Equivalent Circuits for a Hybrid DC/AC System

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***Abstract*—When an inverter-based resource is subject to grid unbalance conditions, the current in the system has a negative 60-Hz component and a positive 180-Hz component along with the fundamental component. The negative-sequence circuit should have both the -60 Hz component and +180 Hz component presented and coupled. This paper explains the interaction between the negative-sequence and third harmonic component and provides a detailed derivation of the negative-sequence circuit. We also provide a generalized equivalent circuit for an inverter-based resource. For validation, we rely on electromagnetic transient (EMT) simulation, to design the experiment, produce time-domain data, and carry out fast Fourier transform (FFT) to perform harmonic analysis and compare those with the analysis results.**

***Index Terms*—Inverter-based resource, circuit analysis, negative sequence, third harmonics.**

I. Introduction

Power electronic converters have become a significant part of the electrical ecosystem. The analysis of the inverter-based resource (IBR) is of huge importance. Among the analysis, the protection study is also a vital part of the power system study. Whenever an IBR is subject to unbalance, the system experiences harmonics. Both $\pm$60-Hz and $+$180-Hz components appear due to phase domain frequency coupling [1]. Due to the IEEE P2800 requirements on negative-sequence current injection, such harmonic conditions will be seen more often. In this research, we aim for quantitative analysis of the inverter characteristics when subject to unbalance. This research can contribute to protection studies.

It has been a major challenge for power system engineers to perform a protection study for IBR-penetrated power grids, as identified by a recent PES task force report TR-81 [2]. So far, the grid-following IBRs are assumed as voltage controlled current sources [3] in fault analysis. Few research has considered the frequency coupling phenomena in circuit representation for fault analysis.

The motivation of this paper is to design accurate IBR representation for fault analysis. In this research, we present an elaborate derivation of a generalized equivalent circuit and analyze the effect of unbalance on the IBR. We also present a 10-Hz harmonic injection experiment and an unbalanced voltage dip case study.

As a first step of this overall research goal of accurate IBR circuit representation, we will derive the circuit representation of a grid-tied inverter, where the inverter is controlled by a constant modulation index. The testbed considered is shown in Fig. 1 and is built in PSCAD v4.6. Our derivation and analysis emphasize the frequency coupling nature of the inverter and are backed up by validation through EMT simulation. Section II shows the derivation of the circuit diagram through equation formations. The EMT simulation results and analysis results are is shown in Section III. Section IV concludes the paper.

II. Generalized Equivalent circuit of a Hybrid DC/AC System

This section focuses on the steady-state analysis of an inverter connecting the dc side to the ac side. The emphasis is on the dc-side and ac-side relationship.

The dc-side circuit is represented by an ideal constant voltage source behind an inductor of 1mH and a dc-link capacitor of 7500μF, as shown in Fig. 1. The measured dc voltage is the voltage across the capacitor, and the measured dc current is the current flowing into the inverter. The ac side has an RLC filter and an RL transmission line connected with an infinite bus.

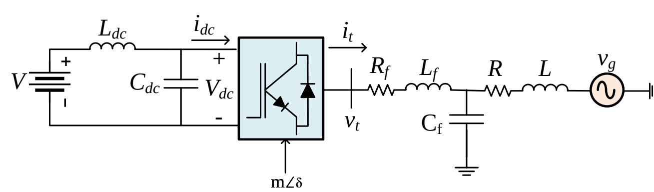


Fig. 1: Topology of an inverter connected to a grid. The inverter has an ideal dc source of 0.78 kV, and is connected to a 0.4799 kV L-L RMS grid voltage.

To convert every variable to per unit variable, we assume, $V_{\mathrm{dc}_{\mathrm{base}}} = 2V_{\mathrm{ac}_{\mathrm{base}}}$, where V_{ac} refers to the amplitude of the per-phase ac voltage. From the above assumption, we can form the dc side current base as $I_{\mathrm{dc}_{\mathrm{base}}} = \frac{3}{4}I_{\mathrm{ac}_{\mathrm{base}}}$.

