Analytical Model of A Grid-Forming Inverter

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Abstract—Recently, inverter-based resources are being integrated into the power grid at a rapid rate. Most of these resources are grid-following inverters, where weak grid operation becomes an issue. The research focus has now shifted towards grid forming inverters, which emulate synchronous generators. Thus proper modeling and analysis are required to understand the grid forming inverters. This paper presents the nonlinear analytical model of a grid forming inverter when operating in grid-connected mode in dq-domain. The developed model is validated in the time domain with the Electromagnetic Transients (EMT) test bed when the system operates under weak grid conditions. The analytical model is also used to perform the eigenvalues analysis. The EMT model is simulated in MATLAB/Simulink and the analytical model is simulated in MATLAB/Simulink. Hardware experimentation is also performed for further benchmarking with the help of laboratory scale hardware setup.

Index Terms—Grid forming inverter, weak grid, analytical model, eigenvalue

I. INTRODUCTION

THE MOTIVATION to achieve the goal of clean energy has led to rapid advances towards power electronics-based sources, known as inverter-based resources (IBRs). IBRs are quickly replacing the traditional synchronous generators based power grids. In fact, in some parts of the world like Tasmania, Ireland, South Australia met more than 90% their energy demand by renewable energy sources [1], [2], [3].

Predominantly these IBRs are grid following inverters (GFLs), which imitate the characteristics of a current source. Recently researchers have shifted their focus on a different type of IBRs, known as “grid forming inverter” (GFM) [4]. Theoretically, the primary function of the GFMs is to act as a controllable voltage source. This characteristic property of GFMs makes them a suitable replacement for synchronous generators in low inertia systems. Furthermore, GFMs can also be used to black start a power grid [5], [6].

Therefore, a proper modeling approach is required to study GFM inverters. The first approach uses EMT simulations packages like MATLAB/Simscape, PSCAD, EMTP, etc. These packages are suitable for performing time-domain simulations. They are a kind of virtual lab for experiments. The second approach is nonlinear analytical model building. The analytical modeling is a widely adopted technique in research community [7], [8], [9], [10]. It can provide many more insights into the system dynamics, by providing linear state-space models for further analysis.

In this paper, we develop the dq-domain analytical model of the GFM when operating in grid-connected mode. The analytical model includes the controller dynamics and circuit modeling. The developed model is validated in the time-domain with the EMT model for weak grid operations. The linear state-space model is also extracted from the analytical model to perform eigenvalue analysis. Furthermore, laboratory-scale hardware experimentation was also conducted to replicate the EMT studies and analytical model results when operating under weak grid conditions.

The rest of the paper is structured as follows. Section II describes the system topology. Section III illustrates the analytical model building. Section IV presents the results and analysis. Section V outlines the hardware implementation. Section VI concludes the paper.

II. SYSTEM TOPOLOGY

The schematic of the GFM presented in Fig. 1. The DC terminal of the three-phase VSC is connected to a DC voltage source, and the converter is connected to the grid through a transmission line, represented by \( R_g \) and \( L_g \). A RLC choke filter is connected at the terminal of the VSC with \( R_f \), \( L_f \), and \( C_f \) as the resistance, inductance, and capacitance respectively. A purely resistive load \( R_{\text{load}} \) is also connected at the PCC bus. The control structure of the GFM is implemented in dq frame. The following notation are used: \( i_{cdq} \), \( v_{dq} \) and \( i_{Ldq} \) are the current flowing through the filter inductor \( (L_f) \), voltage at the PCC bus and the load current in \( dq \) frame respectively. \( P \) and \( Q \) are the real and reactive power at the PCC bus.

The inner loop is PI controller-based current control structure, where the filter inductor current \( (v_{cdq}) \) as the control variable. The outer loop regulates the voltage at the PCC bus \( (v_{dq}) \). Similar to the inner loop, outer loop also implements a PI controller. Two droop control methods are implemented: (1) \( Q - V \) droop and (2) \( P - f \) droop. \( Q - V \) droop generates the \( d \)-axis reference signal \( (v_{dref}) \) for the outer loop. On the other hand, the \( P - f \) droop regulates the PCC bus’s real power and provides the desired angle \( (\omega t) \) for the control. Parameters used in this paper are listed in TABLE. I.

