's dual method. Note that the dual decomposition method fails to converge.

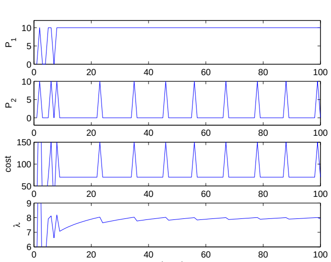

Fig. 2. Dual decomposition method.

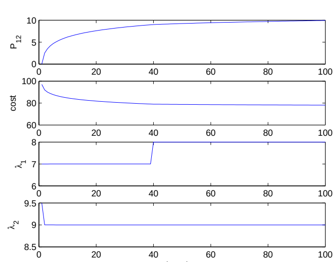

Fig. 3. Dual's dual method.

The advantage of dual's dual method compared to the dual method is listed as follows. For Prob$_1$, dual iteration method applies to the cases where the cost functions is strictly convex [20]. When a generator's cost function is linear, dual method has difficulty to converge and suffers switching oscillations. In order to improve convergence, augmented Lagrangian methods such as method of multipliers and ADMM were proposed [20]. On the other hand, in the dual's dual method, the tie-line flow is updated based on the prices computed from each area. Even with linear cost functions or constant prices at all areas, tie-line flow will be able to reach its limit if the two prices are different or reach convergence if the two prices are the same.

As an extension, the application can be extended to consider AC OPF constraints inside each area. The only assumption that is different from a complete AC OPF is that the tie-line is assumed to be lossless. Therefore, using the proposed problem formulation, an approximate solution for AC OPF can be found.

C. Information Exchange Architectures

The dual's dual problem can be solved in iterative and decomposed ways. In this paper, the classic subgradient updating procedure [19], [26] is adopted for its simplicity.

For the subgradient-based architecture, the tie-line flow is assumed and then updated based on its subgradient. In this case, we find a subgradient of the line flow (P_{12}) is ($\lambda_1 - \lambda_2$) from Prob$_3$ (3). Since the primal problem is a minimization problem, therefore, in the updating procedure, for a positive gradient, the line flow should be reduced. The updating procedure is presented as follows.

$$P_{12}^{k+1} = P_{12}^k - \alpha^k(\lambda_1^k - \lambda_2^k) \tag{9}$$

where α^k is a positive step size. For a given P_{12}^k, the LMPs (λ_1^k and λ_2^k) can be found by solving individual optimization problem for each area. Eq. (5) also complies with economy consideration. If Area 1 has a higher price, then Area 1's export should be reduced. If Area 2 has a higher price, then Area 1's export should be increased.

In terms of information exchange, at every decision making step, Area 1 will broadcast P_{12}^k. Area 2 gets the information and finds its price λ_2^k. Area 2 then sends this information to Area 1. And Area 1 updates the tie-line power flow for the next step. This algorithm can be implemented in any radial network. For example, for a system in Fig. 4 where a utility is connected to multiple communities, the information exchange and updating strategy is demonstrated in Fig. 5.

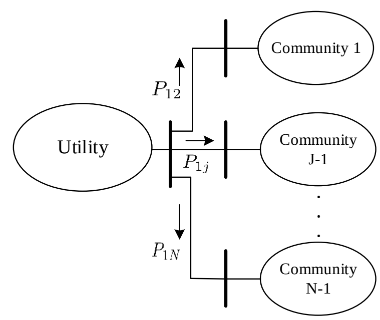

Fig. 4. A radial system.

First, all tie-line flows from the utility to communities are assumed. Then AC OPF is carried out for each entity (the utility or a community). The LMP at the boundary buses are collected by the utility to update the tie-line flows. This procedure will continue until the tie-line flows hit limit or converge to optimal values.

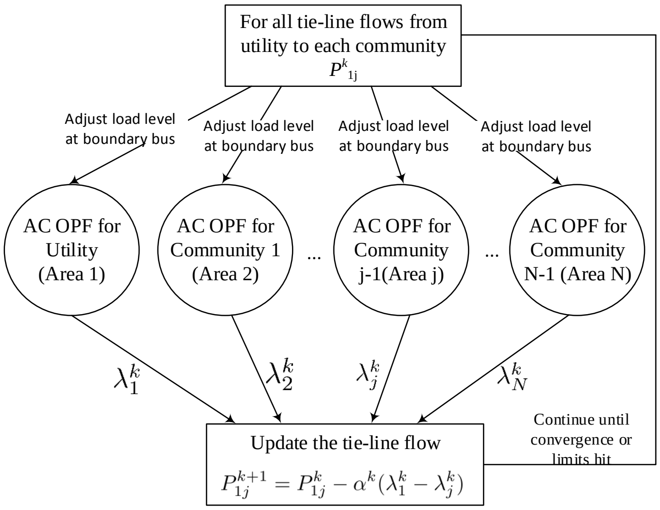

Fig. 5. Utility and community information exchange strategy: Tie-line flow updating based dual's dual method.

As a comparison, the information exchange strategy of Lagrangian dual variable updating based dual method [12] is presented in Fig. 6. Although in both strategies, existing AC OPF solving packages are used, price updating method requires additional manipulation than tie-line flow updating method. Note that for the utility and each community, the connected entities have to be treated as generators with linear cost functions (or constant prices).