A. AC side voltage-current relationship

In this section, we will express the relationship between the converter voltage and the grid voltage as shown in Fig. 1. The total ac side impedance can be written as,

$$R_{\mathrm{tot}} = R_{\mathrm{f}} + R, \;\; L_{\mathrm{tot}} = L_{\mathrm{f}} + L$$

By using Kirchhoff's voltage law, we can write the relation as,

$$\vec{v}_{\mathrm{t}} - \vec{v}_{\mathrm{g}} = (R_{\mathrm{tot}} + sL_{\mathrm{tot}})\vec{i}_{\mathrm{t}} \quad (1)$$

where, $\vec{v}_{\mathrm{t}} = \overline{V}_{\mathrm{t}}e^{j\omega_0 t}$, $\vec{v}_{\mathrm{g}} = \overline{V}_{\mathrm{g}}e^{j\omega_0 t}$, $\vec{i}_{\mathrm{t}} = \overline{I}_{\mathrm{t}}e^{j\omega_0 t}$.

Hence, the relationship of the phasors is as follows.

$$\overline{V}_{\mathrm{t}}(s) - \overline{V}_{\mathrm{g}}(s) = (R_{\mathrm{tot}} + (s + j\omega_0)L_{\mathrm{tot}})\,\overline{I}_{\mathrm{t}}(s) \tag{2}$$

Based on Eq. (2), the relationship between the negative-sequence voltage and current is as follows,

$$\overline{V}^*_{\mathrm{t}}(s) - \overline{V}^*_{\mathrm{g}}(s) = (R_{\mathrm{tot}} + (s - j\omega_0)L_{\mathrm{tot}})\,\overline{I}^*_{\mathrm{t}}(s) \tag{3}$$

B. DC side and DC/AC relationship

1) DC side voltage/current relationship: This section will form the relationship between dc voltage and current, dc and ac voltage, and dc and ac current. From Fig. 1, the dc side contains an ideal dc voltage source, a series inductor, and a dc-link capacitor. From these components, we can express the dc side relationship as,

$$V_{\mathrm{dc}}(s) = V_{\mathrm{th}}(s) - I_{\mathrm{dc}}(s)Z_{\mathrm{dc}}(s) \tag{4}$$

where $V_{\mathrm{th}}(s)$ is the Thévenin equivalent voltage viewed from the dc-bus to the dc side and Z_{dc} is the equivalent impedance of the series inductor and the dc-link capacitor.

$$Z_{\mathrm{dc}}(s) = L_{\mathrm{dc}}s || \frac{1}{C_{\mathrm{dc}}s}$$

s is the Laplacian variable and $s = j\omega$ is used to evaluate the impedance at different frequency.

2) DC/AC relationship: Based on the assumption above and the voltage relationship between the ac and dc side voltages, we can rewrite the expression as

$$\vec{v}_t = \overline{m}V_{\mathrm{dc}}e^{j\omega_0 t}, \;\; \overline{V}_{\mathrm{t}} = \overline{m}V_{\mathrm{dc}} \tag{5}$$

We can also express Eq. (5) as conjugate of V_{t} as shown below,

$$\vec{v}^*_t = \overline{m}^*V_{\mathrm{dc}}e^{-j\omega_0 t}, \;\; \overline{V}^*_{\mathrm{t}} = \overline{m}^*V_{\mathrm{dc}} \tag{6}$$

Now Eqs. (5) and (6) are the relationship between ac and dc side voltage. We can write the relationship as,

$$\vec{v}(s + j\omega_0) = \overline{V}_{\mathrm{t}}(s) = \overline{m}V_{\mathrm{dc}}(s) \tag{7}$$

$$\vec{v}^*(s - j\omega_0) = \overline{V}^*_{\mathrm{t}}(s) = \overline{m}^*V_{\mathrm{dc}}(s) \tag{8}$$

where the modulation index, $\overline{m} = m\angle\delta$.

