Inter-IBR Oscillation Modes
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Abstract—This research addresses the following question: For a weak grid with multiple-IBRs, what is the nature of oscillation modes? The analysis of a system consisting of two identical IBRs offers a glimpse of the picture. It shows that the system has a weak-grid mode and an inter-IBR mode. For the weak-grid mode, two IBRs work as an aggregated one to interact with the grid. This mode is sensitive to IBR exporting power level and grid strength. For the inter-IBR mode, two IBRs oscillate against each other. This mode is less sensitive to grid strength and power level.

Index Terms—Inverter-based resources; weak grids; oscillations.

I. INTRODUCTION

It is well known that a conventional power grid has inter-area oscillation modes, where two groups of synchronous generators oscillate against each other [1]. This type of oscillations have been frequently observed in real world [2]. In 2017, this author re-examined inter-area oscillations using Laplacian graph decomposition [3]. With the two-area four-machine system developed in [1] viewed as four identical dynamic subsystems interconnected through a Laplacian graph (associated with the network admittance matrix), eigenvalue decomposition of the Laplacian graph leads to three decoupled subsystems, each reflecting one mode. The inter-area oscillation mode is associated with the smallest nonzero eigenvalue of the Laplacian graph. The work in [3] clearly demonstrates how the spectrum of the Laplacian graph, relates to the inter-area or local oscillation modes.

For a weak grid with high penetrations of inverter-based resources (IBRs), weak grid oscillations have been reported in real world (e.g., [4]) and can be replicated using a single-IBR infinite-bus model [5], [6]. Weak grid oscillations are sensitive to grid strength and IBR power exporting level; thus, they can be mitigated by ramping down IBR power exporting level.

Recently, 7-Hz oscillations were observed in Australian power grid [7]. The oscillations were identified to be associated with IBR interactions. They did not disappear completely with power levels of IBRs ramped down. Only when some IBRs were taken offline, oscillations disappeared. In addition, oscillations between parallel connected 4-MW type-3 wind turbines in electromagnetic transient simulation have been reported in [8]. Reactive power of one turbine oscillates against that of the other turbine at a frequency of 1.2 Hz. This oscillation mode is shown insensitive to grid impedance.

This letter aims to provide a fundamental understanding on the nature of different types of oscillations modes in a weak grid with multiple IBRs. To start with, an analytical model of a system consisting of two identical IBRs with a weak grid interconnection is built. Eigenvalue and mode shape analysis show that the system presents an aggregated weak-grid mode and an inter-IBR mode. For the weak-grid mode, two IBRs work as an aggregated one. This mode is sensitive to grid strength. For the inter-IBR mode, two IBRs oscillate against each other according to the mode shape. This mode is less sensitive to grid strength. The 1.2-Hz oscillations reported in [8] are apparently of inter-IBR mode nature.

Furthermore, an explanation is offered to show that the two oscillation modes can be viewed using two decomposed systems, each associated with an IBR block and an impedance. This explanation arrives after rigorous mathematic derivation relying on a similar approach adopted in the author’s work on inter-area oscillations [3], i.e., eigenvalue decomposition of the admittance matrix. The circuit associated with the weak-grid mode is essentially the system of an aggregated IBR connected to the grid. Grid strength is reflected by the connecting impedance. The circuit associated with the inter-IBR mode reflects the interconnection between IBRs and has no or much less association with the grid voltage; thus, the inter-IBR mode is less sensitive to grid strength and grid voltage.

Time-domain simulation results are presented to demonstrate that grid voltage dip excites the weak-grid mode only while an increase in a dc-link voltage order can excite the inter-IBR mode for an ideal scenario when the two IBRs and their interconnection to the grid are identical. The dynamic phenomena can be explained by the decomposed circuits.

II. THE TEST BED AND EIGENVALUE ANALYSIS

The test bed of a grid with two identical IBRs is shown in Fig. 1. The two IBRs are connected in parallel to the point of measurement (POM) bus. This bus is connected to the grid (represented by an infinite bus) through a transmission line with an impedance $R_{lg} + jX_{lg}$. The total power from the IBRs is $0.935 \text{ p.u.}$ and each IBR generates half of the total power.