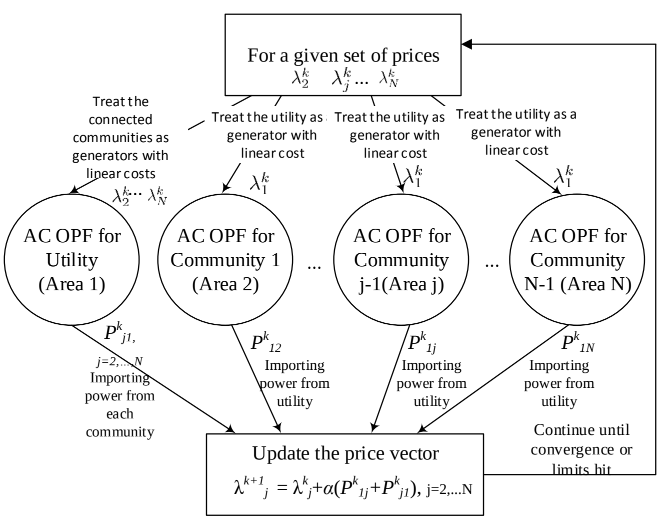

Fig. 6. Utility and community information exchange strategy: Price updating based dual's method.

III. Convergence Analysis

The two-area system is used to illustrate convergence criterion of subgradient update. Assume that the step size α is a constant value.

A. Subgradient Method

For the subgradient (SG) method, the tie-line flow command from Bus 1 to Bus 2 is named as π. Hereafter, we use π instead of P_{12}. This usage is to differentiate the tie-line flow command determined during the iteration process from the physical tie-line flow. At steady-state, these two (command and real value) are the same should the tie-line is lossless. However, considering power system internal dynamics related delays, during the dynamic period, these two are different. Based on π, the LMPs for the two buses can be found. The generator cost functions are assumed to be quadratic, that is:

$$f_1(P_1) = a_1 P_1^2 + b_1 P_1 + c_1, \tag{10}$$
$$f_2(P_2) = a_2 P_2^2 + b_2 P_2 + c_2. \tag{11}$$

When the capacity limits of generators and tie-lines are not hit, the prices should be the same as the marginal prices of the generators. Based on the price difference, the tie-line flow command is been updated. The computation and updating at k-th step can be represented by (12). Here the assumption that $\underline{P}_1 \le \pm d + D_1 <= \bar{P}_1$, and $\underline{P}_2 \le \pm d + D_2 <= \bar{P}_2$ hold, *i.e.*, the generators will not hit the limits before the line hits limit. Therefore, the prices will not hit limit during the updating procedure if the line limit is considered.

$$\begin{aligned}
\lambda_1^k &= \lambda_{10} + 2a_1\pi^k \\
\lambda_2^k &= \lambda_{20} - 2a_2\pi^k \\
\pi^{k+1} &= \begin{cases} d, & \text{if } \pi^k - \alpha(\lambda_1^k - \lambda_2^k) >= d \\ -d, & \text{if } \pi^k - \alpha(\lambda_1^k - \lambda_2^k) \le -d \\ \pi^k - \alpha(\lambda_1^k - \lambda_2^k), & \text{otherwise} \end{cases}
\end{aligned} \tag{12}$$

where $\alpha > 0$ and

$$\lambda_{10} = 2a_1 D_1 + b_1; \quad \lambda_{20} = 2a_2 D_2 + b_2 \tag{13}$$

The price updating scheme shows that once the maximum limits $\bar{\lambda}_i$ are hit, the price should stay at its maximum due to the generator capacity limits. Also, if Area 1 is hitting its maximum limit, or Area 2 is hitting its minimum capacity limits, the tie-line flow from Area 1 to 2 should be kept at its maximum d. On the other hand, if Area 2 is hitting its maximum capacity limit or Area 1 is hitting its minimum capacity limits, the tie-line flow from Area 1 to 2 should be kept at its minimum $-d$.

Once the tie-line flow hits a limit, the iteration procedure will make decision if it should end or continue. For example, if the upper limit of the tie-line from Area 1 to Area 2 is hit, however, $\lambda_1 > \lambda_2$, the iteration should continue and the tie-line flow will be reduced. Otherwise, stop. This implies that when the tie-line flow is hitting the upper limit, LMP at Area 2 should be greater than LMP at Area 1. The starting point of the tie-line flow, of course, should be within the feasible region.

if $\pi^k == d$, $sign(\lambda_1^k - \lambda_2^k) < 0$, stop.
if $\pi^k == -d$, $sign(\lambda_1^k - \lambda_2^k) > 0$, stop.
otherwise, continue.

Except the limit hitting scenarios, the iteration of the tie-line flow command can be found:

$$\begin{aligned}
\pi^{k+1} &= \pi^k - \alpha\left[\lambda_{10} - \lambda_{20} + 2(a_1 + a_2)\pi^k\right] \\
&= (1 - 2(a_1 + a_2)\alpha)\pi^k - \alpha(\lambda_{10} - \lambda_{20}).
\end{aligned} \tag{14}$$

B. Convergence property analysis

Convergence property of the above equation (14) can be both addressed by the convergence condition of an iterative procedure or discrete dynamic system stability criterion. In numerical analysis, to guarantee that the error to the true value is decreasing, the condition for an iteration procedure expressed by $x = \phi(x)$ is that the derivative of $\phi(x)$ at the true value should be less than 1. In linear discrete control systems, the eigenvalues of the system matrix should be within the unit circle. Both indicate that for the above mentioned SG-based iteration, for π to converge to an optimal solution, the following condition should be met:

$$|1 - 2(a_1 + a_2)\alpha| < 1 \tag{15}$$

Therefore, a small α ($\alpha < \frac{1}{a_1+a_2}$) means a converging procedure. When $\alpha = \frac{1}{2(a_1+a_2)}$,

$$\begin{aligned}\pi^{k+1} &= -\alpha(\lambda_{10} - \lambda_{20}) = -\frac{\lambda_{10} - \lambda_{20}}{2(a_1 + a_2)} \\ &= \frac{-2a_1 D_1 - b_1 + 2a_2 D_2 + b_2}{2(a_1 + a_2)}.\end{aligned} \tag{16}$$

The above equation shows that if α is chosen to be $\frac{1}{2(a_1+a_2)}$, the convergence will be realized in one step. If the generator cost functions are the same ($a_1 = a_2$, $b_1 = b_2$), then each generator will generate the same amount of power $\frac{D_1+D_2}{2}$ and the tie-line flow will be $\frac{D_2-D_1}{2}$. Based on the above equation, the same answer can be found.