By using the power balance equation, $P_{\mathrm{dc}} = P_{\mathrm{ac}}$, and the relationship in Eqs. (5) and (6), we can form the relationship between the currents in dc and ac side as

$$V_{\mathrm{dc}}I_{\mathrm{dc}} = 0.5\left(\overline{m}V_{\mathrm{dc}}e^{j\omega_0 t}\cdot\vec{i}^*_{\mathrm{t}} + \overline{m}^*V_{\mathrm{dc}}e^{-j\omega_0 t}\cdot\vec{i}_{\mathrm{t}}\right) \tag{9}$$

From Eq. (9) we can conclude that the dc current depends on both the positive sequence component current and its conjugate. In the Laplace's domain, by expressing the Eq. (9) from the dc side, we can rewrite the equation as

$$\begin{aligned} I_{\mathrm{dc}}(s) &= 0.5\left(\overline{m}\overline{I}^*_{\mathrm{t}}(s) + \overline{m}^*\overline{I}_{\mathrm{t}}(s)\right) \\ &= 0.5\left(\overline{m}\vec{i}^*_{\mathrm{t}}(s - j\omega_0) + \overline{m}^*\vec{i}_{\mathrm{t}}(s + j\omega_0)\right) \end{aligned} \tag{10}$$

From Eqs. (5), (6) and (10), we can express the equivalent circuit as a transformer connected between the dc and ac voltage with modulation index, $\overline{m}$ as the turns ratio and the dc current split to $\overline{m}\overline{I}^*_{\mathrm{t}}$ and $\overline{m}^*\overline{I}_{\mathrm{t}}$. The circuit is shown in Fig. 2(a). In order to satisfy the current relationship based on the transformation ratio, we can rewrite Eqs. (9) and (10) as

$$2I_{\mathrm{dc}} = \overline{m}\overline{I}^*_{\mathrm{t}} + \overline{m}^*\overline{I}_{\mathrm{t}}. \tag{11}$$

Since Eq. (11) shows the dc current is doubled, we need to modify the dc impedance to accommodate the change so that we can rewrite Eq. (4) as,

$$V_{\mathrm{dc}}(s) = V_{\mathrm{th}}(s) - 2I_{\mathrm{dc}}(s)\frac{Z_{\mathrm{dc}}(s)}{2} \tag{12}$$

Our main aim is to keep the dc voltage constant and make changes in dc impedance for any change in dc current to satisfy Eq. (11). Based on Eqs. (2), (3), (5), (6), (11) and (12), we can form the equivalent circuit as shown in Fig. 2(a).

Fig. 2(a) expresses the modulation index as a phasor; it is always preferred to have the transformation ratio as a simple real value. To achieve that, we can designate the converter voltage phase angle as zero while the grid voltage has an angle of $-\delta$. Hence:

$$\overline{V}_{\mathrm{t}}\angle-\delta - \overline{V}_{\mathrm{g}}\angle-\delta = (R_{\mathrm{tot}} + j\omega_0L_{\mathrm{tot}})\overline{I}_{\mathrm{t}}\angle-\delta \tag{13}$$

$$\overline{V}^*_{\mathrm{t}}\angle\delta - \overline{V}^*_{\mathrm{g}}\angle\delta = (R_{\mathrm{tot}} - j\omega_0L_{\mathrm{tot}})\overline{I}^*_{\mathrm{t}}\angle\delta \tag{14}$$

Using Eqs. (13) and (14), Fig. 2(a) can be reformed as Fig. 2(b).To further simplify the circuit, we can use the concept of transformer equivalent circuits. By removing the transformer in Fig. 2(b), we can further simplify the circuit as Fig. 2(c).

