Dynamic Parameter Estimation Based on Rank-Reduced Prony Analysis

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***Abstract*— This paper presents a new least squares estimation (LSE)-based dynamic parameter estimation technique using phasor measurement unit (PMU) data. Generator parameters such as inertia constant, damping coefficients, and regulation speed constant are estimated from captured measurements during transient events. The key idea of this dynamic parameter estimation is based on unknown model structure and reduced-order model. This approach depends on measurement-based methods for ringdown signals. A rank-reduced Prony analysis is employed to accurately identify the system eigenvalues with reduced-order model. Then an optimization problem is formulated to obtain the system matrix and estimate the dynamic parameters. Sensitivity analysis is performed to the optimization problem to find the best parameter estimates.**

***Index Terms*— Dynamic parameter estimation, least squares estimation, phasor measurement unit, Prony analysis**

I. INTRODUCTION

Simulation accuracy can improve the safety and efficiency of the operation of power systems by providing the security margins and transfer limits [1]. Using PMU data and measurement-based methods of the system identification can give dynamic parameter estimation. In practical, the dynamic parameters provided by the manufacturer can be inaccurate due to several reasons such as aging, repairs, or unrecorded gain settings [2]. These inaccurate parameters can cause a deviation in the simulated and actual dynamic response. Therefore, power utilities rely on the dynamic parameter estimation to provide accurate estimation of the generators and enhance the security of power systems.

This paper deals with the dynamic parameter estimation using the PMU data. Dynamic parameter estimation has been investigated in the literature. Based on data, time-domain data based are examined in [3]–[10], while the frequency response data based are discussed in [11]–[13]. Based on measurements, parameter estimation is obtained with methods as digital fault recorder and high sampling interval [7], [9], excitation with step sequence inputs [4], [6], short circuit test methods [3], [8], and offline test approaches [11]–[13]. Based on the parameter type, both electrical and mechanical parameters are estimated in [4], [6], [10], and only electrical parameter estimation is provided in [3], [7]–[9], [11]–[13].

In addition, dynamic parameter estimation can be classified into two categories based on the mathematical estimation approach which are least square estimation and Kalman filter estimation. Parameter estimation is formulated as least squares estimation (LSE) in [3]–[6], [8], [11], [13]. Kalman filter is used to estimate the dynamic parameters in [10], [14]–[16]. LSE-based dynamic parameter identification using discrete autoregression exogenous (ARX) model is presented in [17]. However, many of these techniques assume high order model or known model structure, e.g., [5], [6], [10].

PMU data based estimation is considered as online and time-domain estimation which has the capability of estimating the electrical and mechanical related parameters. The majority of the research in the literature is related to the LSE parameter estimation based on the time-domain or frequency response data. A few research can be found on LSE dynamic parameter estimation based on PMU data identification. PMU data based estimation is limited to state estimation [18] or dynamic state estimation for the second-order mechanical system [15], [19]–[21]. However, those approaches are based on Kalman filter estimation that requires to have a prior information of the transfer function of the power system to formulate the Kalman estimator which may not be available in real applications.

In this paper, a new dynamic parameter estimation technique using rank-reduced Prony analysis methods is proposed. A rank-reduced Prony analysis toolbox has been developed by USF SPS group to identify the system eigenvalues with reduced order [22]. The proposed approach is LSE-based dynamic parameter estimation using PMU data. The model structure is assumed to be unknown, and a reduced-order model is achieved. This proposed technique is solved with two main stages. At Stage 1, the eigenvalues are obtained using rank-reduced Prony analysis which can accurately identify the eigenvalues with reduced-order model. At Stage 2, the A matrix and the system parameters are found by solving a nonlinear optimization problem. The nonlinear optimization problem is solved by MATLAB toolbox YALMIP [23] with IPOPT solver [24].

The remaining sections are organized as follows. Section II analyzes the two-machine power system model. Section III formulates the dynamic parameter estimation. Section IV discuses the parameter estimation case studies and simulation results. Section V concludes the paper.