Fig. 7 illustrates the subgradient method with the extreme case of $\alpha = \frac{1}{a_1+a_2}$.

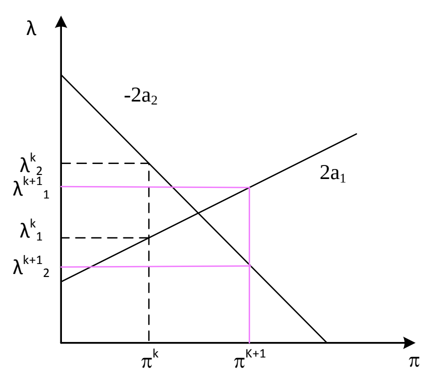

Fig. 7. Subgradient method. An extreme case when $\alpha = \frac{1}{a_1+a_2}$ and $\lambda_2^k - \lambda_1^k = \lambda_1^{k+1} - \lambda_2^{k+1}$. This will cause oscillations.

Remarks: For a nonconstrained convex optimization problem to minimize $f(x)$, where $f : \mathbb{R}^n \to \mathbb{R}$, the subgradient method update is given by

$$x^{(k+1)} = x^{(k)} - \alpha_k g^{(k)},$$

where $\alpha_k > 0$ and $g^{(k)}$ is any subgradient of f at $x^{(k)}$. The subgradient algorithm is guaranteed to converge within some range of the optimal value:

$$\lim_{k\to\infty} f_{\text{best}}^{(k)} - f^* < \epsilon,$$

where f^* is the optimal value of the problem, *i.e.*, $f^* = \inf_x f(x)$. ϵ is a function of the step size α_k. When α_k is a constant step size, the value $f^{(k)}$ converges to a ball with a radius of ϵ [19].

For diminishing step size (i.e., $\alpha_k \to 0$, where $k \to \infty$), the algorithm is guaranteed to converge to the optimal value since ϵ goes to zero when α_k reduces to zero, i.e., $\lim_{k\to\infty} f^{(k)} = f^*$.

The above convergence property is related to the value of the optimization problem or $f(x)$. In this paper, convergence discussion is related to π, which is the decision variable (we can view π as x in the general problem). For a diminishing step size ($\alpha_k \to 0$ when $k \to \infty$), we can be sure that π^k will converge to a point based on the following updating rule

$$\pi^{(k+1)} = \pi^{(k)} - \alpha_k g^{(k)},$$

If the step size α_k is a constant value, convergence of π to an optimal point requires that the subgradient g becomes zero. There is no guarantee for f^k to reach the optimal value when $k \to \infty$ for a constant step size. Therefore, convergence of π^k to π^* is not guaranteed for a general problem.

In this paper, the convergence analysis of π^k is conducted for this specific problem with quadratic cost functions. We found that π^k converges to π^* even with a constant step size. Limits of the step size are identified to have a convergence property. This analysis is one of the contributions of our paper.

IV. Meshed Network Application

The proposed method can be applied to a radial network or a meshed network that can be separated into multiple areas radially connected. It however has difficulty to be implemented into scenarios where areas are connected as a meshed network. This is due to the feature of the proposed method that tie-line flows are given and updated without considering network physics. In this section, we show a meshed network example and indicate that extra care needs to be taken.

Consider a three-area system connected through a network. Each area has a generator and a load. The DC OPF problem is as follows.

$$\min \quad C_1(P_{g1}) + C_2(P_{g2}) + C_3(P_{g3}) \tag{17a}$$

$$\text{s.t.} \quad \lambda_1 : P_{g1} = D_1 + P_{12} + P_{13} \tag{17b}$$

$$\lambda_2 : P_{g2} = D_2 - P_{12} + P_{23} \tag{17c}$$

$$\lambda_3 : P_{g3} = D_3 - P_{13} - P_{23} \tag{17d}$$

$$\underline{d} \le \begin{bmatrix} P_{12} \\ P_{13} \\ P_{23} \end{bmatrix} = C \cdot \begin{bmatrix} P_{g1} - D_1 \\ P_{g2} - D_2 \end{bmatrix} \le \bar{d} \tag{17e}$$

where $\underline{d}$ and $\bar{d}$ are the line lower limit and upper limit vectors, C is the power transfer shifting factor matrix. The element of C matrix (i-th row, j-th column) defines the influence of a power transfer (from Bus j to the reference bus) on the i-th line.

For the test system shown in Fig. 8, the C matrix is as follows when Bus 3 is treated as a reference bus.