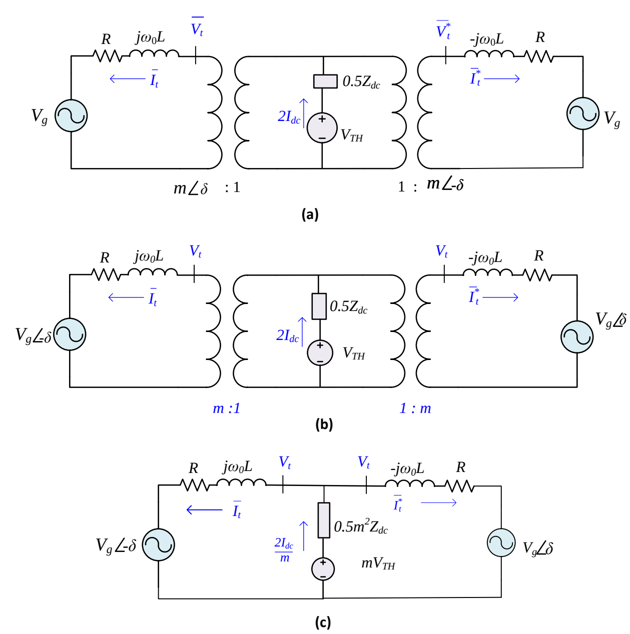


Fig. 2: Equivalent circuit for the positive-sequence components.

Fig. 2 represents the equivalent circuit for a balanced system, where $Z_{\mathrm{dc}} = 0$ and frequency of operation for $\overline{V}_{\mathrm{g}}$ is 60 Hz.

C. Negative-sequence circuit

In this section, we will represent a circuit for an unbalanced condition. Under an unbalanced condition, a second harmonic oscillation will be created in the instantaneous power in the ac side. Based on energy conservation, the dc side also has the 2nd harmonics. We can write the 2nd harmonic component of the dc-link voltage as

$$v_{\text{dc}_2}(t) = 0.5\left(\overline{V}_{\text{dc},+2}e^{j2\omega_0 t} + \overline{V}^*_{\text{dc},+2}e^{-j2\omega_0 t}\right) \quad (15)$$

In the ac side, v_{dc_2} is manifested as

$$\overline{m}e^{j\omega_0 t}v_{\text{dc}_2} = 0.5\overline{m}\overline{V}_{\text{dc},+2}e^{j3\omega_0 t} + 0.5\overline{m}\overline{V}^*_{\text{dc},+2}e^{-j\omega_0 t} \quad (16)$$

Under the unbalanced condition, the ac current can be represented as:

$$\vec{i}_{\text{t}} = \overline{I}_{\text{t},+1}e^{j\omega_0 t} + \overline{I}^*_{\text{t},-1}e^{-j\omega_0 t} + \overline{I}_{\text{t},+3}e^{j3\omega_0 t} \quad (17)$$

Similarly, converter voltage can be represented as,

$$\vec{v}_{\text{t}} = \overline{V}_{\text{t},+1}e^{j\omega_0 t} + \overline{V}^*_{\text{t},-1}e^{-j\omega_0 t} + \overline{V}_{\text{t},+3}e^{j3\omega_0 t} \quad (18)$$

Based on the power balance relationship, it can be seen that the 2nd harmonics components of the dc side and the ac side are the same. Assume that the dc-link voltage ripple is very small compared to its dc component. Hence, the dc-side 2nd harmonic component in the instantaneous power is:

$$p_{dc2} = V_{\text{dc}}i_{\text{dc2}} = V_{\text{dc}} \times 0.5\left(\overline{I}_{\text{dc},+2}e^{j2\omega_0 t} + \overline{I}^*_{\text{dc},+2}e^{-j2\omega_0 t}\right) \quad (19)$$

In the ac side, assume the harmonic components in the ac voltage are small compared to the fundamental component. Hence, the ac-side 2nd harmonic component in the instantaneous power is:

$$\begin{aligned} p_{ac2} &= \text{Re}\left(\overline{V}_{t,+1}\overline{I}^*_{t,+3} + \overline{V}_{t,+1}\overline{I}^*_{t,-1}\right) \\ &= \text{Re}\left(0.5\overline{m}^*V_{\text{dc}}\overline{I}_{t,+3} + 0.5\overline{m}V_{\text{dc}}\overline{I}^*_{t,-1}\right) \end{aligned} \quad (20)$$