II. Two-Machine Power System Model Analysis

The two-machine power system model (shown in Fig. 1) and the simplified turbine model (shown in Fig. 2) are used as a reduced system for the dynamic parameter estimation. The positions of the rotor's q-axis of the two machines are represented by δ_1 and δ_2. If we assume that the speed (ω) in per unit, we can get the following [25]:

$$\dot{\delta}_1 = \omega_0(\omega_1 - 1) \tag{1}$$

where $\Delta\omega = \omega_1 - \omega_0$ and ω_0 is a constant nominal speed. Then (1) can be linearized at an equilibrium point as follows.

$$\Delta\dot{\delta}_1 = \omega_0 \Delta\omega_1 \tag{2}$$

Note that the linearization is evaluated at x_0 as the following:

$$\Delta f = \left.\frac{\partial f}{\partial x}\right|_{x_0} \Delta x \tag{3}$$

The swing equation in per unit can be represented as follows [25].

$$\dot{\omega}_1 = \frac{1}{2H_1}(P_{m1} - P_{e1} - D_1\Delta\omega_1) \tag{4}$$

where H_1 is kinetic energy ratio of the rotor and the power base at a constant nominal speed, P_{m1} is mechanical power, P_{e1} is electrical power, and D_1 is damping coefficient.

Fig. 1. Two-machine power system model.

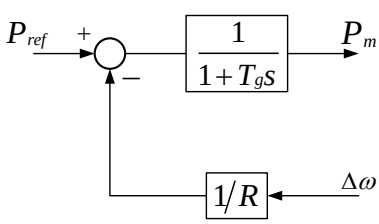


Fig. 2. Turbine model block diagram.

After that, the swing equation (4) is linearized at initial conditions to result in the following [25]:

$$\Delta\dot{\omega}_1 = \frac{1}{2H_1}(\Delta P_{m1} - \Delta P_{e1} - D_1\Delta\omega_1) \tag{5}$$

From the linearized model, the electrical power can be obtained as follows.

$$\Delta P_{e1} = -\Delta P_{e2} = \underbrace{\frac{E_1 E_2}{X}\cos\delta_{12}}_{T}(\Delta\delta_1 - \Delta\delta_2) \tag{6}$$

Then (5) can be expressed as follows.

$$\Delta\dot{\omega}_1 = \frac{1}{2H_1}(\Delta P_{m1} - T(\Delta\delta_1 - \Delta\delta_2) - D_1\Delta\omega_1) \tag{7}$$

From the turbine model as shown in Fig. 2, the mechanical power for Generator 1 can be found as:

$$P_{ref} - \frac{1}{R}\Delta\omega = P_m + T_g\frac{dP_m}{dt} \tag{8}$$

$$\dot{P}_{m1} = \frac{1}{T_{g1}}(P_{1,ref} - \frac{1}{R_1}\Delta\omega_1 - P_{m1}) \tag{9}$$

(9) can be linearized at an equilibrium point as follows.

$$\Delta\dot{P}_{m1} = \frac{1}{T_{g1}}(-\frac{1}{R_1}\Delta\omega_1 - \Delta P_{m1}) \tag{10}$$

Thus, swing equations for Generator 1 are obtained. Similarly, the equations for Generator 2 can be provided. For simplification, δ_2 can be treated as the reference angle, or $\delta_1 - \delta_2$ can be used as state variables to get the following:

$$\begin{cases} \Delta\dot{\delta}_{12} = \omega_0(\Delta\omega_1 - \Delta\omega_2) \\ \Delta\dot{\omega}_1 = \frac{1}{2H_1}(\Delta P_{m1} - T\Delta\delta_{12} - D_1\Delta\omega_1) \\ \Delta\dot{P}_{m1} = \frac{1}{T_{g1}}(-\frac{1}{R_1}\Delta\omega_1 - \Delta P_{m1}) \\ \Delta\dot{\omega}_2 = \frac{1}{2H_2}(\Delta P_{m2} + T\Delta\delta_{12} - D_2\Delta\omega_2) \\ \Delta\dot{P}_{m2} = \frac{1}{T_{g2}}(-\frac{1}{R_2}\Delta\omega_2 - \Delta P_{m2}) \end{cases} \tag{11}$$

Therefore, the A matrix can be obtained as follows.