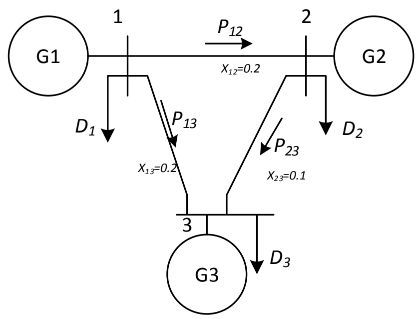

Fig. 8. A meshed network.

$$C = \begin{bmatrix} 0.4 & -0.2 \\ 0.6 & 0.2 \\ 0.4 & 0.8 \end{bmatrix} \tag{18}$$

Introduce the dual's dual formulation, we have:

$$\begin{aligned} \min_{P_{12},P_{13},P_{23}} \max_{\lambda_i} \quad \min_{P_{gi}} \; & C_1(P_{g1}) + C_2(P_{g2}) + C_3(P_{g3}) \\ & + \lambda_1(D_1 + P_{12} + P_{13} - P_{g1}) \\ & + \lambda_2(D_2 - P_{12} + P_{23} - P_{g2}) \\ & + \lambda_3(D_3 - P_{13} - P_{23} - P_{g3}) \end{aligned} \tag{19}$$

The above dual's dual optimization problem can be solved iteratively by applying the subgradient update method. Note the gradient of P_{12} can be found as $(\lambda_1 - \lambda_2)$. P_{12} is a primal variable. The iteration procedure is given as follows.

$$\begin{aligned} P_{12}^{k+1} &= P_{12}^k - \alpha_1^k(\lambda_1^k - \lambda_2^k) \\ P_{13}^{k+1} &= P_{13}^k - \alpha_2^k(\lambda_1^k - \lambda_3^k) \\ P_{23}^{k+1} &= P_{23}^k - \alpha_3^k(\lambda_2^k - \lambda_3^k) \end{aligned} \tag{20}$$

The dual's dual problem formulation has not taken into the network characteristic into consideration. The tie-line power flow vector has to be feasible for the network. For example, the following tie-line power flow vector $[250, 0, 0]^T$ MW is not feasible. Given this tie-line power flow vector, we can find the net power injection at each bus. The tie-line power flow vector is dependent on the net power injection (17e). The example is shown below.

$$\begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & -1 & -1 \end{bmatrix}}_{A} \begin{bmatrix} P_{12} \\ P_{13} \\ P_{23} \end{bmatrix} = \begin{bmatrix} 250 \\ 0 \\ -250 \end{bmatrix} \text{MW} \tag{21}$$

where P_i is the i-th bus net power injection.

In the above equation, the net power from Bus 3 is included. This power can be excluded. Therefore, we have:

$$\begin{bmatrix} P_1 \\ P_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}}_{A_1} \begin{bmatrix} P_{12} \\ P_{13} \\ P_{23} \end{bmatrix}. \tag{22}$$

Given the net power injection at the each bus, we can find the tie-line power flow applying (17e).

$$\begin{bmatrix} P_{12} \\ P_{13} \\ P_{23} \end{bmatrix} = C \begin{bmatrix} P_1 \\ P_2 \end{bmatrix} = C \cdot A_1 \begin{bmatrix} P_{12} \\ P_{13} \\ P_{23} \end{bmatrix} = \begin{bmatrix} 100 \\ 150 \\ 100 \end{bmatrix} \text{MW} \tag{23}$$

Therefore, in order for the tie-line power flow vector to be feasible, certain restriction has to be applied. Hereafter, we notate the tie-line power flow vector as π.

$$\pi = C \cdot A_1 \cdot \pi \tag{24}$$

Therefore, the tie-line power flow vector has to meet the above requirement to be feasible. For this particular example, we find the matrix $C \cdot A_1$ as

$$C \cdot A_1 = \begin{bmatrix} 0.4 & -0.2 \\ 0.6 & 0.2 \\ 0.4 & 0.8 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0.6 & 0.4 & -0.2 \\ 0.4 & 0.6 & 0.2 \\ -0.4 & 0.4 & 0.8 \end{bmatrix}.$$

The above matrix has a rank of 2 and three eigenvalues as $1, 1, 0$. Re-examining (24), we find that in order to be feasible towards the network characteristic, π should be an eigenvector of CA_1. This eigenvector is related to eigenvalue of 1. For this case, there are two sets of vectors that are related to eigenvalue 1. The two sets are:

$$\pi = k \begin{bmatrix} 0.7276 \\ 0.4851 \\ -0.4851 \end{bmatrix} \quad \text{or} \quad \pi = k \begin{bmatrix} 0.1173 \\ 0.5355 \\ 0.8364 \end{bmatrix} \tag{25}$$

Due to the feasibility requirement (25), the step size α_i in the update procedure (20) has to be selected with care. Multiplying $CA_1 - I$ at the left and right of (20), we have

$$(CA_1 - I) \begin{bmatrix} \alpha_1^k(\lambda_1^k - \lambda_2^k) \\ \alpha_2^k(\lambda_1^k - \lambda_3^k) \\ \alpha_3^k(\lambda_2^k - \lambda_3^k) \end{bmatrix} = 0 \tag{26}$$

Therefore, for meshed network topology, additional requirements are posed for step size. Extra care is to be taken.

a) Alternative method: Due to the difficulty of the updating method based on tie-line flow. An alternative method is proposed.Consider the DC OPF formulation as follows.