Hence, the relationship between the dc and ac side current and voltage can be written as,

$$2\overline{I}_{\text{dc},+2} = \overline{m}\overline{I}^*_{\text{t},-1} + \overline{m}^*\overline{I}_{\text{t},+3} \quad (21)$$

$$\overline{V}^*_{\text{t},-1} = 0.5\overline{m}^* \cdot \overline{V}_{\text{dc},+2} \quad (22)$$

$$\overline{V}_{\text{t},+3} = 0.5\overline{m} \cdot \overline{V}_{\text{dc},+2} \quad (23)$$

In the above equation, $(+2)$ represents the second harmonic, (-1) represents the negative sequence component, and $(+3)$ represents the positive sequence third harmonics components.

Based on the dc/ac voltage and current relationship, an equivalent circuit representing the 120-Hz dc component vs. the ac side 180-Hz component and -60-Hz component has been derived and presented in Fig. 3.

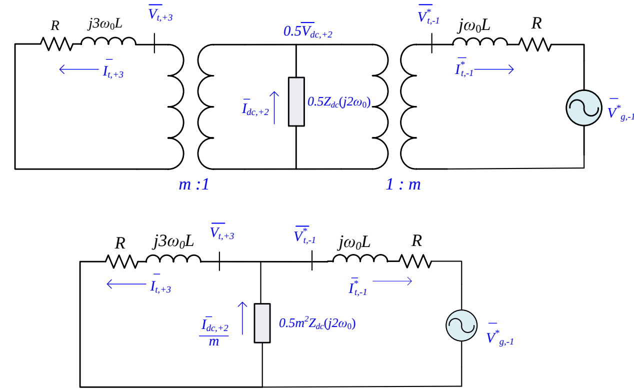


Fig. 3: Negative-sequence equivalent circuit for the hybrid dc/ac circuit.

D. The generalized circuit

Based on Fig. 3, a generalized circuit can also be found, as shown in Fig. 4.

For the negative-sequence circuit, s is substituted by $j2\omega_0$ and the resulting circuit is the same as that in Fig. 3. Note that the dc Thévenin voltage source at 120 Hz is 0. Also the grid voltage does not have a 180-Hz component. Thus, Fig. 3 does not present the dc Thévenin voltage source and the $v_g(s + j\omega_0)$ component.

For the positive-sequence circuit, in Fig. 4(a), s is substituted by 0 and the resulting circuit is similar as that in Fig. 2, except that both the voltage and current scaled by half.

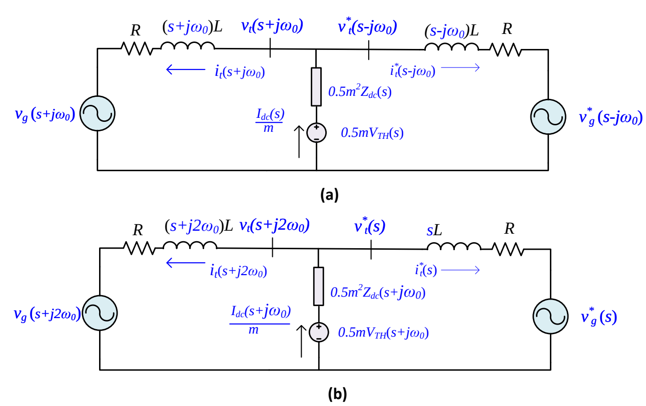


Fig. 4: Generalized equivalent circuit.

The generalized circuit represents the circuit of an inverter connected to a grid. The circuit can be viewed based on the dc system as Fig. 4(a) or based on the ac system Fig. 4(b). This circuit in Fig. 4(b) represents the frequency coupling phenomenon explicitly. The positive-sequence circuit indeed reflects the coupling between the 60-Hz component and itself: 60-Hz component's conjugate. s should be substituted by $-j2\pi \times 60$ rad/s.

