Day-Ahead Distribution Market Analysis Via Convex Bilevel Programming

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Outline

- Introduction
- Bilevel Programming Problem Formulation And Solving Strategy
  - The Bilevel Problem Formulation
  - The Solution Strategy
- Case Study
- Conclusion

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Introduction

- In the past, the distribution system was quite passive that is limited to serving the load.

- Recently, the system characteristics have begun to vary with the advent of distributed energy resource (DER) technologies:
  - Microgrids (MGs).
  - Residential consumers with rooftop photovoltaic panels (PVs).
  - Distributed storage (DS).
  - Even fleets of electric vehicles (EVs).

Introduction

- The DER contribution to the system:
  - Load profile peak shaving and valley filling.
  - Ancillary services.
  - Voltage support.
  - Investment deferral.

- The main operation challenges:
  - Voltage fluctuation.
  - Supply-demand imbalance.

- The distribution market ensures reliable operation.

Introduction

- In the US, distribution market policies are under reforming [5].
- California state has already initiated plans regarding DER participation [6].
- Other states as Massachusetts, Hawaii and Minnesota are preparing the layout [7].

Introduction

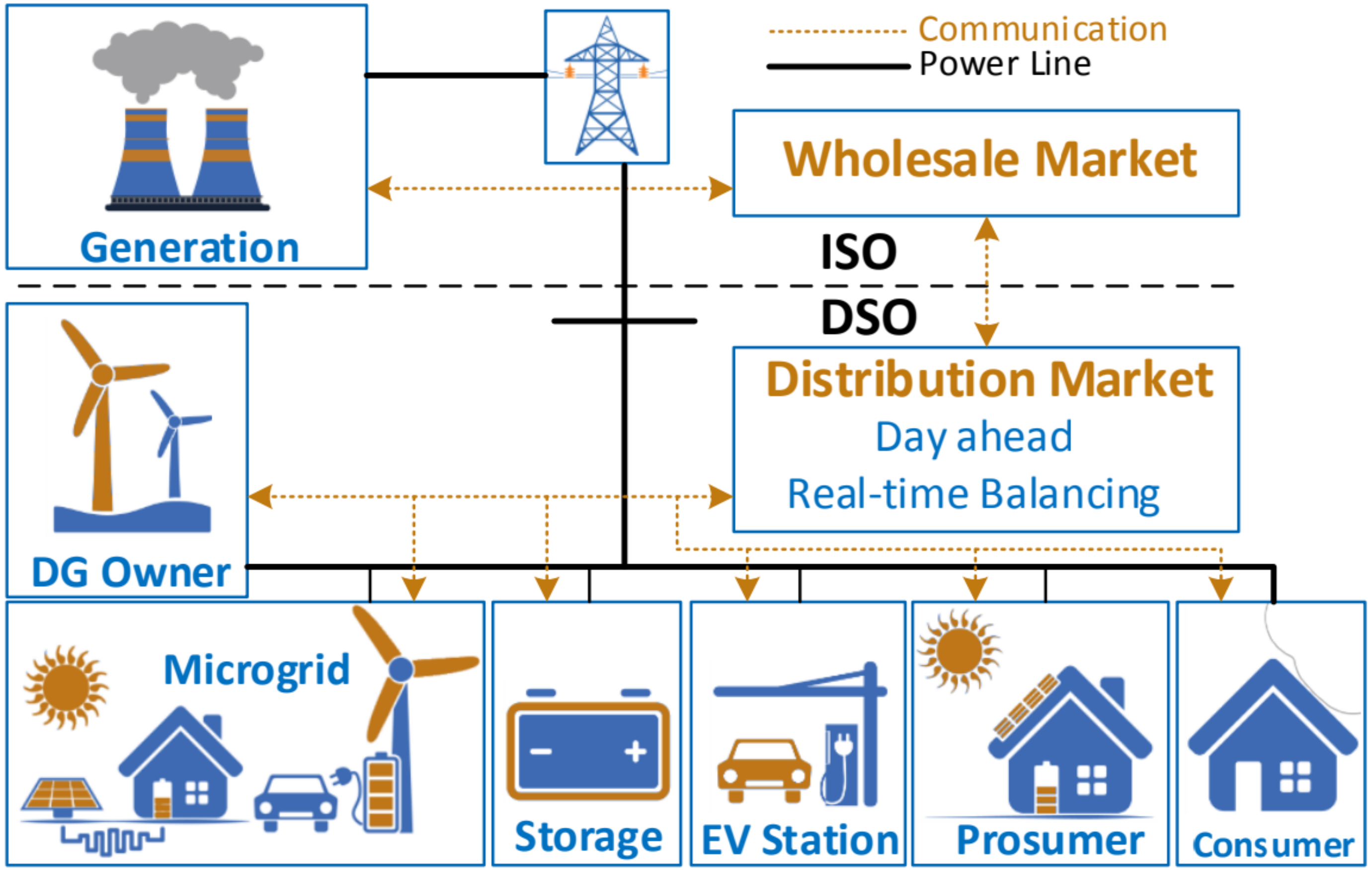

Fig. 1. Power flow and data exchange between DSO and DERs.