III. dq-DOMAIN ANALYTICAL MODEL

In this paper, the analytical model of the GFM presented in Fig. 1 is developed in \( dq \)-domain [11], and is based on per unit system. The block diagram of the analytical model is presented in Fig. 2. As per Fig. 2, the system is modeled with four main blocks: 1) droop control, 2) inner loop, 3) outer loop, 4) circuit dynamics, and 5) frame conversion. It is noted that, while the signals at the circuit level are based on grid frame (superscript "g"), the signals associated with the control structure are based on the reference frame or reference angle provided by the the \( P - f \) droop. Hence, the control structure is in the \( P - f \) droop frame of reference (superscript "c"). Furthermore, there
TABLE I: Parameters used for grid forming converter. Note: The PI controller gains are based on per unit system.

<table>
<thead>
<tr>
<th>Description</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Power Base</td>
<td>$S_b$</td>
<td>50 VA</td>
</tr>
<tr>
<td>Voltage Base</td>
<td>$V_{base}$</td>
<td>20 V</td>
</tr>
<tr>
<td>Nominal Frequency</td>
<td>$f_0$</td>
<td>60 Hz</td>
</tr>
<tr>
<td>Grid Voltage</td>
<td>$V_g$</td>
<td>11.5 V</td>
</tr>
<tr>
<td>DC Voltage</td>
<td>$V_{DC}$</td>
<td>40 V</td>
</tr>
<tr>
<td>DC Link Capacitor</td>
<td>$C_{DC}$</td>
<td>260 µF</td>
</tr>
<tr>
<td>Choke Filter</td>
<td>$L_f, R_f, C_f$</td>
<td>1.5 mH, 70 mΩ, 47 µF</td>
</tr>
<tr>
<td>Transmission Line Load</td>
<td>$R_{Load}$</td>
<td>20 W</td>
</tr>
<tr>
<td>Switching Frequency</td>
<td>$f_{SW}$</td>
<td>5 kHz</td>
</tr>
<tr>
<td>Inner Loop: PI gains</td>
<td>$k_{pi}, k_{iq}$</td>
<td>1, 5</td>
</tr>
<tr>
<td>Outer Loop: PI gains</td>
<td>$k_{pL}, k_{qL}$</td>
<td>3, 50</td>
</tr>
<tr>
<td>Droop Parameters</td>
<td>$m, n$</td>
<td>0.08, 0.1</td>
</tr>
</tbody>
</table>

are 11 state variables in the analytical model. $x_1$ is associated with the $P - f$ droop, $x_2 - x_5$ are related to the outer loop and the inner loop, $i_{g0dq}^*$ are the state-variables related to the circuit dynamics.

A. Circuit Dynamics

The circuit dynamics includes the grid current $i_{g0dq}^*$, the current through the choke filter $i_{dq}^*$ and the PCC bus voltage $v_{g0}^*$. The model of the circuit dynamics is developed in the grid $dq$ frame that rotates at a speed of $\omega_0$. The differential equations for the circuit dynamics are presented as follows:

$$\begin{align*}
\frac{dv_{g0}^*}{dt} &= -\frac{R_{gpu}}{L_{gpu}}v_{g0}^* + \frac{1}{L_{gpu}}v_{g0}^* - \frac{1}{L_{gpu}}v_{g0}^* + \omega_0 i_{g0}^* \\
\frac{di_{dq}^*}{dt} &= -\frac{R_{gpu}}{L_{gpu}}i_{dq}^* + \frac{1}{L_{gpu}}v_{g0}^* - \frac{1}{L_{gpu}}v_{g0}^* + \omega_0 i_{dq}^* \\
\frac{dv_{cd}^*}{dt} &= -\frac{R_{fpu}}{L_{fpu}}i_{cd}^* + \frac{1}{L_{fpu}}v_{cd}^* - \frac{1}{L_{fpu}}v_{cd}^* + \omega_0 i_{cd}^* \\
\frac{dv_{c0}^*}{dt} &= \frac{1}{C_{fpu}}(v_{c0}^* - v_{c0}^*) + \omega_0 i_{c0}^* \\
\frac{dv_{q0}^*}{dt} &= \frac{1}{C_{fpu}}(v_{q0}^* - v_{q0}^*) + \omega_0 i_{q0}^*
\end{align*}$$

(1)

Here $R_{fpu}, L_{fpu}$, and $C_{fpu}$ are the per unit values of the $RLC$ components of the choke filter, and similarly $R_{gpu}$ and $L_{gpu}$ are the per-unit values of the transmission line parameters. Also, $i_{cd}^*$ and $i_{c0}^*$ are the dq components of the current $i_{g0}$ (marked in Fig. 1) in grid frame, and $v_{cd}^*$ and $v_{c0}^*$ are the dq components of the terminal voltage $v_t$. The per unit expressions for the real power and the reactive power at the PCC bus are:

$$P = v_{d0}^* i_{d0}^* + v_{q0}^* i_{q0}^* \quad \text{and} \quad Q = -v_{d0}^* i_{q0}^* + v_{q0}^* i_{d0}^*$$

(2)

And the load current in $dq$ frame (grid frame) is given by:

$$i_{Ld}^* = \frac{v_{d0}^*}{R_{Loadpu}} \quad \text{and} \quad i_{Lq}^* = \frac{v_{q0}^*}{R_{Loadpu}}$$

(3)

Hence, (1) - (3) completes the analytical model of the circuit dynamics.