The negative-sequence component is coupled with the positive-sequence 180-Hz component. This representation can be extended to any frequency. For the -60 Hz and +180-Hz coupling, s is substituted by $2\pi \times 60$ rad/s.

If the dc-system has an oscillation component at 10 Hz, the ac system will show +70 Hz and +50 Hz oscillation components. In the generalized circuit representation Fig. 4(b), s should be substituted by $-j2\pi \times 50$ rad/s to accurately reflect the frequency coupling. While the circuit representation appears simple, the physical meaning of the ac components needs careful contemplation.

Using the dc system view circuit, we can find that the ac impedance viewed from the dc side becomes:

$$\frac{2}{m^2}\left[\frac{1}{Z_{ac}(s+j\omega_0)}+\frac{1}{Z_{ac}(s-j\omega_0)}\right]^{-1} \tag{24}$$

A similar expression without per unit can also be found in the literature [4].

III. EMT MODEL AND VALIDATION

This section describes the EMT testbed and validates the circuit model described in the previous section. The EMT model is built in PSCAD v4.6. A simple IBR system is built as an open-loop model with the inverter controlled by a constant modulation index, $\overline{m}$. Fig. 1 shows the topology of the EMT model. Table I lists the parameters of the EMT testbed.

TABLE I: Parameters.

Parameters	Values (SI)
V_{grid}	391.9 V
R_{line}, X_{line}	0.0219 Ω , 582 μH
L_{filter}, C_{filter}	125 μH, 200 μF
R_{damp}, L_{damp}, C_{damp}	2.5 Ω, 625 μH, 100 μF

To validate the circuit in Fig. 4, we performed two experiments in EMT simulation and validated the result using circuit calculation.

A. Experiment 1: 70 Hz injection in the grid

A 70 Hz component at 0.1 pu is injected into grid voltage, the DC side voltage, and current, and converter side voltage and current are measured. A constant modulation index of $1\angle 70°$ is applied to the inverter. From Fig. 4, if the grid voltage has a 70 Hz component, the converter voltage should have a 70 Hz and 50 Hz component, and the dc voltage should have a 10 Hz component.

Fig. 5 shows the instantaneous and FFT plot for grid voltage and current, validating the presence of 50 Hz and 70 Hz components in the current. The spectrum plot in Fig. 6(a)shows the presence of a 50 Hz and a 70 Hz component in converter voltage and current. Fig. 6(b) shows the instantaneous voltage and current plot in dc side, and the FFT plot verifies the presence of a 10 Hz component in the dc side voltage and current. It can be seen that a 10 Hz ripple on the dc side is reflected as a 50 Hz and 70 Hz harmonic component on the ac side.

The above result from the EMT simulation was verified using circuit analysis. Fig. 8 was used to validate the result. From the figure, it can be verified that the 50 Hz and 70 Hz components of the converter voltage share the same node and are equal, which can be validated from Fig. 6. From Fig. 6 it can be validated that the dc voltage and current have a 10 Hz component. Table II shows the comparison of the EMT and circuit analysis, and it can be seen that the results are closely matched.

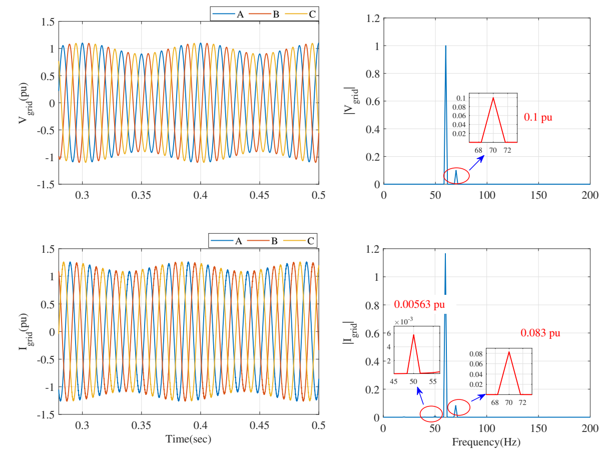


Fig. 5: Instantaneous plot for grid voltage and current with a 70 Hz voltage injection in grid voltage. The FFT plot shows 70 Hz component of magnitude 0.1 pu in grid voltage and the grid current has both 70 Hz and 50 Hz component.