Introduction

- In the distribution system, the DSO assumes the role of the ISO by receiving the prosumer’s bids, and clearing the market.

- The increased DER adoption justifies the use of unit commitment for the economic dispatch.

- Since the locational marginal pricing (LMP) is an effective, it can therefore be extended to the distribution market.

Introduction

- The LMP is attained from DCOPF, which ignores the voltage and reactive power.
- The DCOPF approximation is unsuitable for the distribution system.
- The Distribution locational marginal pricing (DLMP) takes into account the voltage and reactive power.
- Our methodology adopts the SOCP convex relaxation, which provides AC-OPF solution.

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The Bilevel Problem Formulation

$$\min_{u \in \mathcal{U}} \sum_{k \in J} \sum_{t \in T} C^g_{\text{fixed},k} u^g_{kt} + \min_{w \in \mathcal{W}} \sum_{k \in J} \sum_{t \in T} C^g_{pk} p^g_{kt} + C^g_{qk} q^g_{kt} \tag{1a}$$

subject to

$$\mathcal{U} = \left\{ u^g_{kt} \in \{0,1\}; \forall k \in \mathcal{N}^g \right\} \tag{1b}$$

$$\mathcal{W} = \Big\{ \underline{P}^g_k u^g_{kt} \leq p^g_{kt} \leq \overline{P}^g_k u^g_{kt} : (A^{\min}_{kt}, A^{\max}_{kt}); \forall k \in \mathcal{N}^g \tag{1c}$$

$$\underline{Q}^g_k p^g_{kt} \leq q^g_{kt} \leq \overline{Q}^g_k p^g_{kt} : (R^{\min}_{kt}, R^{\max}_{kt}); \forall k \in \mathcal{N}^g \tag{1d}$$

$$p^g_{kt} - P^d_{it} = G_{ii} e_{it} + \sum_{j=1, j \neq i} (G_{ij} c_{ijt} - B_{ij} s_{ijt}) : (\lambda^p_{it}); \forall i \in \mathcal{N} \tag{1e}$$

$$q^g_{kt} - Q^d_{it} = -B_{ii} e_{it} - \sum_{j=1, j \neq i} (B_{ij} c_{ijt} + G_{ij} s_{ijt}) : (\lambda^q_{it}); \forall i \in \mathcal{N} \tag{1f}$$

The Bilevel Problem Formulation

$$\underline{V}_i^2 \le e_{it} \le \overline{V}_i^2 : (X_{it}^{\min}, X_{it}^{\max}); \forall i \in \mathcal{N} \quad (1g)$$

$$D_{ijt}^1 = 2c_{ijt} : (\alpha_{ijt}); \forall (i,j) \in \mathcal{L}, \quad (1h)$$

$$D_{ijt}^2 = 2s_{ijt} : (\beta_{ijt}); \forall (i,j) \in \mathcal{L} \quad (1i)$$

$$D_{ijt}^3 = e_{it} - e_{jt} : (\phi_{ijt}); \forall (i,j) \in \mathcal{L} \quad (1j)$$

$$D_{ijt}^4 = e_{it} + e_{jt} : (\theta_{ijt}); \forall (i,j) \in \mathcal{L} \quad (1k)$$

$$(D_{ijt}^1)^2 + (D_{ijt}^2)^2 + (D_{ijt}^3)^2 \le (D_{ijt}^4)^2; \forall (i,j) \in \mathcal{L}\Big\} \quad (1l)$$

- Where

$$e_{it} = V_{it}^2,$$
$$c_{ijt} = V_{it} V_{jt} \cos \angle \overline{V}_{it} - \overline{V}_{jt},$$
$$s_{ijt} = V_{it} V_{jt} \sin \angle \overline{V}_{it} - \overline{V}_{jt}.$$

The Solution Strategy

- The bilevel problem is solved by merging the lower- level problem with the upper level one in order to have a single-equivalent model.

- This is approached by replacing the lower level problem by its primal and dual constraints with an additional constraint ensuring an equal primal-dual objective.

