Dynamic Mode Decomposition in Various Power System Applications

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Outline

- Introduction
- Dynamic Mode Decomposition
  - The DMD General Concept
  - The DMD Algorithm
- The DMD Applications In Power Systems
  - RLC Circuit
  - PMU Measurements
  - Abnormal Operation Analysis
- Conclusion

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Introduction

- Power system oscillations can be identified by two methods
  - Modeling-based
  - Measurement-based (ringdown analysis)

- Classical ringdown analysis methods in power systems:
  - Prony
  - Matrix Pencil (MP)
  - Eigensystem Realization Algorithm (ERA)

Introduction

- A recent method called Dynamic Mode Decomposition (DMD) has been proposed in:
  - Fluid field
  - Brain modeling
- It was used to decompose the high-dimensional data into spatial and temporal structures.
- The DMD algorithm is a combination of several techniques [9]:
  - Proper orthogonal decomposition (POD)
  - Fourier transform
  - Koopman operator
  - Least-square
  - Singular value decomposition (SVD)

Introduction

- The DMD applications:
  - The DMD has the ability to decompose the spatial and temporal dynamic modes.
  - The system dynamical model is constructed based on the dominant spatiotemporal structures.
  - The current system states can be achieved from the dynamical model. The future state could be predicted.
  - Since the DMD has the prediction ability, there is a chance of applying control strategies on the system.
- Our work provides a concise review of DMD algorithm and further demonstrates DMD implementation in power system applications.

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The DMD General Concept

- The DMD takes the collected measurements as an input.
- The discrete-time dynamics is considered

$$\mathbf{x}_{k+1} = \mathbf{F}\left(\mathbf{x}_k\right)$$

- If the system is a linear system, it can be represented by linear relationship using the dynamics matrix

$$\mathbf{x}_{k+1} = \mathbf{A}\mathbf{x}_k$$

- The time-domain expression of x(t) can be found if the eigenvalues of A and its eigenvector matrix are known

$$\mathbf{x}(t) = \sum_{k=1}^{n} \phi_k \exp\left(\omega_k t\right) b_k = \mathbf{\Phi} \exp(\mathbf{\Omega} t)\mathbf{b}$$

The DMD General Concept

- The measurements are set in two matrices, one of them has a time shift, in order that the DMD is enabled to approximate the dynamics matrix, A:

$$\mathbf{X_1} = \begin{bmatrix} | & | & & | \\ \mathbf{x}_1 & \mathbf{x}_2 & \cdots & \mathbf{x}_{m-1} \\ | & | & & | \end{bmatrix} \qquad \mathbf{X_2} = \begin{bmatrix} | & | & & | \\ \mathbf{x}_2 & \mathbf{x}_3 & \cdots & \mathbf{x}_m \\ | & | & & | \end{bmatrix}$$

- The augmented data matrix form involves both shift-stacking and time-delay

$$\mathbf{X_{aug,1}} = \begin{bmatrix} x_1 & x_2 & \cdots & x_{m-s} \\ x_2 & x_3 & \cdots & x_{m-s+1} \\ \vdots & \vdots & \ddots & \vdots \\ x_s & x_{s+1} & \cdots & x_{m-1} \end{bmatrix} \qquad \mathbf{X_{aug,2}} = \begin{bmatrix} x_2 & x_3 & \cdots & x_{m-s+1} \\ x_3 & x_4 & \cdots & x_{m-s+2} \\ \vdots & \vdots & \ddots & \vdots \\ x_{s+1} & x_{s+2} & \cdots & x_m \end{bmatrix}$$

The DMD General Concept

- Relying on the linear approximation is described in terms of the data measurement matrices

$$\mathbf{X_2} \approx \mathbf{A}\mathbf{X_1}$$

- The best-fit dynamics matrix, A, is given by

$$\mathbf{A} = \mathbf{X_2}\mathbf{X_1}^{\dagger}$$

The DMD Algorithm

- For large-scale systems, evaluating the dynamics matrix, A, could be not only computationally expensive, but intractable.