Both IBRs employ grid-following converters, which adopt the DC-link voltage control and the point of common coupling (PCC) voltage control. Analytical models developed in [9] are adopted for IBRs with their dc-side power assumed as known parameters. The system parameters and control parameters are listed in Table I. This case is Case 1 where the two IBRs and their connecting line impedances are identical. For comparison, Case 2 with different line impedances ($R_{l1} + jX_{l1} = 0.02 + j0.2$, $R_{l2} + jX_{l2} = 0.06 + j0.6$) is also studied. The eigenvalue loci of the two test beds are presented in Fig. 2a and Fig. 2b. The R/X ratio is kept at 10%. For both cases, it can be seen that the two-IBR test bed has two modes at
7.8-Hz. One mode moves to the right-half-plane (RHP) when \( X_{lg} \) is increasing. The other mode is less sensitive to \( X_{lg} \). For Case 1, this mode appears not responding to the change of \( X_{lg} \) at all.

Mode shape analysis results of Case 1 are presented in Fig. 2. Mode shape is associated to the system matrix’s right eigenvector corresponding to a mode and it reflects how this mode is reflected in a state. A mode shape, or an element of the eigenvector, is a complex number and can be plotted in the complex plane as a vector. For an oscillation mode, if the mode shapes associated to two state variables have a phase angle difference close to 180 degree, these two states will oscillate against each other. Otherwise, if the phase angle difference is close to 0 degree, the two states will oscillate in phase.

Fig. 3 presents the mode shapes of four states of two IBRs. Using the information of the phase angle difference between the same state of two IBRs, we can differentiate a mode as an aggregated mode or an inter-IBR mode. For the aggregated mode, the two IBRs work in phase. For the inter-IBR mode, the two IBRs oscillate against each other.

It can be clearly seen that the two IBRs work coherently for the aggregated mode, or the weak-grid mode and the two IBRs oscillate against each other for the inter-IBR mode.

From the analytical model, observability of the measurements can be found with linear state-space models extracted. Observability of the two modes in the measurements is presented in Table II. It can be seen that the inter-IBR mode is most observable in the real power measurement.

**Remarks:** The simple system with two identical IBRs has two dominant modes at 7.8 Hz for \( X_{lg} = 0.2 \) p.u. One mode is the weak-grid mode, where the two IBRs working as an aggregated one to interact with the grid. The other mode is the inter-IBR mode, where the two IBRs will oscillate against each other. The weak grid mode is sensitive to grid strength while the inter-IBR mode is less sensitive to grid strength.

### III. Two Decoupled Circuits

In this section, an approach is sought to offer better insights on the two modes. The original study system of two identical

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**TABLE I:** System Parameters

<table>
<thead>
<tr>
<th>Description</th>
<th>Parameters</th>
<th>Value (p.u.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Power level</td>
<td>( P_{PCC1}, P_{PCC2} )</td>
<td>0.935/2</td>
</tr>
<tr>
<td>System frequency</td>
<td>( f_0 )</td>
<td>60 Hz</td>
</tr>
<tr>
<td>Converter filter</td>
<td>( R_1, R_2 )</td>
<td>0.003 × 2</td>
</tr>
<tr>
<td>DC-link capacitor</td>
<td>( C_{dc1}, C_{dc2} )</td>
<td>0.108 F</td>
</tr>
<tr>
<td>Shunt capacitor</td>
<td>( C_L )</td>
<td>0.232</td>
</tr>
<tr>
<td>Network line</td>
<td>( L_{11}, L_{12} )</td>
<td>0.2</td>
</tr>
<tr>
<td>Inner loop</td>
<td>( X_{ii} )</td>
<td>0.3, 5</td>
</tr>
<tr>
<td>DC-link control loop</td>
<td>( K_{iL1}, K_{iL2} )</td>
<td>0.1, 80</td>
</tr>
<tr>
<td>PLL</td>
<td>( K_{p, iL1}, K_{p, iL2} )</td>
<td>60, 1400</td>
</tr>
<tr>
<td>Feedforward filter</td>
<td>( X_{f} )</td>
<td>0.001</td>
</tr>
</tbody>
</table>

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**TABLE II:** Observability of the modes in measurements