The Lower-Level Dual Formulation

$$\max_{z\in\mathcal{Z}} \sum_{i\in\mathcal{N}}\sum_{t\in T} \lambda^p_{it}P^d_{kt} + \lambda^q_{it}Q^d_{kt} + \underline{V}_i^2 X^{\min}_{it} - \overline{V}_i^2 X^{\max}_{it} + \sum_{k\in\mathcal{N}^g}\sum_{t\in T}(\underline{P}^g_k A^{\min}_{kt}u^g_{kt} - \overline{P}^g_k A^{\max}_{kt}u^g_{kt} + \underline{Q}^g_k R^{\min}_{kt} - \overline{Q}^g_k R^{\max}_{kt}) \tag{2a}$$

subject to

$$A^{\min}_{kt} - A^{\max}_{kt} + \lambda^p_{it} = C^g_k; \forall k\in\mathcal{N}^g \tag{2b}$$

$$R^{\min}_{kt} - R^{\max}_{kt} + \lambda^q_{it} = C^g_k; \forall k\in\mathcal{N}^g \tag{2c}$$

$$X^{\min}_{it} - X^{\max}_{it} - G_{ii}\lambda^p_{it} + B_{ii}\lambda^q_{it} + \phi_{ijt} + \theta_{ijt} = 0; \forall i\in\mathcal{N} \tag{2d}$$

$$-G_{ij}\lambda^p_{it} + B_{ij}\lambda^q_{it} + 2\alpha_{ijt} = 0; \forall ij\in\mathcal{L} \tag{2e}$$

$$B_{ij}\lambda^p_{it} + G_{ij}\lambda^q_{it} + 2\beta_{ijt} = 0; \forall ij\in\mathcal{L} \tag{2f}$$

$$(\alpha_{ijt})^2 + (\beta_{ijt})^2 + (\phi_{ijt})^2 \le (\theta_{ijt})^2; \forall ij\in\mathcal{L} \tag{2g}$$

$$\lambda^p_{it}, \lambda^q_{it}, X^{\min,\max}_{it} \ge 0; \forall i\in\mathcal{N} \tag{2h}$$

$$A^{\min}_{kt}, A^{\max}_{kt}, R^{\min}_{kt}, R^{\max}_{kt} \ge 0; \forall k\in\mathcal{N}^g \tag{2i}$$

$$\alpha_{ijt}, \beta_{ijt}, \theta_{ijt} \ge 0; \forall ij\in\mathcal{L} \tag{2j}$$

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The Single-Equivalent Model

$$
\begin{aligned}
\max_{j\in\mathcal{J}} \sum_{i\in\mathcal{N}}\sum_{t\in T} & \lambda^p_{it}P^d_{kt} + \lambda^q_{it}Q^d_{kt} + \underline{V}_i^2 X^{\min}_{it} - \overline{V}_i^2 X^{\max}_{it} \\
+ \sum_{k\in\mathcal{N}^g}\sum_{t\in T} & (\underline{P}^g_k A^{\min}_{kt} u^g_{kt} - \overline{P}^g_k A^{\max}_{kt} u^g_{kt} + \underline{Q}^g_k R^{\min}_{kt} \qquad \text{(3a)} \\
& - \overline{Q}^g_k R^{\max}_{kt}) - \sum_{k\in J}\sum_{t\in T} C^g_{\text{fixed},k} u^g_{kt}
\end{aligned}
$$

subject to

$$
\begin{aligned}
&\text{Constraints} \quad (1b) && \text{(3b)} \\
&\text{Constraints} \quad (1c)-(1l) && \text{(3c)} \\
&\text{Constraints} \quad (2b)-(2j) && \text{(3d)}
\end{aligned}
$$

$$
\begin{aligned}
\sum_{i\in\mathcal{N}}\sum_{t\in T} & \lambda^p_{it}P^d_{kt} + \lambda^q_{it}Q^d_{kt} + \underline{V}_i^2 X^{\min}_{it} - \overline{V}_i^2 X^{\max}_{it} \\
+ \sum_{k\in\mathcal{N}^g}\sum_{t\in T} & (\underline{P}^g_k A^{\min}_{kt} u^g_{kt} - \overline{P}^g_k A^{\max}_{kt} u^g_{kt} + \underline{Q}^g_k R^{\min}_{kt} \qquad \text{(3e)} \\
& - \overline{Q}^g_k R^{\max}_{kt}) = \sum_{k\in J}\sum_{t\in T} C^g_{pk} p^g_{kt} + C^g_{qk} q^g_{kt}
\end{aligned}
$$

The Linearized Single-Equivalent Model

$$\max_{j\in\mathcal{J}} \sum_{i\in\mathcal{N}} \sum_{t\in T} \lambda^p_{it} P^d_{kt} + \lambda^q_{it} Q^d_{kt} + \underline{V}^2_i X^{\min}_{it} - \overline{V}^2_i X^{\max}_{it}$$
$$+ \sum_{k\in\mathcal{N}^g} \sum_{t\in T} (\underline{P}^g_k AD^{\min}_{kt} - \overline{P}^g_k AD^{\max}_{kt} + \underline{Q}^g_k R^{\min}_{kt} - \overline{Q}^g_k R^{\max}_{kt})$$
$$- \sum_{k\in J} \sum_{t\in T} C^g_{\text{fixed},k} u^g_{kt} \tag{4a}$$