$$\begin{aligned} \min_{P_{gi}} \quad & C_1(P_{g1}) + C_2(P_{g2}) + C_3(P_{g3}) \\ \text{subject to} \quad & \underbrace{\begin{bmatrix} P_{g1} \\ P_{g2} \\ P_{g3} \end{bmatrix}}_{P_g} - \underbrace{\begin{bmatrix} D_1 \\ D_2 \\ D_3 \end{bmatrix}}_{D} = - \underbrace{\begin{bmatrix} -10 & 5 & 5 \\ 5 & -15 & 10 \\ 5 & 10 & -15 \end{bmatrix}}_{B} \underbrace{\begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix}}_{\theta} \end{aligned} \tag{27}$$

$$\underline{d} \leq K\theta \leq \overline{d} \tag{28}$$

where K is a coefficient matrix and $K\theta$ is the tie-line power flow vector. The dual problem is expressed as follows.

$$\max_{\lambda} \quad \min_{P_{gi}} \quad \sum_i C_i(P_{gi}) + \lambda^T(P_g - D + B\theta)$$

$$\underline{d} \leq K\theta \leq \overline{d} \tag{29}$$

where λ is the vector of dual variables associated with $P_g - D = B\theta$. If θ can be treated in an additional layer similar as the tie-line flow, the above problem becomes:

$$\min_{\theta} \max_{\lambda} \min_{P_{gi}} \quad \sum_i C_i(P_{gi}) + \lambda^T(-P_g + D - B\theta)$$
$$\underline{d} \le K\theta \le \overline{d} \tag{30}$$

The gradient for the phase angle vector θ is $-B^T\lambda$. Since B is a symmetric matrix, the gradient is $-B\lambda$. The θ based iterative procedure can be summarized as:
Step 1: Given a vector of θ^k at k-step, find out the net injection at each bus P^k_{ni}, where $P_n = -B\theta^k$.
Step 2: For each area, carry out OPF solving by treating the net injection to the grid P^k_{ni} as a load. For each area, the following optimization problem is to be solved.

$$\max_{\lambda_i} \min_{P_{gi}} \quad C_i(P_{gi}) + \lambda_i(-P_{gi} + D_i + P^k_{ni}) \tag{31}$$

The price λ^k_i at every bus will be found.
Step 3: Update: $\theta^{k+1} = \theta^k - \alpha B\lambda^k$, where α is the step size.
We also conducted a numerical test for a three-bus system and the results over iterations are shown in Fig. 9. The case study results show that the three prices converge when the line limits are not considered. For a step size $\alpha = 0.0001$, the algorithm converges in less than 200 iterations.

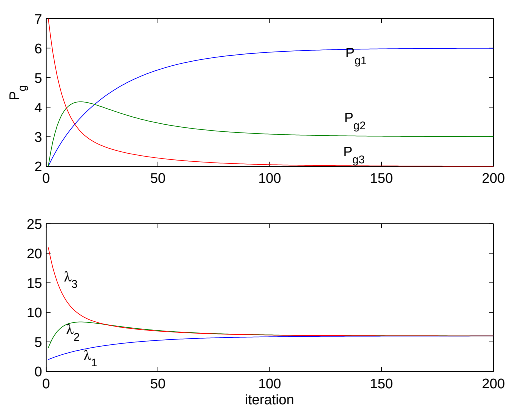

Fig. 9. Generator power and prices at each bus solved by dual's dual method.

A. Comparison with ADMM

The above DCOPF problem for the meshed network is also solved by ADMM. The type of consensus ADMM with a global variable z is implemented. For each area, $\theta^i = z^k$ is a constraint. The updating procedure for z, P_{gi} and λ_i are given as follows. Note that λ_i are the dual variable vector related to $\theta^i = z^k$ constraint. It is not the same as the LMP price vector.

$$P^k_{gi}, \theta^{i,k+1} = \text{argmin}\{C_i(P_{gi}) + (\lambda^k_i)^T(\theta^i - z^k) + \frac{\rho}{2}\|\theta^i - z^k\|^2$$
$$\text{s.t. (17b) or (17c) or (17d)}\}$$
$$z^{k+1} = \frac{1}{3}\sum P^{k+1}_{gi}$$
$$\lambda^{k+1}_i = \lambda^k_i + \rho(\theta^{i,k+1} - z^{k+1})$$

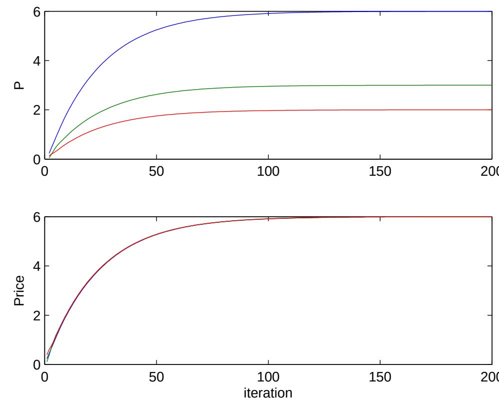

Fig. 10. Generator power and prices at each bus solved by ADMM method.

Fig. 10 gives the iterative results for ADMM method. Note that the dual's dual method and ADMM method can achieve comparable convergence for this DCOPF problem. To adopt ADMM, each area's objective function has to be modified.

V. More Case Studies

In this section, two case studies are conducted. The first case study is to find the approximate AC OPF solution of an IEEE 14-bus system using the proposed multi-agent solving method. The second case study is to show how to implement iterative decision making procedure in a more realistic platform. The procedure is implemented in a dynamic simulation platform in Power System Toolbox (PST) [27] for a two-area four-machine system.