<table>
<thead>
<tr>
<th>Measurement</th>
<th>Inter-IBR</th>
<th>Weak grid</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_1 )</td>
<td>0.1426/( -101.5^\circ )</td>
<td>0.0377/( -161.9^\circ )</td>
</tr>
<tr>
<td>( Q_1 )</td>
<td>0.0061/( 61.7^\circ )</td>
<td>0.0048/( 157.9^\circ )</td>
</tr>
<tr>
<td>( V_1 )</td>
<td>0.0004/( -128.3^\circ )</td>
<td>0.0068/( -1.6^\circ )</td>
</tr>
<tr>
<td>( V_2 )</td>
<td>0.0004/( -128.3^\circ )</td>
<td>0.0068/( -1.6^\circ )</td>
</tr>
<tr>
<td>( \theta_1 )</td>
<td>0.0309/( 154.2^\circ )</td>
<td>0.0329/( 154.2^\circ )</td>
</tr>
<tr>
<td>( \theta_2 )</td>
<td>0.0309/( -25.7^\circ )</td>
<td>0.0329/( 154.2^\circ )</td>
</tr>
</tbody>
</table>
IBRs with a symmetric topology have four nodes as shown in Fig. 3. Node 1 and Node 2 are the two PCC buses that connect the IBRs. Each IBR can be viewed as an impedance connected at the PCC bus. Node 4, the POM bus, can be eliminated and the resulting 3-node network topology can be found via Kron reduction of the admittance matrix representing the network. For simplicity, a lossless network is assumed in the reduction process.

The 3-node topology shows that the two IBRs are connected to the grid bus as well as each other. Based on the three-node system, it can be seen that

\[
\begin{bmatrix}
T_1 \\
T_2
\end{bmatrix} = \begin{bmatrix}
Y_{11} & Y_{12} \\
Y_{21} & Y_{22}
\end{bmatrix} \begin{bmatrix}
V_1 \\
V_2
\end{bmatrix} + \begin{bmatrix}
1 \\
1
\end{bmatrix} \frac{V_g}{j(X_l + 2X_{lg})},
\]

(1)

where \(T_1\) and \(T_2\) are the current injection from the two IBRs to the network, \(V_1\) and \(V_2\) are the IBR PCC voltages, and \(Y_{11}, Y_{12}, Y_{21},\) and \(Y_{22}\) are the element of the admittance matrix of the 3-node system.

Through eigenvalue decomposition of the \(2 \times 2\) matrix and coordinate conversion, (1) can be transformed to the following:

\[
\begin{bmatrix}
-T_1 + T_2 \\
-T_1 + T_2 \\
T_1 + T_2 \\
T_1 + T_2
\end{bmatrix} = \begin{bmatrix}
\frac{1}{jX_l} & 0 \\
0 & \frac{1}{j(X_l + 2X_{lg})}
\end{bmatrix} \begin{bmatrix}
-V_1 + V_2 \\
V_1 + V_2 \\
V_1 + V_2 \\
V_1 + V_2
\end{bmatrix} + \begin{bmatrix}
0 \\
0 \\
0 \\
0
\end{bmatrix} \begin{bmatrix}
1 \\
1 \\
1 \\
1
\end{bmatrix} V_g,
\]

(2)

Equation (2) shows that the network can be viewed as two decoupled circuits. The first circuit describes the relationship between the IBRs’ current difference and voltage difference from the network perspective. The second circuit describes the relationship between the sum of IBRs’ currents and the sum of the IBRs’ voltages. Furthermore, if the two IBRs are identical, then, the newly defined currents and voltages have the following relationship:

\[
\begin{bmatrix}
-T_1 + T_2 \\
T_1 + T_2
\end{bmatrix} = \begin{bmatrix}
Y_{11} \\
Y_{21}
\end{bmatrix} \begin{bmatrix}
-V_1 + V_2 \\
V_1 + V_2
\end{bmatrix},
\]

(3)

where \(Y_{11}\) represents the admittance of the IBR.

Thus, the two decoupled circuits can be found as shown in Fig. 4. It can be clearly seen that the first decoupled circuit represents the difference between two IBRs. Hence this circuit can be viewed as associated with the inter-IBR mode. The second decoupled circuit represents the sum of two IBRs. Hence, this circuit can be viewed as associated with the weak-grid mode. For the ideal scenario with two identical IBRs and a symmetric topology, it can be seen that the inter-IBR mode is not associated with the grid voltage. Further, varying grid strength or the grid impedance \(X_{lg}\) will influence the weak-grid mode, but not the inter-IBR mode.