subject to

$$\text{Constraints} \quad (3b) - (3d) \tag{4b}$$
$$0 \le AD^{\min}_{kt} \le M u^g_{kt}; \forall k \in \mathcal{N}^g \tag{4c}$$
$$0 \le A^{\min}_{kt} - AD^{\min}_{kt} \le M(1 - u^g_{kt}); \forall k \in \mathcal{N}^g \tag{4d}$$
$$0 \le AD^{\max}_{kt} \le M u^g_{kt}; \forall k \in \mathcal{N}^g \tag{4e}$$
$$0 \le A^{\max}_{kt} - AD^{\max}_{kt} \le M(1 - u^g_{kt}); \forall k \in \mathcal{N}^g \tag{4f}$$

$$\sum_{i\in\mathcal{N}} \sum_{t\in T} \lambda^p_{it} P^d_{kt} + \lambda^q_{it} Q^d_{kt} + \underline{V}^2_i X^{\min}_{it} - \overline{V}^2_i X^{\max}_{it}$$
$$+ \sum_{k\in\mathcal{N}^g} \sum_{t\in T} (\underline{P}^g_k AD^{\min}_{kt} - \overline{P}^g_k AD^{\max}_{kt} + \underline{Q}^g_k R^{\min}_{kt} \tag{4g}$$
$$- \overline{Q}^g_k R^{\max}_{kt}) = \sum_{k\in J} \sum_{t\in T} C^g_{pk} p^g_{kt} + C^g_{qk} q^g_{kt}$$

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The Case Study

- The proposed model is tested on the on 69-bus system.

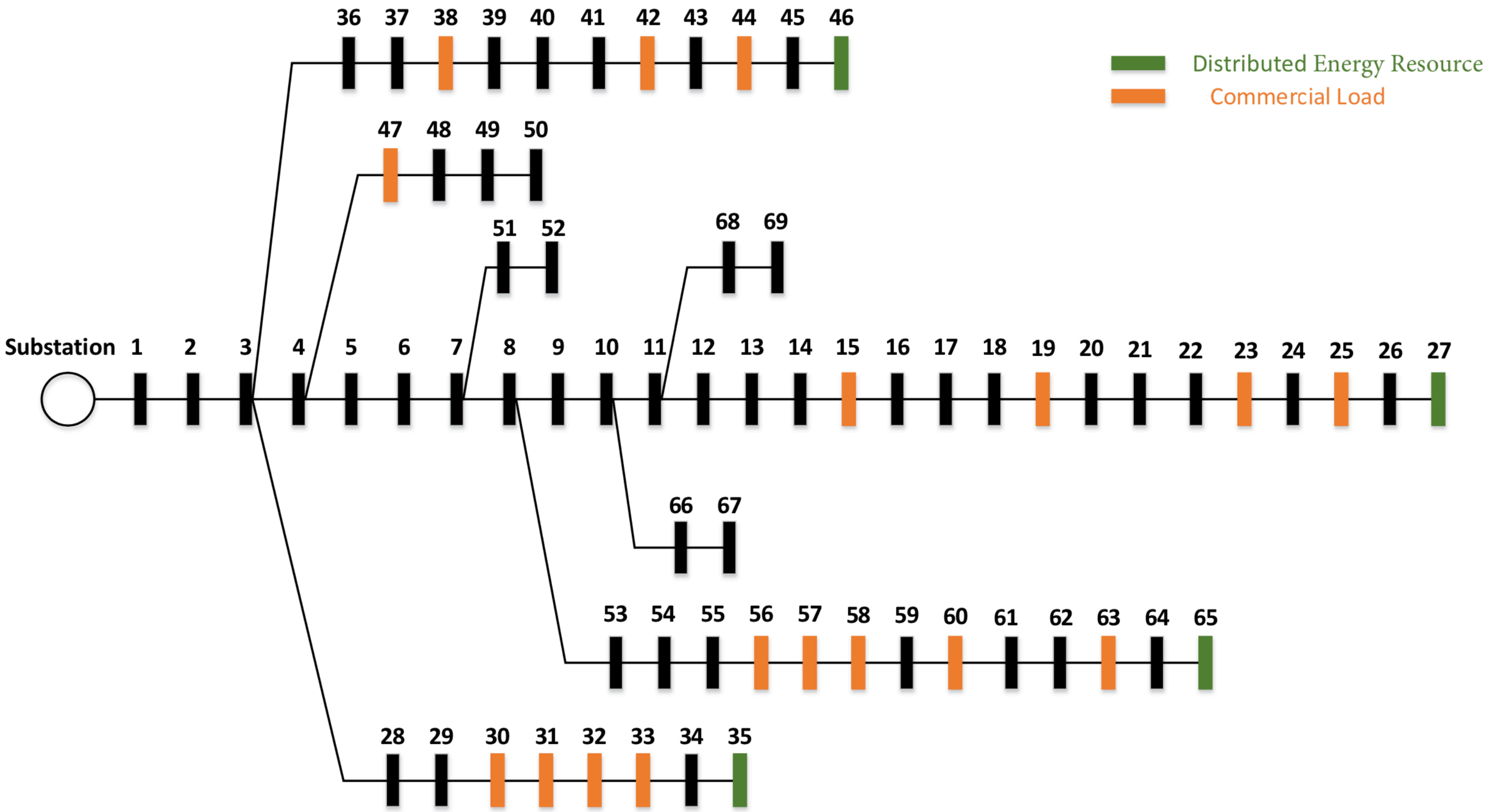
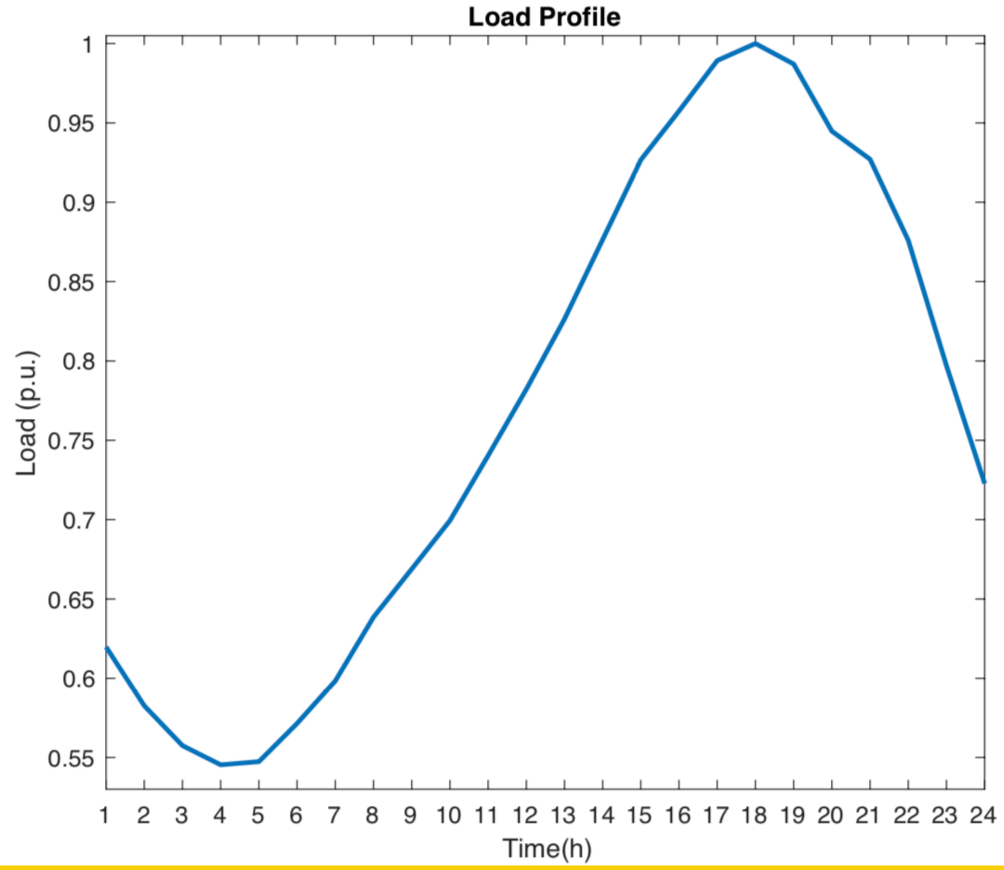