$$\dot{x} = \underbrace{\left[\begin{array}{ccc:cc} 0 & \omega_0 & 0 & -\omega_0 & 0 \\ -\frac{T}{2H_1} & -\frac{D_1}{2H_1} & \frac{1}{2H_1} & 0 & 0 \\ 0 & -\frac{1}{T_{g1}R_1} & -\frac{1}{T_{g1}} & 0 & 0 \\ \hdashline \frac{T}{2H_2} & 0 & 0 & -\frac{D_2}{2H_2} & \frac{1}{2H_2} \\ 0 & 0 & 0 & -\frac{1}{T_{g2}R_2} & -\frac{1}{T_{g2}} \end{array}\right]}_{A} \underbrace{\begin{bmatrix} \Delta\delta_{12} \\ \Delta\omega_1 \\ \Delta P_{m1} \\ \Delta\omega_2 \\ \Delta P_{m2} \end{bmatrix}}_{x} \tag{12}$$

III. Formulation of the Dynamic Parameter Estimation

The proposed optimization problem can estimate the model structure and parameter from the eigenvalues that obtained from rank-reduced Prony analysis. A single Hankel matrix is formed from the measurement data. By identifying the real coefficients from the characteristic polynomial function through least square estimation (LSE), the identification of the system eigenvalues can be achieved. It is well known that Prony analysis is sensitive to the order assumption especially if it is low. Therefore, Singular value decomposition (SVD)-based rank reduction technique is applied to the Prony Hankel matrix. Further, an eigenvalue reduction is performed to accurately identify the system eigenvalues. A toolbox for rank-reduced Prony analysis has been developed in [22] and used in this proposed approach.

The model has nine parameters which are H_1, H_2, T, D_1, D_2, T_{g1}, T_{g2}, R_1, and R_2. Thus, the optimization problem can be formulated as the following:

$$\min \quad \sum_{i=1}^{n} e_i^2 \tag{13a}$$

$$\text{s.t.:} \quad \det(\lambda_i I - A) + e_i = 0, \quad i = 1, 2, \cdots, n. \tag{13b}$$

$$H_1 \geq 0, H_2 \geq 0, T \geq 0, D_1 \geq 0, \tag{13c}$$

$$D_2 \geq 0, T_{g1} \geq 0, T_{g2} \geq 0, \tag{13d}$$

$$R_1 \geq 0, R_2 \geq 0. \tag{13e}$$

where n is the number of the system order. If four of the parameters are fixed to be true values, the other five parameters can be found. On the other hand, if all parameters are unknown, the solution can also be found with prior information of multiple parameters range.

This is a nonlinear optimization problem which is solved by MATLAB toolbox YALMIP [23] with IPOPT solver [24]. The unknown parameters of the A matrix are preferred to be on the numerator to avoid zero initial conditions, then the estimated values of H, T_g, and R parameters will be inversed. Therefore, the A matrix is re-written as follows.

$$\left[\begin{array}{ccc:cc} 0 & \omega_0 & 0 & -\omega_0 & 0 \\ -\frac{TH_1'}{2} & -\frac{D_1 H_1'}{2} & \frac{H_1'}{2} & 0 & 0 \\ 0 & -T_{g1}' R_1' & -T_{g1}' & 0 & 0 \\ \hdashline \frac{TH_2'}{2} & 0 & 0 & -\frac{D_2 H_2'}{2} & \frac{H_2'}{2} \\ 0 & 0 & 0 & -T_{g2}' R_2' & -T_{g2}' \end{array}\right] \tag{14}$$

IV. Case Studies and Simulation Results

The proposed dynamic parameter estimation is applied to two systems: a simulated two-area two-machine system and the two-area four-machine case system. The measurement data are generated using the power system toolbox (PST) [26]. The nonlinear optimization problem is solved using MATLAB toolbox YALMIP [23] with IPOPT solver [24].

A. Two-Area Two-Machine System

A two-area two-machine system as shown in Fig. 3 is simulated using power system toolbox (PST) [26]. A three-phase fault is applied on Line 1-2 at Bus 1 side. The fault is cleared in 0.01 s, and the line is tripped after 0.05 s. Dynamic simulation data after the line tripping are used for eigenvalue identification. Voltage signals of Bus 1 and Bus 3 are chosen for eigenvalue identification and signal reconstruction. Those two signals are resampled with sampling time interval of 0.02 s. The time periods for the simulation data are 10 s.

Rank reduced Prony analysis is applied for identifying the system eigenvalues. The system order is assumed to be 5 with two pairs of complex conjugate eigenvalues and a real one. The reconstructed signals of Prony analysis are shown in Fig. 4. It can be seen that the rank reduced Prony analysis can provide accurate reconstructed signals with reduced order model. The eigenvalues resulted from Prony analysis are listed in Table I.