TABLE II: Comparison between EMT and analysis results.

	EMT(pu)	Analysis(pu)
$V_{\text{dc},_{10\text{Hz}}}$	$0.0096\angle-1.13$	$0.0096\angle-1.149$
$I_{\text{dc},_{10\text{Hz}}}$	$0.088\angle 0.434$	$0.089\angle 0.422$
$I_{\text{conv},_{50\text{Hz}}}$	$0.0056\angle-2.243$	$0.0059\angle-2.223$
$I_{\text{conv},_{70\text{Hz}}}$	$-1.478\angle 1.67$	$0.0832\angle-1.4891$
$V_{\text{conv},_{50\text{Hz}}}$	$0.0046\angle 2.356$	$0.0048\angle 2.37$
$V_{\text{conv},_{70\text{Hz}}}$	$0.0049\angle-0.14$	$0.0048\angle 0.0732$

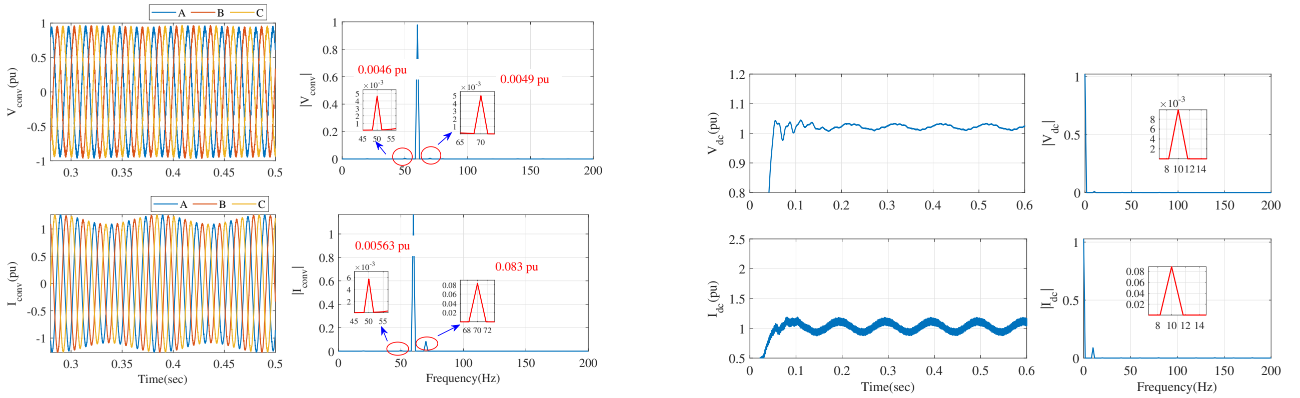
B. Experiment 2: Voltage dip in grid voltage

The EMT testbed is used to generate an unbalance in the system. To create the event, a fixed modulation index of $1\angle 70°$ is set, and a voltage dip of 0.3 pu is provided at 0.3 s.

With the unbalanced voltage dip, the current is also unbalanced. The current has a positive sequence 180 Hz component, a positive sequence at 60 Hz, and a negative sequence 60-Hz component. The spectrum plot in Fig. 7a shows the presence of a third harmonic and negative sequence component in the converter voltage and current.