- The DMD avoids this complication by using Singular Value Decomposition (SVD).

- First, SVD and rank reduction is implemented on X1:

$$\mathbf{X_1} \approx \mathbf{U \Sigma V^*}$$

- In terms of SVD components, A becomes

$$\mathbf{A} = \mathbf{X_2 V \Sigma^{-1} U^*}$$

The DMD Algorithm

- The full matrix A is projected onto the POD modes in order to reduce its rank

$$\tilde{\mathbf{A}} = \mathbf{U}^{*}\mathbf{A}\mathbf{U} = \mathbf{U}^{*}\mathbf{X}_{2}\mathbf{V}\boldsymbol{\Sigma}^{-1}$$

- The lower-rank dynamical model is defined

$$\tilde{\mathbf{A}}\mathbf{W} = \mathbf{W}\boldsymbol{\Lambda}$$

- The DMD exact modes are defined as following

$$\boldsymbol{\Phi} = \mathbf{X}_{2}\mathbf{V}\boldsymbol{\Sigma}^{-1}\mathbf{W}$$

- It depends on the initial measurement column vector. The mode initializations

$$\mathbf{b} = \boldsymbol{\Phi}^{\dagger}\mathbf{x}_{1}$$

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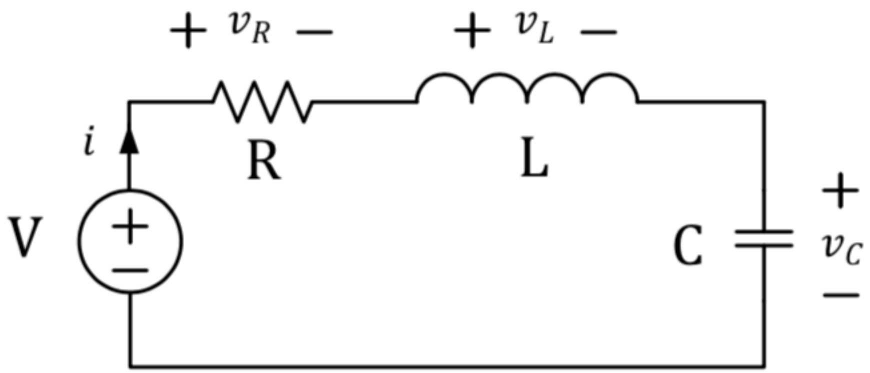
RLC Circuit