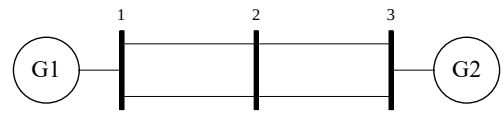


Fig. 3. Two-area two-machine system simulated in PST.

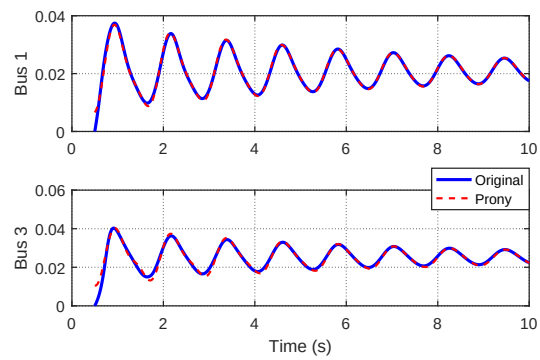


Fig. 4. Prony analysis reconstructed signals for the two machine system.

TABLE I
Two-Machine System Prony Eigenvalues

Prony Eigenvalues
-5.7129×10^{-5}
$-0.13364 + 5.1774i$
$-0.13364 - 5.1774i$
$-0.31111 + 10.359i$
$-0.31111 - 10.359i$

Then the constraints for the nine parameters are added with the prior information of the system. The feasible region can be reduced with adding an acceptable range for some parameters. Also, it is assumed that $H_1' = H_2'$ and $D_1 = D_2$ since the two generators have the same size and damping coefficient. Thus, eleven constraints are added to the optimization problem as follows.

$$\begin{aligned} \min \quad & \sum_{i=1}^{5} e_i^2 \\ \text{s.t.:} \quad & \det(\lambda_i I - A) + e_i = 0, \quad i = 1, 2, \cdots, 5. \\ & H_1' \geq 0, \quad H_2' \geq 0, \\ & 20 \geq T \geq 1.6, \\ & 100 \geq D_1 \geq 1, \quad 100 \geq D_2 \geq 1, \\ & T_{g1}' \geq 0, \quad T_{g2}' \geq 0, \\ & 100 \geq R_1' \geq 1, \quad 100 \geq R_2' \geq 1, \\ & H_1' = H_2', \quad D_1 = D_2. \end{aligned}$$

The above nonlinear optimization problem is solved using MATLAB toolbox YALMIP [23] with IPOPT solver [24]. The actual values and estimated values of the two-area

two-machine system parameters are shown in Table II. The number of iterations in this optimization problem is 196, while the overall nonlinear problem error is 1.18×10^{-9}.

TABLE II
TWO-MACHINE SYSTEM PARAMETER ESTIMATION RESULT

Parameter	Actual Value	Estimated Value
H_1	6.5	6.5569
D_1	6	3.5271
T_{g1}	0.5	0.49294
R_1	0.04	0.01
H_2	6.5	6.5569
D_2	6	3.5271
T_{g2}	0.5	0.49294
R_2	0.04	0.01
T	2	1.6

Sensitivity Analysis: Parameter estimation can be improved by having some of the parameters fixed or reducing the feasible range with prior information of the system. For example, if the lower bound of parameter T is set to be 2 ($20 \geq T \geq 2$), this can result in a better estimation. In this case, the reduced feasible regions of D_1 and D_2 are not required (i.e., $D_1 \geq 0$ and $D_2 \geq 0$). Therefore, only three feasible regions of T, R_1', and R_2' are reduced by an acceptable range with prior information of the system. Table III shows the result of the parameter estimation with 2 as a lower bound of parameter T feasible region. The number of iterations and the overall nonlinear problem error are 276 and 5.91×10^{-12}, respectively.

In addition, the estimation result can also be more accurate when the lower bounds of parameters D_1 and D_2 are set to be 6 ($100 \geq D_1 \geq 6$ and $100 \geq D_2 \geq 6$) as can be seen from Table IV. With those two constraints, the constraint $D_1 = D_2$ will be no longer needed. Therefore, the number of constraints is reduced to 10 instead of 11. The resulted number of iterations in this case is 107, and the nonlinear problem error is 1.24×10^{-8}. Hence, it can be seen that the dynamic parameter estimation using the rank-reduced Prony analysis can give an accurate estimation.