![Fig. 2: Block diagram of the analytical model for GFM. “FC” stands for “frame conversion.”](image-url)
B. Frame conversion

As discussed earlier, the control system is operating in a different reference rotating at a speed of “ω” which is provided by the \( P - f \) droop. The control structure regulates the converter current \( i_c \), and the voltage at the PCC bus \( v \) is \( dq \) frame. Since the variables from the circuit level model and the control level model are in a different frame of reference, a proper transformation is required to capture the analytical model accurately. For the transformation, the concept of space vector is used \([12]\). For example, the space vector for the PCC bus voltage \( \vec{V} \), is presented as:

\[
\vec{V} = (v_d^c + j v_q^c)e^{j\omega t} = (v_d^g + j v_q^g) e^{j\omega_0 t}
\]

Hence:

\[
(v_d^c + j v_q^c) = (v_d^g + j v_q^g) e^{-j\Delta \theta}.
\]

From \( \Box \) the signals in grid frame of reference can be transformed to the \( P - f \) droop frame of reference, and vice-versa, and \( \Delta \theta = \omega t - \omega_0 t \).

C. Analytical Model Initialization

As discussed earlier, there are 11 state variables in the analytical model. A proper initialization procedure is needed to develop the analytical model accurately.

The complex power at the PCC bus is given: \( S = V I^* \), where \( V \) is the PCC bus voltage phasor and \( I_c \) is the current phasor through the filter inductor. The real power is: \( P = \text{real}(S) \), and the reactive power is: \( Q = \text{imag}(Q) \). The PCC bus voltage phasor is defined as \( V = V_{\text{mag}} e^{j\theta} \) and the current \( I_c \) is obtained as:

\[
I_g = \frac{V - V_d}{R_{\text{gpu}} + j X_{\text{gpu}}}, \quad I_{\text{Load}} = \frac{V}{R_{\text{Load,pu}}}, \quad I_{Cf} = \frac{V}{-i X_{\text{Cpu}}}, \quad I_c = I_g + I_{\text{Load}} + I_{Cf}
\]

Here, \( I_g \) is the current in the transmission line, \( I_{Cf} \) is the current through the filter capacitor \( C_f \), and \( V_g = 1 \). Since, the GFM is in grid connected mode the system is always at the nominal frequency \( (f_0) \). The steady-state computing can be formulated as an optimization problem and solved using YALMIP \([13]\). Part of the code is is shown below:

```matlab
Vmag = sdpvar(1,1);
theta = sdpvar(1,1);
P = sdpvar(1,1);
Q = sdpvar(1,1);
assign(Vmag,1);
assign(P, 1);
assign(Q, 0);
assign(theta, 0);
Constraints = [P==Pref;
Vmag == n*(Qref-Q)];
Objective = 0;
```

Here \( P_{\text{ref}} \) and \( Q_{\text{ref}} \) are the desired operating conditions for the system. The optimization problem solves for the value of \( V_{\text{mag}} \) and \( \theta \). Given \( V_{\text{mag}} \) and \( \theta \), the state variables are calculated for initialization. For \( P_{\text{ref}} = 2.22 \) pu and \( Q_{\text{ref}} = 0 \), steady-state values for \( V_{\text{mag}} \) \( \theta \), and \( Q \) are listed in Table \( \Box \).

### IV. Analysis and Simulation-Based Validations

The EMT test bed and the analytical model are used to study the VSC system under weak grid conditions \( (X_g = 0.35 \) pu). The EMT test-bed is implemented in MATLAB/Simscape, and the analytical model using MATLAB/Simulink environment. First, we present the time-domain results from the EMT test-bed and the analytical model. The analytical model is then used to extract the eigenvalues for further analysis.