- The DMD is implemented on a series RLC circuit

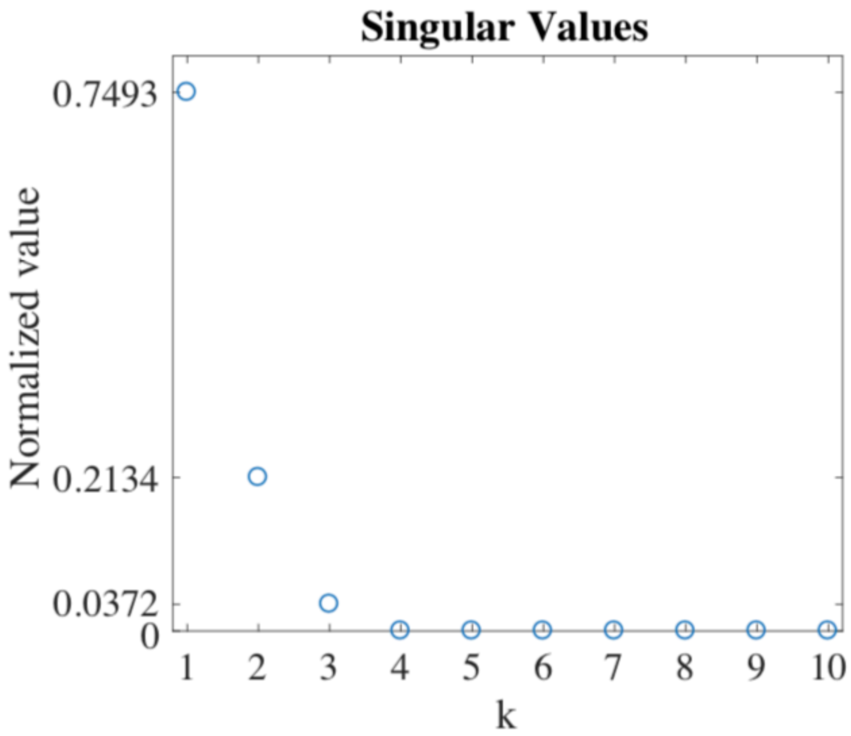


- The dynamics matrix, A

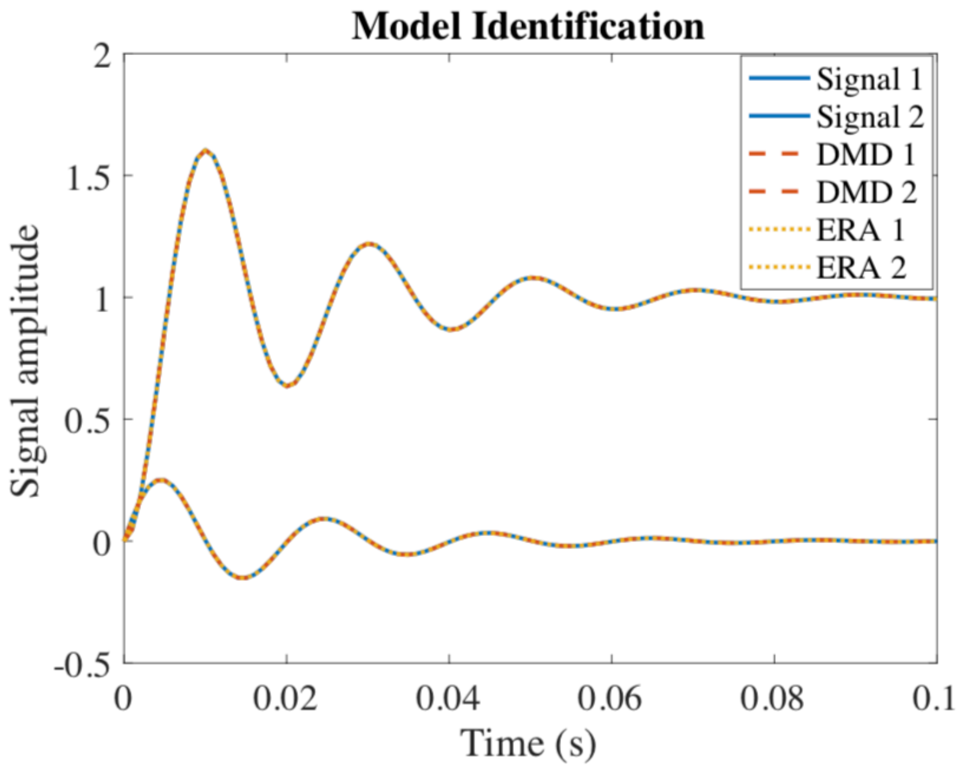
$$\mathbf{A} = \begin{bmatrix} \frac{-R}{L} & \frac{-1}{L} \\ \frac{1}{C} & 0 \end{bmatrix} = \begin{bmatrix} -100 & -100 \\ 1000 & 0 \end{bmatrix}$$