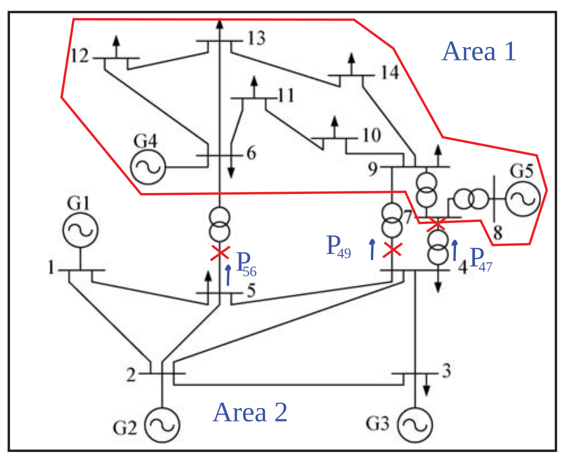

Fig. 11. IEEE 14-bus system.

A. Multi-Agent AC OPF

The system in Fig. 11 is separated into two areas if the tie-lines Line56, Line47 and Line49 are open. The original system has Bus 1 designated as the slack bus. For the two islanded systems, Bus 1 and Bus 6 are designated as the slack buses for each area respectively. Since these three branches are for transformers modeled as reactance only, the line flows at the sending end and the receiving end are exactly the same. The reactive power flows are approximated to be zero. This approximation will result in discrepancy between the solution of the proposed method and the solution from AC OPF.

Area 1 consists of two generators at bus 6 and bus 8 and loads on the following buses: 11, 12, 13, 14, 10, 6, 7, and 9. In addition, the existing loads at buses 6,7,9 will be modified to include the line flow injections from P_{56}, P_{47}, and P_{49} respectively. The reactive power injections due to the three tie-lines are assumed to be zero. Similarly, for Area 2, the tie-line flows are also reflected in load modification. AC OPF can then be carried out for Area 1 and Area 2. MATPOWER [28] is used for this research to carry out AC OPF calibration at each step. The AC OPF formulation can be found in MATPOWER manual [25], with generator power output, voltage magnitudes and phase angles included in decision variables, generator capacity, line flow limits, and voltage magnitude limits considered. The LMPs at the boundary buses (4, 5, 6, 7, 9) are exchanged between the two areas to update the line flows. With updated line flows, the two areas again carry out AC OPF until the line flows converge or limits are hit.

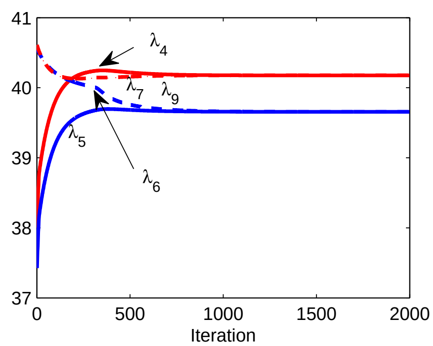

Fig. 12. Boundary bus prices. Step size: 0.1.

Fig. 12 presents the boundary bus prices. The step size is chosen as 0.1 for iteration. It can be observed that the prices at Bus 5 and Bus 6 will eventually be the same while the prices at Bus 4, Bus 7 and Bus 9 are also the same.

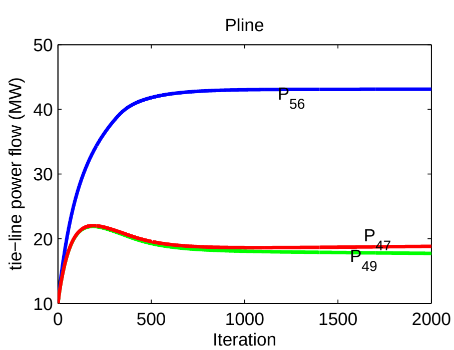

Fig. 13. Tie-line power flow. Step size: 0.1.

Fig. 13 presents the tie-line power flows. The flows are observed to achieve contant values after initial adjustment.

The solutions using the proposed method are compared with conventional AC OPF solutions from MATPOWER. Table I gives the comparison on generator power dispatch and tie-line flows. It is observed that the real-power dispatch levels for the five generators are close with less than 1.5% error. The individual tie-line real-power flows have 26% discrepancy due to the omission of reactive power injection in the proposed method. However, the total tie-line power flow has only 0.12% error.

TABLE I
COMPARISON OF POWER DISPATCH AND POWER FLOW

	Centralized AC OPF		Proposed Method		error (%)
	P (MW)	Q (Mvar)	P (MW)	Q (MVar)	in P
G1	194.3302	0.0008	194.3333	0.0049	0.0016
G2	36.7192	23.6850	36.7162	12.6079	0.0082
G3	28.7426	24.1269	28.5841	22.6538	0.5514
G4	0.0003	11.5455	0.0004	20.3407	–
G5	8.4949	8.2730	8.5857	4.8770	1.0689
Line56	42.0553	15.1379	43.1045	0	2.4948
Line47	22.8471	-3.9936	17.7270	0	22.4103
Line49	14.8406	1.1668	18.8173	0	26.7961
Total	79.743	12.3111	79.6488	0	0.1181

Further, the LMPs of each bus are listed in Table II. The greatest error is for Bus 6 at 0.19%. Therefore, overall, the proposed method can provide a very close approximation of the optimal power flow solution.

TABLE II
COMPARISON OF LMP ($/MWH).