Remarks: The above decomposition analysis can be extended to an \(n\)-IBR parallel connection system and leads to \(n\) modes manifested in \(n\) circuits. While one mode is the aggregated weak-grid mode, the rest modes are inter-IBR modes.

a) Non-ideal scenario: In reality, even two IBRs from the same manufacturer may have slight difference. Also the line impedances cannot be exactly same. Consider a scenario that the IBRs are identical while the line impedances are not the same. The line impedances are different: \(X_{l1} = 0.1, X_{l2} = 0.2,\) and \(X_{lg} = 0.5.\)

The three-node system will be found from Kron reduction of the original four-node system. After eigenvalue decomposition of the \(2 \times 2\) admittance matrix, the resulting two circuits can be expressed as follows.

\[
\begin{bmatrix}
-0.5I_1 + 0.4I_2 \\
0.4I_1 + 0.5I_2
\end{bmatrix} = \begin{bmatrix}
\frac{0.128}{1 + j278} & 0 \\
0 & \frac{0.128}{j278}
\end{bmatrix} \begin{bmatrix}
-V_1 + 0.4V_2 \\
0.4V_1 + 0.5V_2
\end{bmatrix} + \begin{bmatrix}
\frac{0.064}{1 + j278} \\
0
\end{bmatrix} V_g.
\]

(4)

Based on (4), it can be clearly seen that the first circuit reflects the relationship between the weighted difference of two IBRs’ current/voltage, while the second circuit reflects the relationship between the weighted sum of the two IBRs’ current/voltage. Thus, the first circuit will manifest the inter-IBR mode while the second circuit will manifest the aggregated weak-grid mode.

Remarks: When the line impedances are different, the inter-IBR mode is also influenced by the grid voltage and the grid impedance \(X_{lg}\). Through a much less degree compared to the weak-grid mode. This explains the effect of topology on inter-IBR mode’s response to a varying grid impedance as shown in the eigenvalue loci in Fig. 2a and Fig. 2b.

IV. SIMULATION RESULTS

Time-domain simulation is conducted using the analytical model built in MATLAB/Simulink. At the initial condition, the two IBRs are sending out the same level of power with a total of \(0.935\) pu injected to the grid. The transmission line is assumed to have a reactance of \(0.20\) pu.

Two events are triggered in Case 1. The first event is grid voltage dip. The second event is IBR1’s dc-link voltage order increase. The simulation results for grid voltage dip are shown in Fig. 5a. It can be seen that the two IBRs behave exactly the same. This indicates that only the weak-grid mode is excited. Fig. 5b presents the simulation results for a dc-link voltage order increase in IBR1. This event excited both modes. In the real power measurements, the inter-IBR mode has a higher observability index compared to the weak-grid mode according to Table II. Thus, it can be seen that in the real power measurement, IBR1 is oscillating against IBR2 in a second after the event.
Comparison of identical versus nonidentical line impedances is presented in Fig. 6 for an event of grid strength reduction followed up by IBRs power ramping down to their halves. It can be seen that for Case 1 when both IBRs and line impedances are identical, the two IBRs have the same dynamic performance, indicating that the oscillations are associated to the weak-grid mode. This also indicates that $X_{lG}$ change and ramping down the same level of power does not excite the inter-IBR mode.

For Case 2, the system has an unstable oscillation after $X_{lG}$ changes to 0.55 p.u.. This is due to the dominant weak-grid mode moving to the RHP, as shown in the eigenvalue loci in Fig. 2b. After power ramping down, the system becomes stable. For this case with nonidentical line impedances, both the weak-grid mode and the inter-IBR mode are influenced and excited by change in the grid strength and power levels. The inter-IBR mode can be clearly seen in the real power measurement after 2.5 s: IBR1 is oscillating against IBR 2.

V. CONCLUSION

This letter investigates the characteristics of oscillation modes in a weak grid with multi-IBRs. The analysis shows that there are two types of oscillation modes: weak-grid modes and inter-IBR modes. The weak-grid mode can be viewed as the interaction of the aggregated IBR with the grid. The inter-IBR mode can be viewed as the interaction among IBRs. Compared to the weak-grid mode, the inter-IBR model is less sensitive to grid strength.

REFERENCES