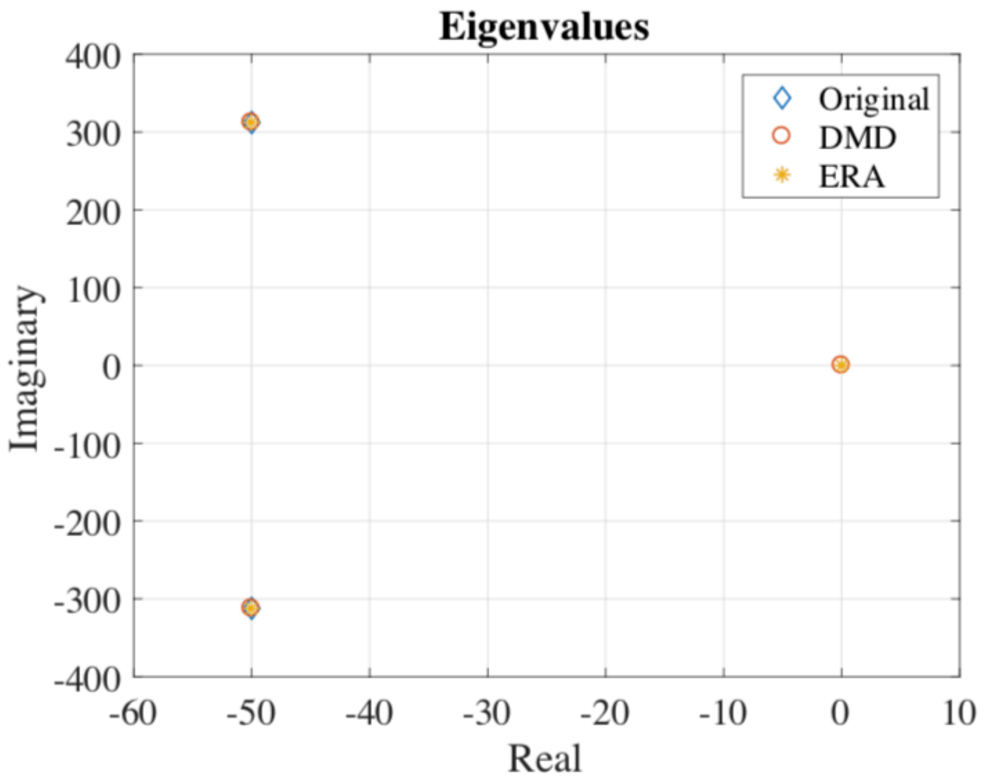
RLC Circuit

- The size of both Xaug1 and Xaug2 is (20 x 10).
- Even though Xaug1 rank is 10, the rank- reduction is primarily based on the effective singular values.
- This case has three non-zero effective singular values that have most the data information.
- The DMD rank, r, is set 3.