TABLE III
ESTIMATION WITH DIFFERENT LOWER BOUND OF T

Parameter	Actual Value	Estimated Value
H_1	6.5	7.9835
D_1	6	5.1725
T_{g1}	0.5	0.48802
R_1	0.04	0.01
H_2	6.5	7.9835
D_2	6	5.1725
T_{g2}	0.5	0.48802
R_2	0.04	0.01
T	2	2.00

B. Two-Area Four-Machine System

The classic two-area four-machine system as shown in Fig. 5 is used to estimate the two-area two-machine system parameters to achieve the reduced order model. Each generator is assumed to have a second-order swing dynamic. Ten seconds simulation is conducted for a short-circuit transient

TABLE IV
ESTIMATION WITH DIFFERENT LOWER BOUNDS OF DAMPING COEFFIECIENTS

Parameter	Actual Value	Estimated Value
H_1	6.5	6.4665
D_1	6	6.00
T_{g1}	0.5	0.55315
R_1	0.04	0.01
H_2	6.5	6.4665
D_2	6	6.00
T_{g2}	0.5	0.55315
R_2	0.04	0.01
T	2	1.6

event. The measurements are resampled with 0.02 s sampling rate to have equal time steps. Two voltage signals of Bus 13 and Bus 101 are selected to identify the system eigenvalues using rank-reduced Prony analysis. The number of samples is $N = 458$ for each measurement data. The Hankel matrix width for the Prony analysis is set to be 110. Two case studies related to the system order assumption are examined. In Case 1, the number of the order is selected to be 5, while the number of the system order is 8 in Case 2. The reconstructed signals of the Prony analysis for the two cases are shown in Fig. 6 and Fig. 7. Table V shows the resulted eigenvalues from Prony analysis of this system for Case 1 and Case 2.

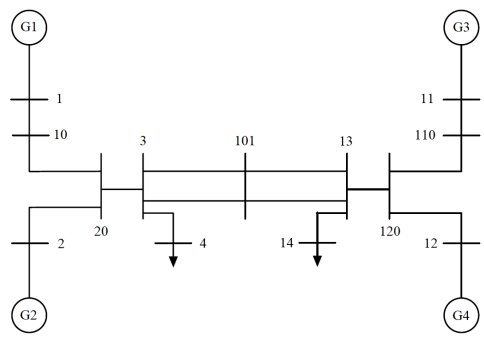


Fig. 5. The two-area four-machine test case in PST.

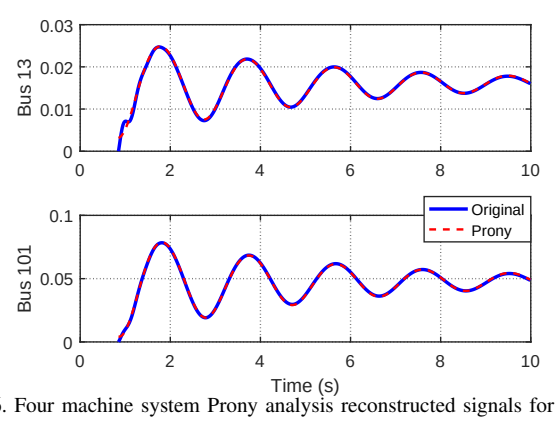


Fig. 6. Four machine system Prony analysis reconstructed signals for Case 1 (Order is 5).

Since the resulted eigenvalues are eight in Case 2, only five eigenvalues should be chosen to estimate the two-machine system. The assumption is to choose the smallest two pairs

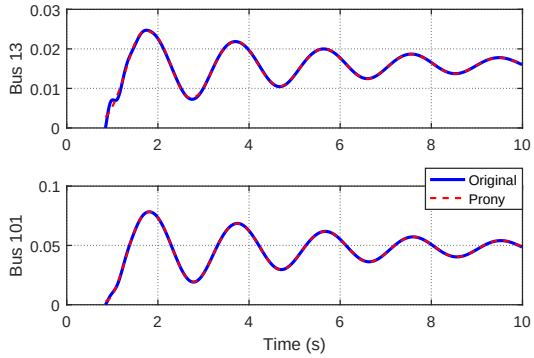


Fig. 7. Four machine system Prony analysis reconstructed signals for Case 2 (Order is 8).