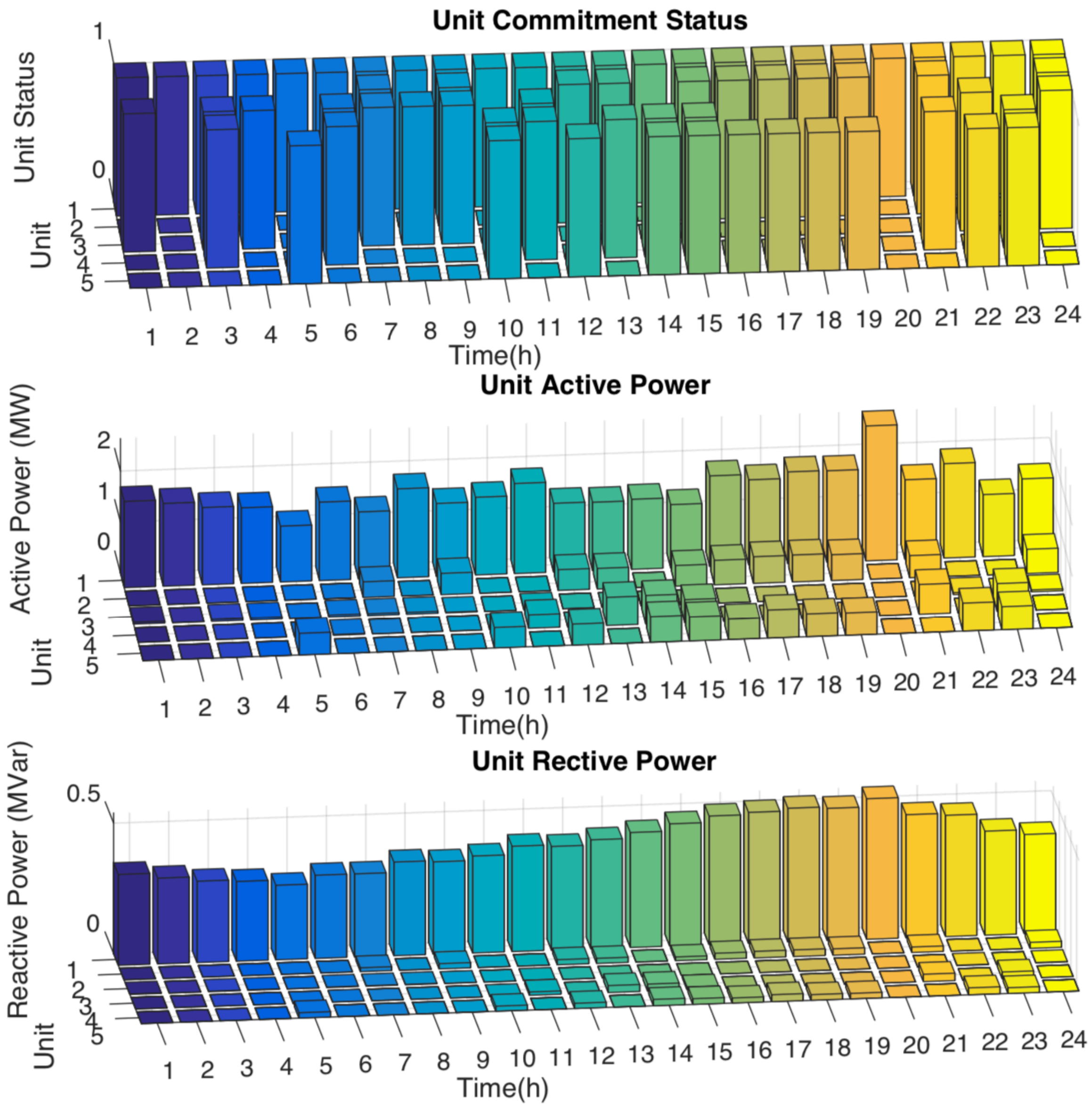
Fig. 2. 69-node distribution system [20].