	Centralized OPF	Proposed method	error (%)
Bus 1	36.7238	36.7240	0.0005
Bus 2	38.3596	38.3581	0.0039
Bus 3	40.5749	40.5717	0.0079
Bus 4	40.1902	40.1754	0.0368
Bus 5	39.6608	39.6561	0.0119
Bus 6	39.7337	39.6561	0.1953
Bus 7	40.1715	40.1727	0.0030
Bus 8	40.1699	40.1717	0.0045
Bus 9	40.1662	40.1780	0.0294
Bus 10	40.3178	40.3156	0.0055
Bus 11	40.1554	40.1191	0.0904
Bus 12	40.3791	40.3028	0.1890
Bus 13	40.5755	40.5111	0.1587
Bus 14	41.1975	41.1713	0.0636

In the next study, the transformer in Line56 is assumed to have limited capacity of 40 MW. The iterative solutions of the proposed method are presented in Fig. 14 and 15. It is observed that when the line limit is hit, the LMPs at Bus 5 and Bus 6 are no longer the same. The proposed method can take care of tie-line limits.

B. Dynamic simulation platform implementation and frequency response demonstration

The discrete decision making architectures will be implemented into a dynamic simulation platform to examine their impact on system frequency response. Power System Toolbox [27] is selected as the dynamic simulation platform. The classic two-area four-machine power system [29] is modified slightly to have shortened tie-lines and well-damped electromechanical dynamics. Generators are modeled as classical

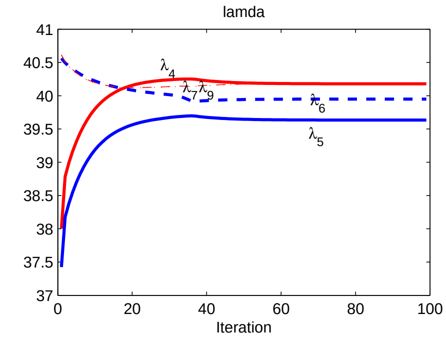

Fig. 14. Tie-line power flow. Note Line56 hits its limit at 40 MW. Iteration step size: 1.

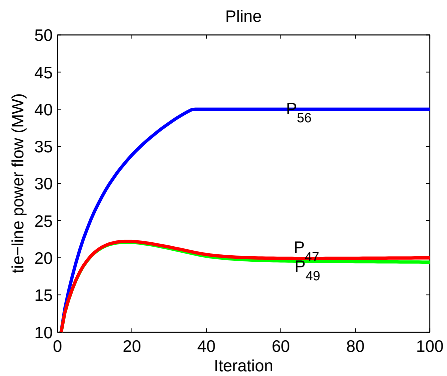

Fig. 15. Tie-line power flow. LMPs for Bus 5 and Bus 6 are different due to the binding constraint of Line56. Iteration step size: 1.

generators with turbine-governor blocks. Primary frequency droops with the regulation constant at 4% are all included.

The power system and its privacy-preserving decision making architecture are shown in Fig. 16. The discrete decision making will take place every 5 seconds. The power commands from Agent 1 and Agent 2 will be sent to change the turbine-governors' power reference inputs. Among the two agents, the information exchanged includes the tie-line power flow command and the price signal. Area 1 consists of Gen 1 and Gen 2 and Load 1. Area 2 consists of Gen 3, Gen 4 and Load 2. The two areas are connected through tie-lines. Initially, the four generators are dispatched at 6.8776 pu, 7.00 pu, 7.16 pu and 7.00 pu. Assume that in Area 1 the two generators

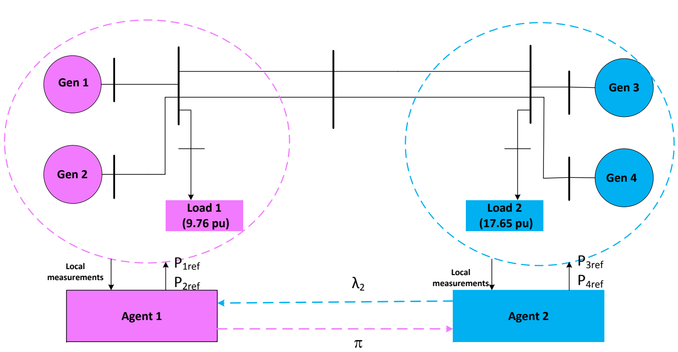

Fig. 16. The two-area system: physical topology and the information exchange architecture.

are having the same quadratic cost functions: $1.5P_1^2$, $1.5P_2^2$ and in Area 2 the two generators are also having the same quadratic cost functions P_3^2 and P_4^2. The total load is 27.41 pu. If the tie-line transfer limit is very high, then the LMPs at two areas are the same when tie-line power loss is ignored. In addition, if the generators' limits are ignored, then LMP equals marginal cost of each generator.

Initially the four generators' dispatch levels are similar. After the decision making procedures, Area 2's generators will have higher dispatch levels as Gen 3 and Gen 4 are much cheaper than Gen 1 and Gen 2.

The economic dispatch problem is expressed in (32).

$$\begin{aligned} \min \quad & 1.5P_{g1}^2 + 1.5P_{g2}^2 + P_{g3}^2 + P_{g4}^2 \\ \text{s.t.} \quad & \sum_i P_{gi} = 27.41 \end{aligned} \tag{32}$$

where P_{gi} is the power dispatch level of ith generator.

The economic dispatch pattern for the four generators from the optimization problem is: 548.2 MW, 548.2 MW, 822.3 MW, and 822.3 MW. The tie-line flow should be 102 MW or 1.02 pu.

The proposed decision architecture will be implemented. At every 5 seconds, Area 1 will send Area 2 the tie-line flow command and Area 2 sends Area 1 its LMP λ_2. The two areas also computes LMPs and dispatch patterns based on the tie-line flow command π. Area 1 updates the tie-line flow command π based on the two LMPs. At the next time interval, Area 1 sends out the updated tie-line flow command.