TABLE V
FOUR-MACHINE SYSTEM PRONY EIGENVALUES FOR THE TWO CASES

Case 1 Prony Eigenvalues	Case 2 Prony Eigenvalues
-0.004562	-0.00189
$-0.21683 \pm 3.263i$	$-0.21894 \pm 3.2603i$
$-0.33351 \pm 6.614i$	$-0.34378 \pm 6.5680i$
	$-0.52516 \pm 10.471i$
	-0.29177

of the complex conjugate eigenvalues. It should be noted that the real eigenvalue has also to be the smallest one. On the other hand, the largest real or pair of complex conjugate eigenvalue may lead to an inaccurate parameter estimation.

From the 2-area 4-machine data, the reduced order model can be achieved to estimate the 2-area 2-machine dynamic parameters. Each generator of the four-machine system has the same size, damping coefficient, and turbine government of the two-machine system. Therefore, the expected estimation of the machine is the aggregation of the two machines. Then the optimization problem is formulated with eleven constraints related to the machine parameters and specifications. Sensitivity analysis is performed to provide the best dynamic estimates by reducing the feasible regions of some parameters with prior information of the system. The lower bound of parameter T is set to be 1.4, while the upper bounds and lower bounds of the parameters R_1' and R_2' are 150 and 1, respectively. Therefore, the nonlinear optimization problem is formulated as the following:

$$
\begin{aligned}
\min \quad & \sum_{i=1}^{5} e_i^2 \\
\text{s.t.:} \quad & \det(\lambda_i I - A) + e_i = 0, \quad i = 1, 2, \cdots, 5. \\
& H_1' \geq 0, \quad H_2' \geq 0, \\
& 20 \geq T \geq 1.4, \\
& D_1 \geq 0, \quad D_2 \geq 0, \\
& T_{g1}' \geq 0, \quad T_{g2}' \geq 0, \\
& 150 \geq R_1' \geq 1, \quad 150 \geq R_2' \geq 1, \\
& H_1' = H_2', \quad D_1 = D_2.
\end{aligned}
$$

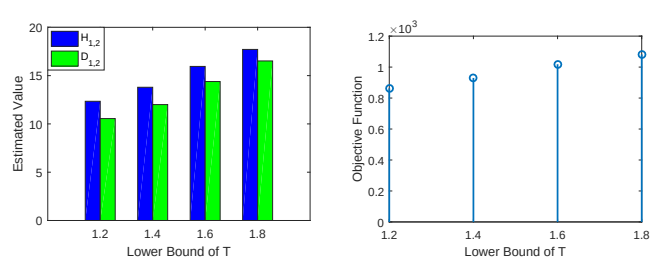
The estimated values of the two-machine system parameters from the four-machine system data are shown in Table VI. It can be seen that the proposed technique can provide an accurate parameter estimation with reduced orders. Each machine is resulted in the size of the two machines due to the estimation of the reduced system. Hence, the reduced order model and the two-machine system parameters can be estimated using the four-machine system data. The number of iterations in Case 1 is 129, whereas the overall nonlinear problem error is 1.40×10^{-10}. On the other hand, the number of iterations and overall nonlinear problem error in Case 2 are 226 and 2.01×10^{-11}, respectively. For further validation, the estimated system eigenvalues are compared to the Prony eigenvalues as shown in Table VII. For the two cases, the estimated system eigenvalues can give an accurate two complex conjugate eigenvalues and another approximate two. It should be noted that the parameter T can be sensitive to the estimation of parameters H and D which can increase the objective function. Fig. 8 and Fig. 9 show the estimated values of $H_{1,2}$ and $D_{1,2}$ with different lower bounds of T and its effect on the optimization objective function for Case 1 and Case 2, respectively. As a final step, the estimated system is simulated in PST and compared to the measurements as shown in Fig. 10. It can be seen that the estimated system has the ability to obtain a well estimation to the measurements. Thus, the proposed parameter estimation can provide superior results for the reduced systems.