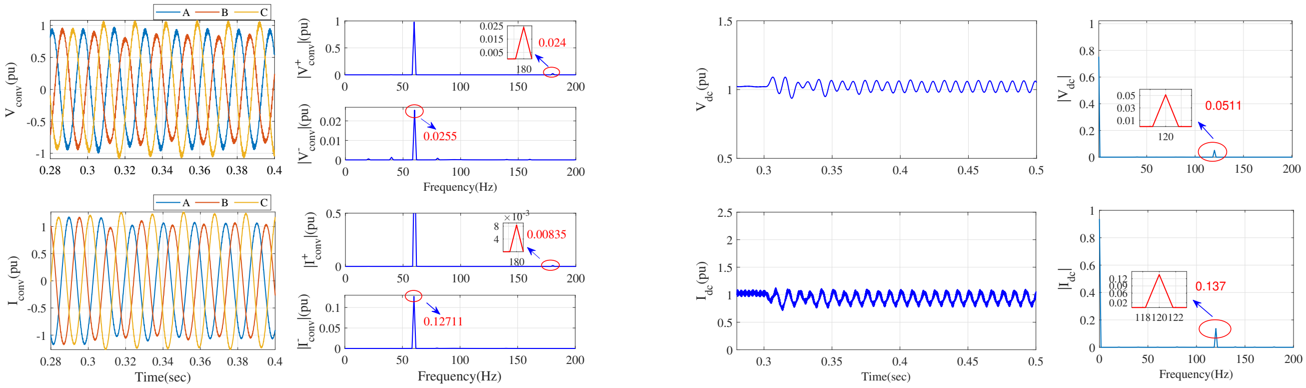
Fig. 7b shows the instantaneous voltage and current plot in the dc side, and the FFT plot verifies the presence of second harmonics in the dc side voltage and current. It can be seen that the 120-Hz ripple in the dc reflects as the 180-Hz and -60-Hz harmonic components in the ac side.

The above results from the EMT model were validated using circuit analysis. Fig. 3 was used for validating the result. Just by looking at the circuit, we can see that $\overline{V}_{\text{t},,-1}$ and $\overline{V}_{\text{t},,+3}$ are equal which can also be verified from the



(a) (b)

Fig. 6: Experiment 1. (a) Converter voltage and current and the FFT plots. (b)dc side voltage and current and the FFT plots.



(a) (b)

Fig. 7: Experiment 2. (a) Converter voltage and current and the FFT plots (b) dc side voltage and current and the FFT plots.

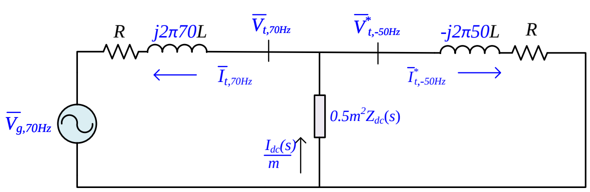


Fig. 8: Equivalent circuit for 10 Hz injection.

FFT plot for converter voltage in Fig. 7a. Fig. 7b shows the instantaneous curve for dc side voltage and current, which shows the oscillation and the FFT plot verifies the oscillation is the second harmonic component. Table III validates the EMT result with analysis, and it can be seen that results are very closely matched.

IV. Conclusion

This paper presents a detailed mathematical derivation that leads to a generalized circuit representation of a hybrid dc/ac system consisting of an inverter-based resource. The circuit explains the frequency-coupling phenomena that are seen in an inverter connected to a grid. The result from the circuit analysis is validated using the EMT simulation results. This type of circuit can be further expanded to include inverter control and used in fault analysis.

TABLE III: Comparison between EMT and analysis results.

	EMT(pu)	Analysis(pu)
$V_{\mathrm{dc},+2}$	$0.0511\angle-1.778$	$0.0498\angle-1.7930$
$I_{\mathrm{dc},+2}$	$0.1369\angle2.9316$	$0.1358\angle2.9194$
$I_{\mathrm{conv},-1}$	$0.127\angle-1.4292$	$0.1273\angle-1.446$
$I_{\mathrm{conv},+3}$	$0.00835\angle1.0418$	$0.0085\angle1.0328$
$V_{\mathrm{conv},-1}$	$0.0255\angle0.178$	$0.0249\angle0.1259$
$V_{\mathrm{conv},+3}$	$0.0240\angle2.58$	$0.0249\angle2.5704$

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