When implemented into dynamic simulation, the discrete change for every step should not be too large. This can be achieved by selecting a varying α in the π updating procedure. In the first few steps, α should be small. α can be increased when the price difference becomes smaller.

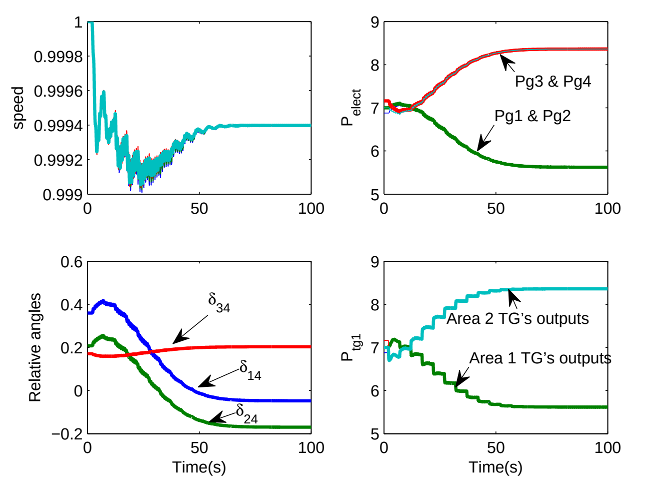

Fig. 17. System dynamic responses. Clockwise: a) Generators' speeds in pu; b) Generators' power based on the system power base (100 MW); c) Generators' turbine governor unit power based on the system power base (100 MW); d) Gen 1-3's angles relative to Gen 4 in radians.

The dynamic responses of the system are presented in Fig. 17. It can be seen that within 100 seconds, the system achieves a new steady-state. The power dispatch levels of the generators are all changed to new levels. The system has a frequency deviation of -0.0006 pu. This is due to the assumption of lossless line in developing the decision making strategy. The power dispatched by the generators only takes care of loads. The total generation is less than the total consumption including loads and tie-line power loss. Therefore, the steady-state frequency is below the nominal frequency.

Table III lists the generator dispatch levels at 100 second. These values are compared with the solutions from the ideal economic dispatch problem in (32).

TABLE III
GENERATOR DISPATCH LEVELS

	Dynamic Simulation	Economic Disptach
P_{g1}	5.62 pu	5.482 pu
P_{g2}	5.62 pu	5.482 pu
P_{g3}	8.36 pu	8.223 pu
P_{g4}	8.36 pu	8.223 pu

It is found that the generator dispatch levels are all higher than the optimal values from economic dispatch (0.135 pu). This discrepancy is due to the droop control. For the four 900 MW generators, each is equipped with a droop frequency controller with the droop parameter R at 0.04 pu. Therefore, for a -0.0006 pu frequency variation, the turbine governors should provide additional power (ΔP_{gi}, where $\Delta P_{gi} = -\frac{1}{R}\Delta f$). In this case, $\Delta P_{gi} = \frac{1}{0.04} \times 0.0006$ pu based on the generators' power base (900 MW) and $\Delta P_{gi} = 0.135$ pu based on the system base (100 MW).

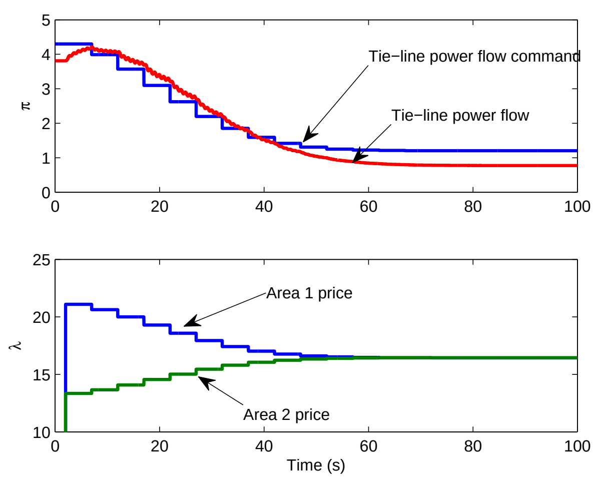

Fig. 18. The Lagrangian multipliers. a) $\pi = \pi - \alpha(\lambda_1 - \lambda_2)$ - the line flow command, where $\alpha = 0.2 * n$ and n the decision making step; b) λ - Area 1 and Area 2 prices computed based on given π.

The discrete decision variables are also recorded for every simulation step and presented in Fig. 18. It can be found that in ten steps, both π and LMPs converge to constant values. The tie-line flow command and the tie-line power flow from dynamic simulation are plotted against each other. The two are different due to power system dynamics and the loss in the network. Initially, all generators are dispatched at the same level, therefore, generators in Area 1 incur higher marginal cost. Area 1's LMP is much higher than Area 2's LMP. Therefore, based on the updating mechanism, the tie-line power flow command from Area 1 to Area 2 should decrease. In turn, Area 1's generators should dispatch less while Area 2's generators should dispatch more power. The dynamic simulation results reasonably reflect the system behavior during its decision making process.

VI. Conclusion

In this paper, a novel multi-agent decision making architecture is designed based on dual's dual problem formulation. Subgradient updating solving procedure renders an architecture incredibly easy to be implemented. This paper investigates the convergence property of the algorithm and applicability in meshed networks. Case studies demonstrates the ability of multi-agent AC OPF solving. This paper is the first kind to introduce dual's dual problem formulation in AC OPF solving. The method provides a close approximation to AC OPF solutions. Further, the discrete decision making architectures are implemented in a dynamic simulation platform to illustrate the architecture setup in a real-world scenario.

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