TABLE VI
PARAMETER ESTIMATION FROM THE FOUR-MACHINE DATA

	$H_{1,2}$	$D_{1,2}$	$T_{g1,2}$	$R_{1,2}$	T
Case 1	13.794	12.00	0.81061	0.00667	1.4
Case 2	13.988	12.71	0.81096	0.00667	1.4

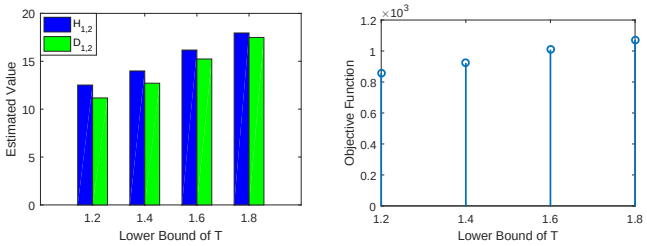
TABLE VII
PRONY EIGENVALUES COMPARED TO THE ESTIMATED SYSTEM EIGENVALUES

Case	Prony Eigenvalues	Estimated System Eigenvalues
1	-0.004562	-1.0477
	$-0.21683 \pm 3.263i$	$-0.83431 \pm 2.5589i$
	$-0.33351 \pm 6.614i$	$-0.31048 \pm 6.6060i$
2	-0.00189	-1.0472
	$-0.21894 \pm 3.2603i$	$-0.84372 \pm 2.5417i$
	$-0.34378 \pm 6.5680i$	$-0.32010 \pm 6.5594i$



(a) (b)

Fig. 8. Parameter T sensitivity for Case 1. (a) with estimated $H_{1,2}$ and $D_{1,2}$. (b) with the objective function.



(a) (b)

Fig. 9. Parameter T sensitivity for Case 2. (a) with estimated $H_{1,2}$ and $D_{1,2}$. (b) with the objective function.

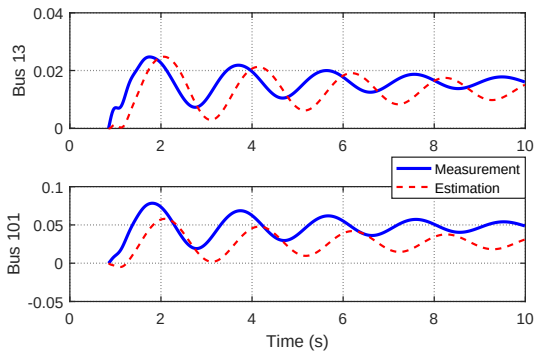


Fig. 10. Estimated system versus the measurements.

V. CONCLUSION

A new dynamic parameter estimation based on LSE using PMU data is presented. A rank reduced Prony analysis is employed to accurately identify the system eigenvalues for a reduced order model. An optimization problem is formulated to estimate the generator parameters such as inertia constant, damping coefficients, and regulation speed constant. Sensitivity analysis is implemented to find the best dynamic estimates. This new technique can estimate the dynamic parameters with a reduced-order model and without a requirement of known model structure. It can provide superior estimated results using rank-reduced Prony analysis.

REFERENCES

[1] E. P. T. Cari and L. F. C. Alberto, "Parameter estimation of synchronous generators from different types of disturbances," in *2011 IEEE Power and Energy Society General Meeting*. IEEE, 2011, pp. 1–7.

[2] M. Shen, V. Venkatasubramanian, N. Abi-Samra, and D. Sobajic, "A new framework for estimation of generator dynamic parameters," *IEEE Transactions on Power Systems*, vol. 15, no. 2, pp. 756–763, 2000.

[3] J. Sanchez-Gasca, C. Bridenbaugh, C. Bowler, and J. Edmonds, "Trajectory sensitivity based identification of synchronous generator and excitation system parameters," *IEEE Transactions on Power Systems*, vol. 3, no. 4, pp. 1814–1822, 1988.

[4] Z. Zhao, F. Zheng, J. Gao, and L. Xu, "A dynamic on-line parameter identification and full-scale system experimental verification for large synchronous machines," *IEEE Transactions on Energy Conversion*, vol. 10, no. 3, pp. 392–398, 1995.

[5] M. Burth, G. C. Verghese, and M. Vélez-Reyes, "Subset selection for improved parameter estimation in on-line identification of a synchronous generator," *IEEE Transactions on Power Systems*, vol. 14, no. 1, pp. 218–225, 1999.

[6] M. Karrari and O. Malik, "Identification of physical parameters of a synchronous generator from online measurements," *IEEE transactions on energy conversion*, vol. 19, no. 2, pp. 407–415, 2004.

[7] E. Kyriakides, G. T. Heydt, and V. Vittal, "Online parameter estimation of round rotor synchronous generators including magnetic saturation," *IEEE Transactions on Energy Conversion*, vol. 20, no. 3, pp. 529–537, 2005.