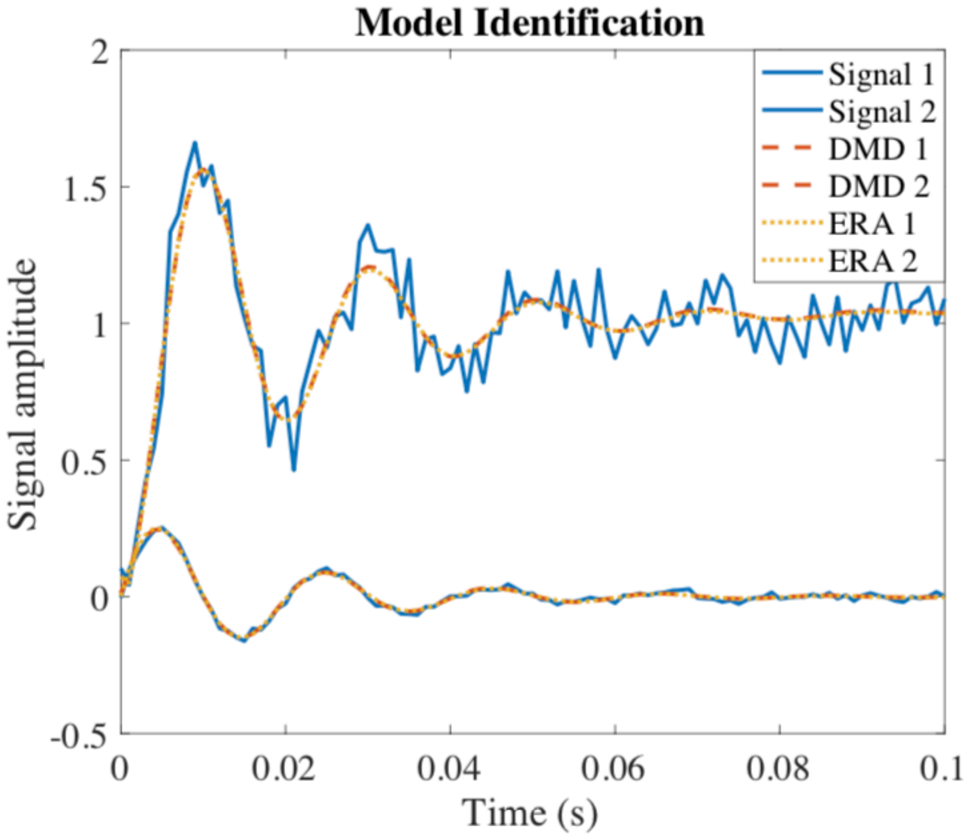
RLC Circuit DMD Results

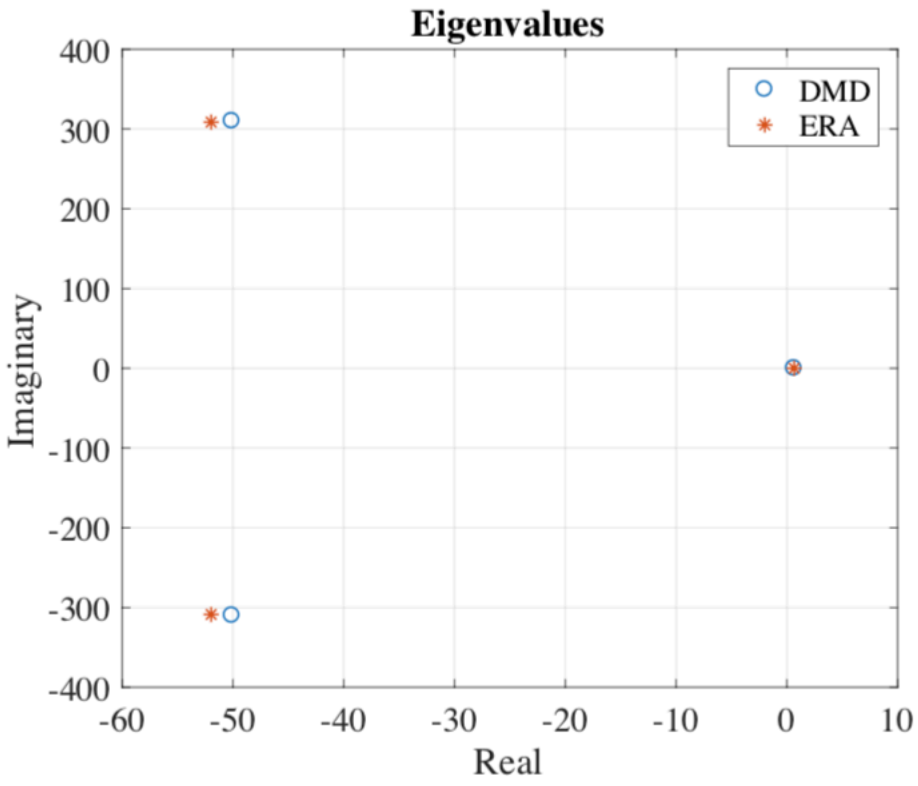