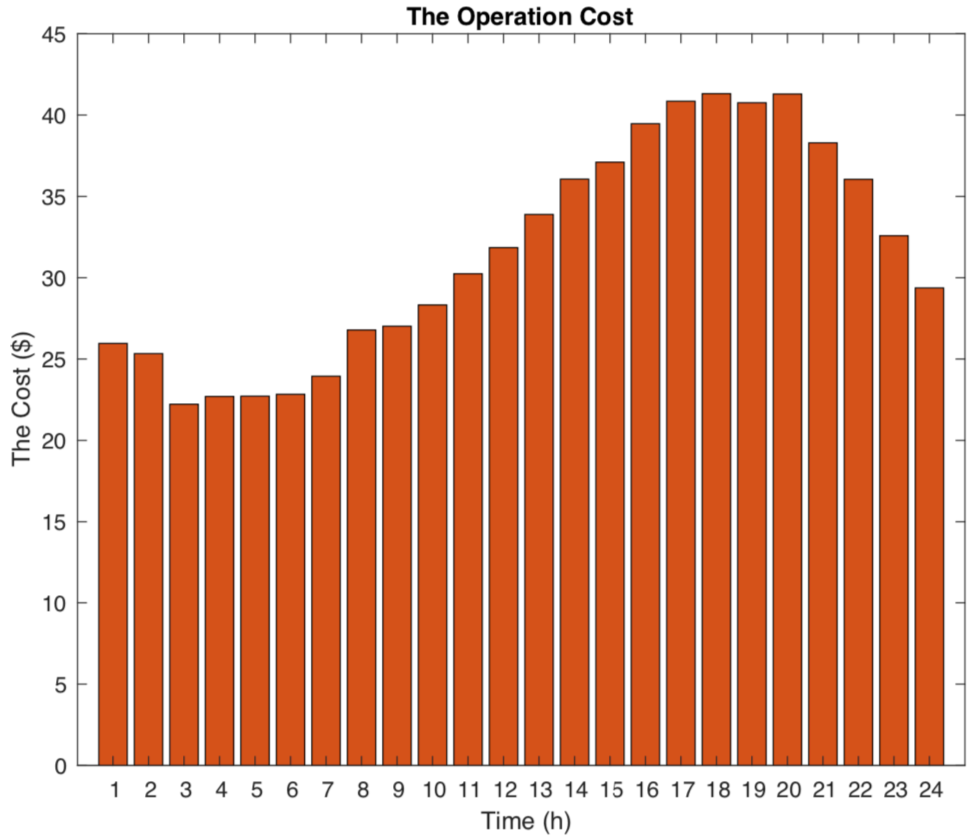
The Case Study

- The model is implemented using CVX toolbox with MATLAB R2015a, and solved by Gurobi solver.
- The load profile used in the case study is taken from CAISO.