[8] C. Lee and O. T. Tan, "A weighted-least-squares parameter estimator for synchronous machines," *IEEE Transactions on Power Apparatus and Systems*, vol. 96, no. 1, pp. 97–101, 1977.

[9] J. R. Melgoza, G. T. Heydt, A. Keyhani, B. L. Agrawal, and D. Selin, "Synchronous machine parameter estimation using the hartley series," *IEEE transactions on energy conversion*, vol. 16, no. 1, pp. 49–54, 2001.

[10] M. Namba, T. Nishiwaki, S. Yokokawa, and K. Ohtsuka, "Identification of parameters for power system stability analysis using kalman filter," *IEEE Transactions on Power Apparatus and Systems*, no. 7, pp. 3304–3311, 1981.

[11] E. Eitelberg and R. Harley, "Estimating synchronous machine electrical parameters from frequency response tests," *IEEE transactions on energy conversion*, no. 1, pp. 132–138, 1987.

[12] I. Kamwa, P. Viarouge, H. Le-Huy, and E. Dickinson, "A frequency-domain maximum likelihood estimation of synchronous machine high-order models using ssfr test data," *IEEE Transactions on Energy Conversion*, vol. 7, no. 3, pp. 525–536, 1992.

[13] R. Escarela-Perez, T. Niewierowicz, and E. Campero-Littlewood, "Synchronous machine parameters from frequency-response finite-element simulations and genetic algorithms," *IEEE Transactions on Energy Conversion*, vol. 16, no. 2, pp. 198–203, 2001.

[14] E. Ghahremani and I. Kamwa, "Dynamic state estimation in power system by applying the extended kalman filter with unknown inputs to phasor measurements," *IEEE Transactions on Power Systems*, vol. 26, no. 4, pp. 2556–2566, 2011.

[15] L. Fan and Y. Wehbe, "Extended kalman filtering based real-time dynamic state and parameter estimation using pmu data," *Electric Power Systems Research*, vol. 103, pp. 168–177, 2013.

[16] M. Ariff, B. Pal, and A. K. Singh, "Estimating dynamic model parameters for adaptive protection and control in power system," *IEEE Transactions on Power Systems*, vol. 30, no. 2, pp. 829–839, 2015.

[17] B. Mogharbel, L. Fan, and Z. Miao, "Least squares estimation-based synchronous generator parameter estimation using pmu data," in *2015 IEEE Power & Energy Society General Meeting*. IEEE, 2015, pp. 1–5.

[18] E. Ghahremani and I. Kamwa, "Online state estimation of a synchronous generator using unscented kalman filter from phasor measurements units," *IEEE Transactions on Energy Conversion*, vol. 26, no. 4, pp. 1099–1108, 2011.

[19] Z. Huang, K. Schneider, and J. Nieplocha, "Feasibility studies of applying kalman filter techniques to power system dynamic state estimation," in *2007 International Power Engineering Conference (IPEC 2007)*. IEEE, 2007, pp. 376–382.

[20] K. Kalsi, Y. Sun, Z. Huang, P. Du, R. Diao, K. K. Anderson, Y. Li, and B. Lee, "Calibrating multi-machine power system parameters with the extended kalman filter," in *2011 IEEE Power and Energy Society General Meeting*. IEEE, 2011, pp. 1–8.

[21] Z. Huang, P. Du, D. Kosterev, and B. Yang, "Application of extended kalman filter techniques for dynamic model parameter calibration," in *2009 IEEE Power & Energy Society General Meeting*. IEEE, 2009, pp. 1–8.

[22] A. Almunif, L. Fan, and Z. Miao, "A tutorial on data-driven eigenvalue identification: Prony analysis, matrix pencil and eigensystem realization algorithm," *submitted, Electric Power Systems Research*, 2019.

[23] J. Löfberg, "Yalmip : A toolbox for modeling and optimization in matlab," in *In Proceedings of the CACSD Conference*, Taipei, Taiwan, 2004.

[24] A. Wächter and L. T. Biegler, "On the implementation of an interior-point filter line-search algorithm for large-scale nonlinear programming," *Mathematical programming*, vol. 106, no. 1, pp. 25–57, 2006.

[25] L. Fan, *Control and dynamics in power systems and microgrids*. CRC Press, 2017.

[26] J. H. Chow and K. W. Cheung, "A toolbox for power system dynamics and control engineering education and research," *IEEE transactions on Power Systems*, vol. 7, no. 4, pp. 1559–1564, 1992.