RLC Circuit DMD Results

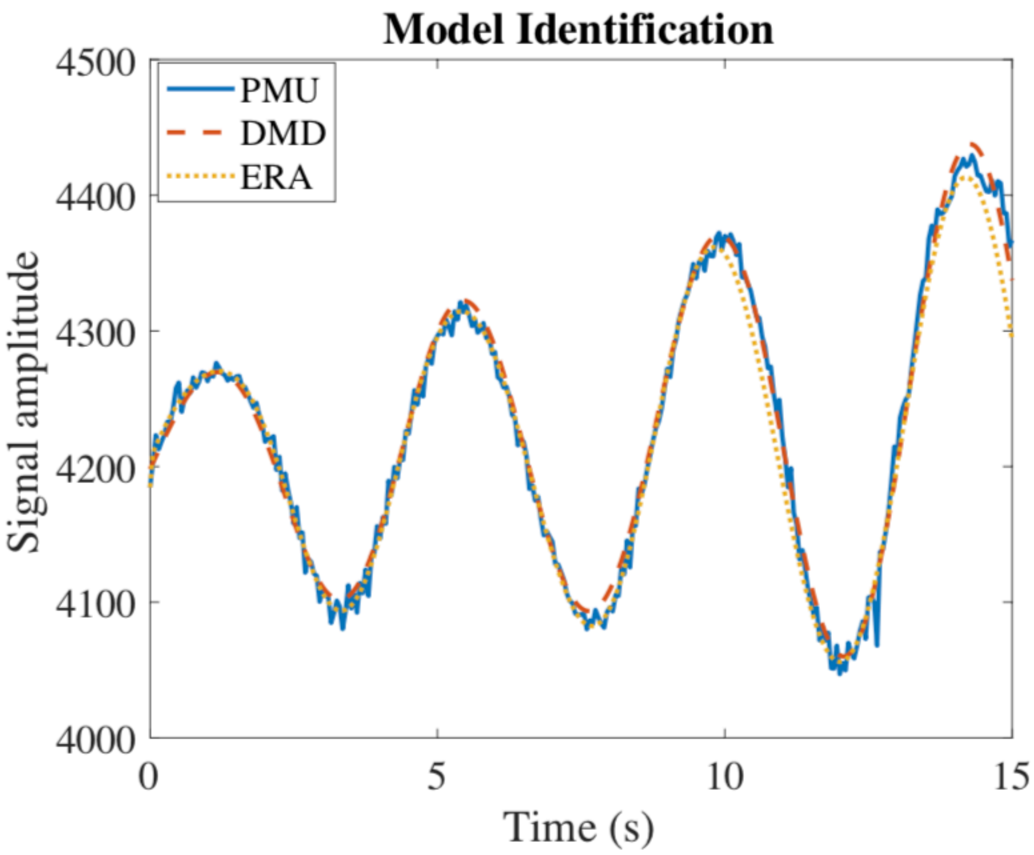
- 10% random noises are added to the measurement signals.