A. Time-domain results

\( P_{\text{ref}} \) is increased in steps, and the system response is recorded for analysis. The time domain results are presented in Fig. \( \Box \). It is observed that as the reference real power is increased the system undergoes oscillations and the frequency of the oscillations is 3-Hz. Also, the system is marginally stable when \( P_{\text{ref}} = 2.23 \) pu. The results from the developed analytical model match well with the EMT simulations. Moreover, when \( P_{\text{ref}} = 2.22 \) pu the steady state values match very well with the results tabulated in Table \( \Box \) Please note: \( Q_{\text{ref}} = 0 \) for the simulation results.

B. Eigenvalue Analysis

The linearized state-space model is obtained from the developed \( dq \)-domain analytical model. This is done with the help of MATLAB’s “linmod” command, which computes the linear state-space model. The eigenvalues are then computed from the obtained state-space model, by using the “A” matrix.

The eigenvalue loci when \( P_{\text{ref}} \) is varied from 2.0 pu to 2.25 pu is presented in Fig. \( \Box \). It can be observed from Fig. \( \Box \) that, when \( P_{\text{ref}} \) is increased, the eigenvalue associated with the 3-Hz component moves to the right-half plane (RHP), with the marginal condition as \( P_{\text{ref}} = 2.23 \) pu. The eigenvalue trajectory is in coherence with the time-domain results, presented in Fig. \( \Box \).

Fig. \( \Box \) presents the mode shape of all the state-variables. The mode shape is obtained from the right eigenvector \( (V) \) corresponding to a particular mode (for this paper 3-Hz mode). Mode shape provides the information on how a particular mode is reflected in the system. It is observed from Fig. \( \Box \) that 3-Hz oscillation mode is due to the state-variable associated with the \( q \) axis grid current \( i_{dq}^g \). Furthermore, the state variables \( i_{dq}^g \) and \( i_{dq}^g \) (red line) have a phase difference of about 180 degrees and hence oscillate against each other. Additionally, state variables \( i_{dq}^g \) and \( i_{dq}^g \) (blue dashed line) have a phase difference of about 90 degrees.

Moreover, from the linear state-space model we can also determine the mode under investigation is observable in which measurement. In this paper the measurements are \( P, Q, |V| \) and \( \Delta \theta \). Here \( |V| \) is the magnitude of the PCC bus voltage,
P_{ref} = 2.23pu
P_{ref} = 2.25pu
P_{ref} = 2.00pu

Q = 0.7882 pu
Q = 0.9408 pu
Q = 0.3064 pu

|V| = 0.9212 pu
|V| = 0.9300 pu
|V| = 0.1027 pu

|Δθ| = 67.7°
|Δθ| = 67.5°
|Δθ| = 5.24°

Fig. 3: Side-by-side comparison of results obtained from the EMT test-bed and developed analytical model. (a) Time domain results from the EMT test-bed. (b) Time domain results from analytical model.

and Δθ is angle from the P − f droop. Table. III lists the observability of the 3-Hz mode in the measurements. It can be seen that the 3-Hz oscillation mode is most observable in the reactive power measurement (Q).

From the time domain responses presented in Fig. 3 we can observe that the ripple magnitude (in % with respect to steady state value) of the 3-Hz oscillation for measurement Q is about 5.1%, for |V| = 1.16%, P = 0.9%, and Δθ = 2.89%. The conclusion can be made that Q measurement has the largest presence of 3 Hz oscillations in its response.

Additionally, on comparison of the response of Q and |V| at t = 45s, when P_{ref} is increased from 2.22 pu to 2.23 pu, it is observed that the phase shift between the two waveform is ≈ 120°, with Q leading |V|. These observations match closely

with the results tabulated in Table. III

<table>
<thead>
<tr>
<th>Measurement</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td>0.1404−94.08°</td>
</tr>
<tr>
<td>Q</td>
<td>0.3064−30.08°</td>
</tr>
<tr>
<td></td>
<td>V</td>
</tr>
<tr>
<td>Δθ</td>
<td>0.227−5.24°</td>
</tr>
</tbody>
</table>
The parameters used in the experimentation are listed in Table. The hardware test-bed, when connected to a weak grid and the frequency of oscillations is ≈ 3-Hz. The results from the EMT studies, analytical model, and the hardware experimentation are a very good match.

VI. CONCLUSION

This paper presents the analytical model of the GFM when operating in grid-connected mode. The analytical model is developed in dq frame and includes the control dynamics network model. First, the developed analytical model is validated with the EMT model in the time domain. Eigenvalue analysis is also conducted for further validations of the developed analytical model. Laboratory-scale hardware experimentation was also performed for additional validations.

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REFERENCES