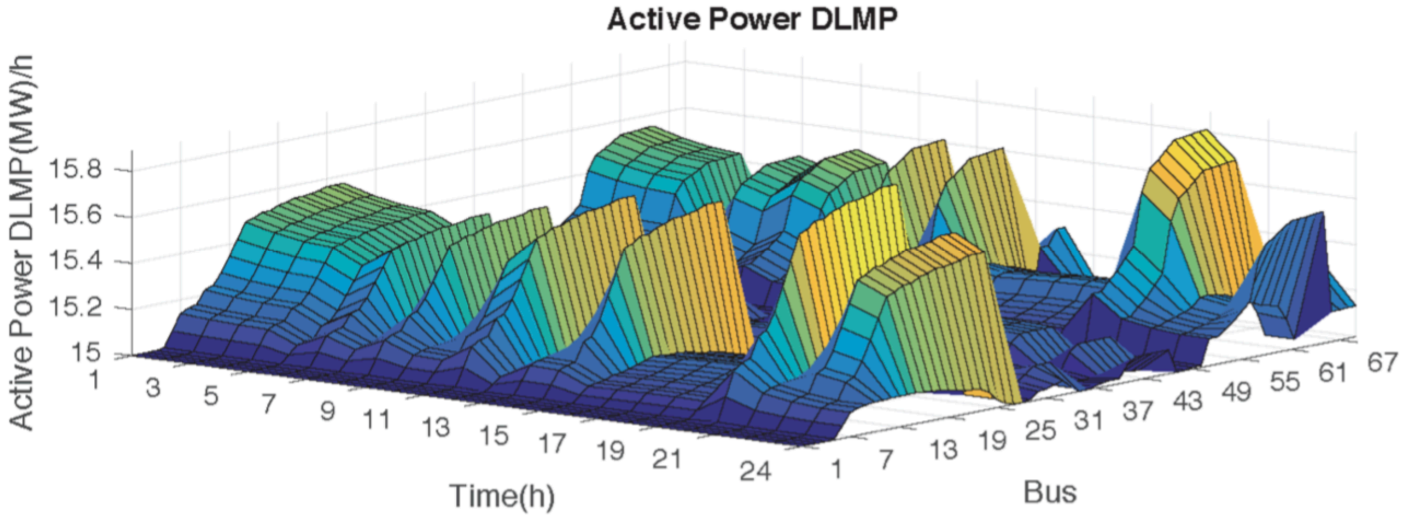


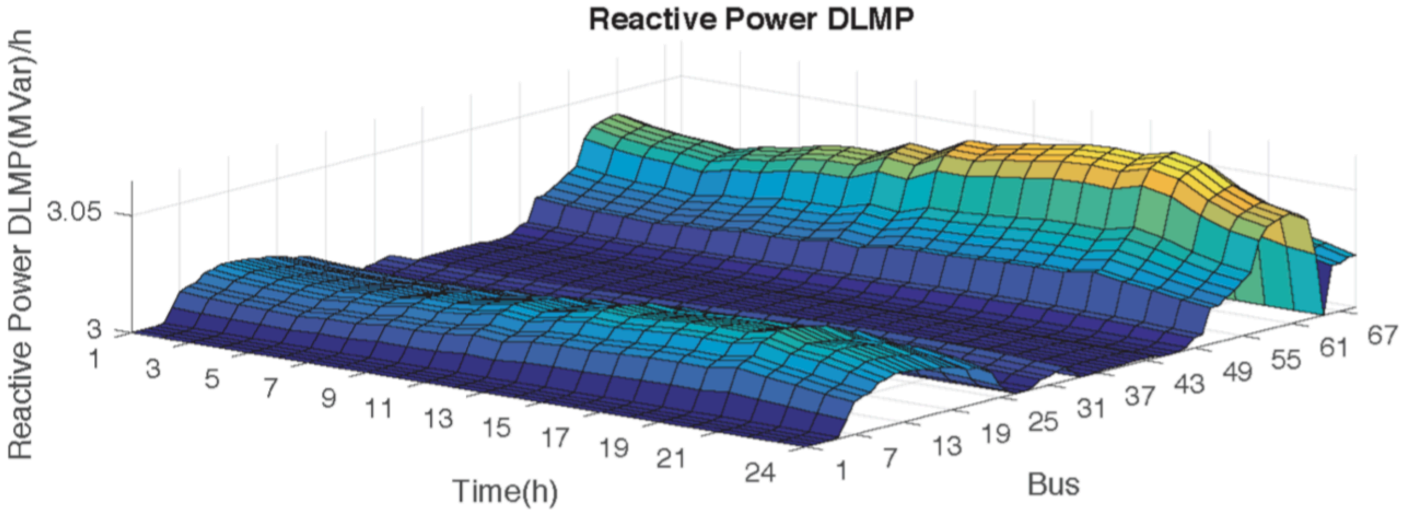
The Case Study Result



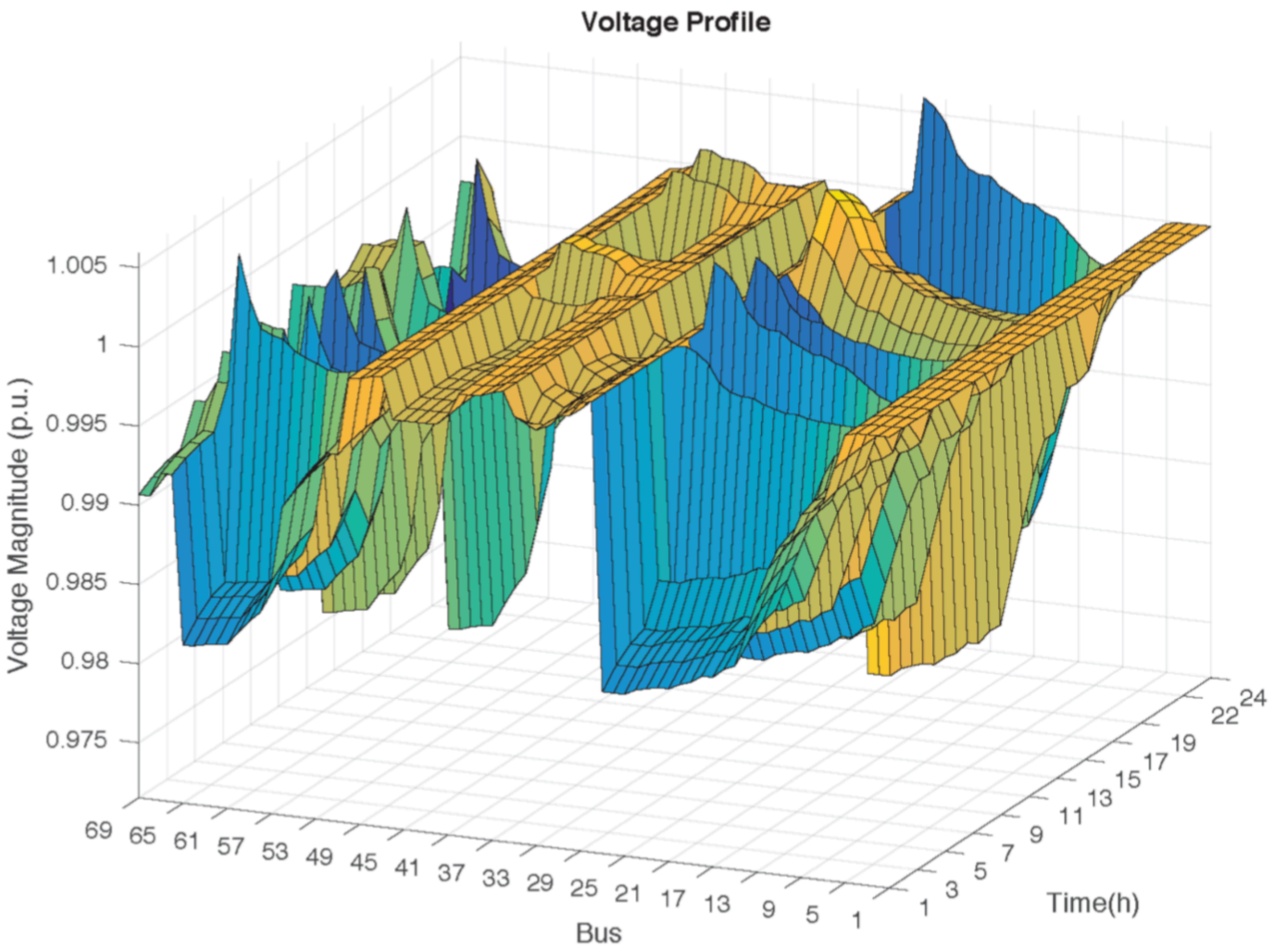


The Case Study Result





The Case Study Result



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Conclusion

- Our model is bilevel in which its upper level problem commits the optimal DER unit statuses with respect to the lower level problem that takes into account the AC optimal power flow.
- The proposed model exploits the duality theory and big-M method to arrive at a comprehensive primal- dual MISOCP formulation.
- The resultant optimization problem shows potential for eliciting both active- and reactive-power DLMPs.
- For future work, the model can be extended to incorporate demand elasticity with either price-based or incentive-based implementations.

Thank you.