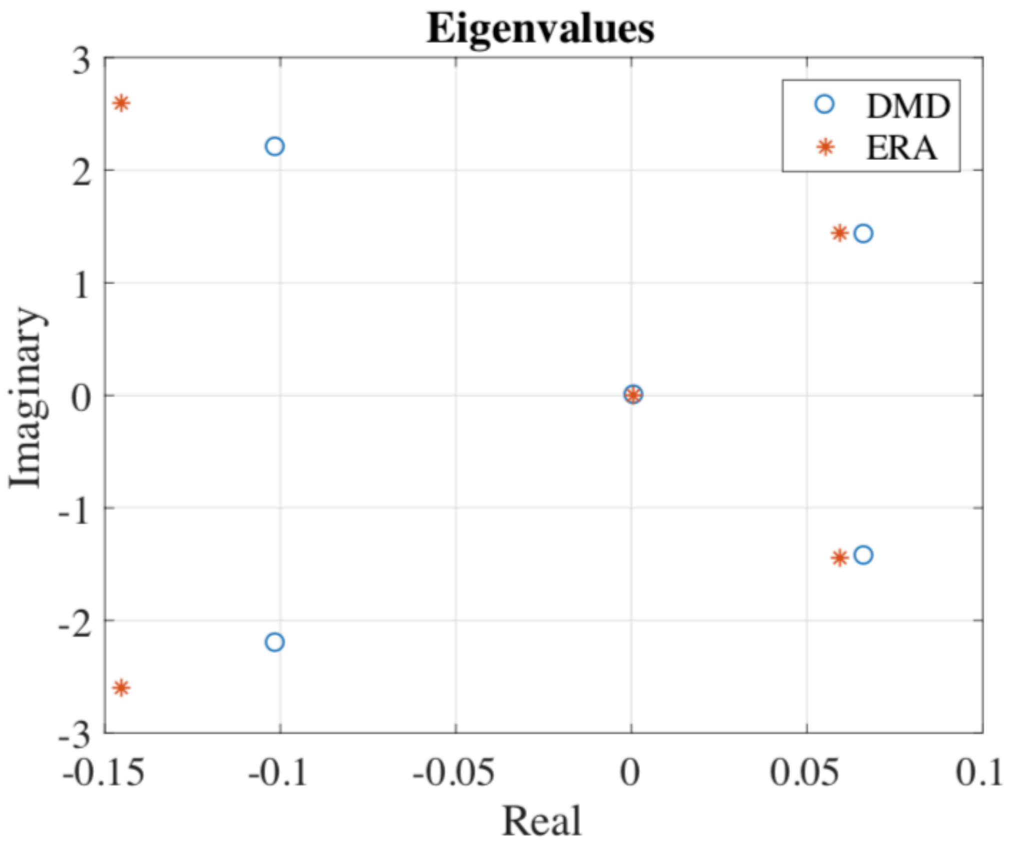




PMU Measurements

- The DMD and ERA algorithms are implemented on PMU data of power measurements during an oscillatory event.





Abnormal Operation Analysis

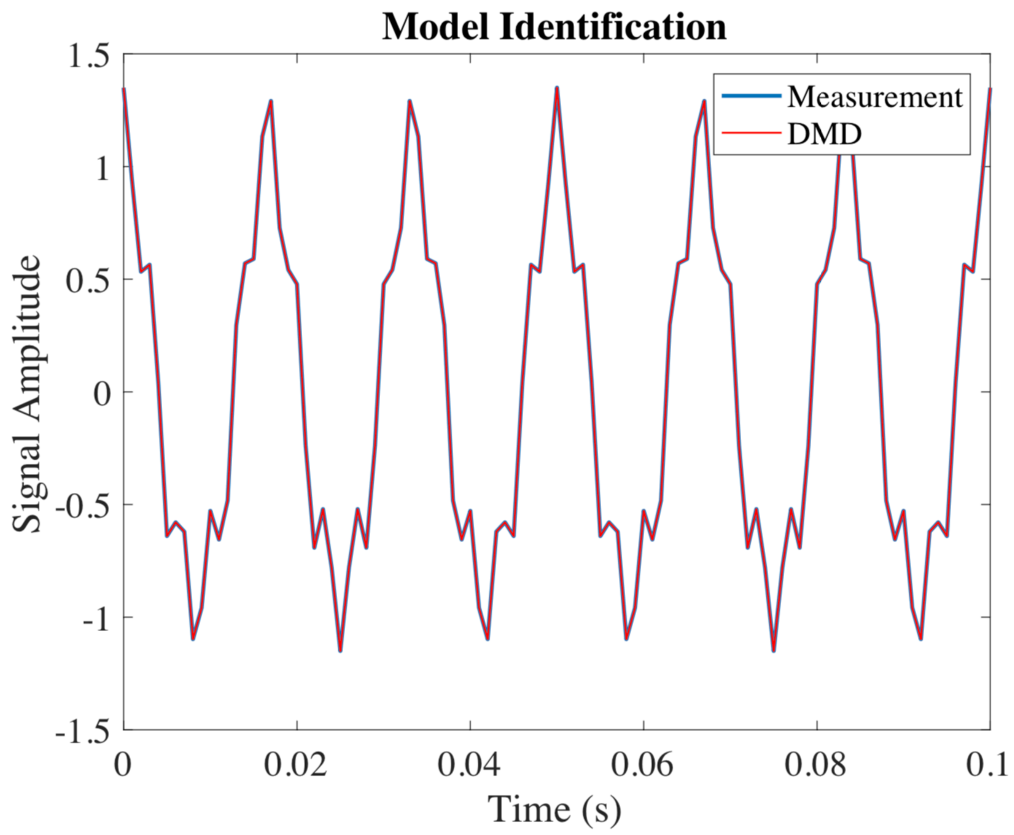
- The distribution-level system typically has unbalanced and harmonic components.

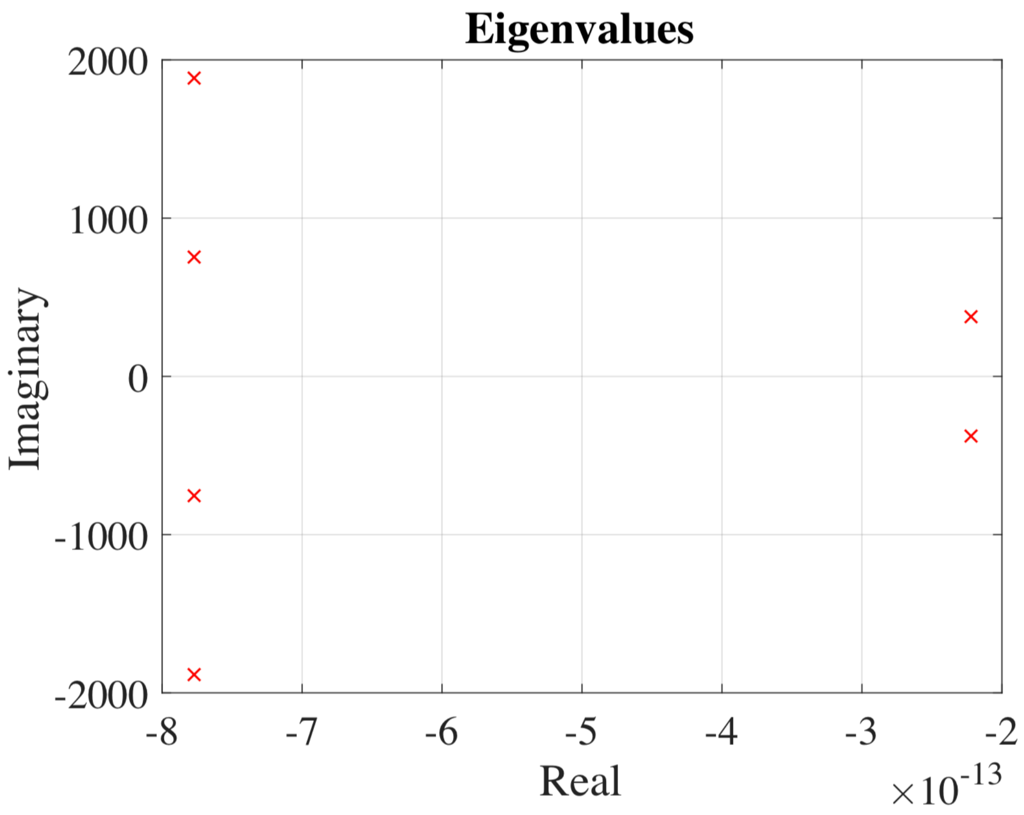
- As an example, if the voltage has a second-order harmonic and a fifth-order harmonic components, the phase ”a” voltage is represented as

$$V_{sa}(t) = \widehat{V}_s \cos\left(\omega_0 t\right) + k_2 \widehat{V}_s \cos\left(2\omega_0 t\right) + k_5 \widehat{V}_s \cos\left(5\omega_0 t\right)$$

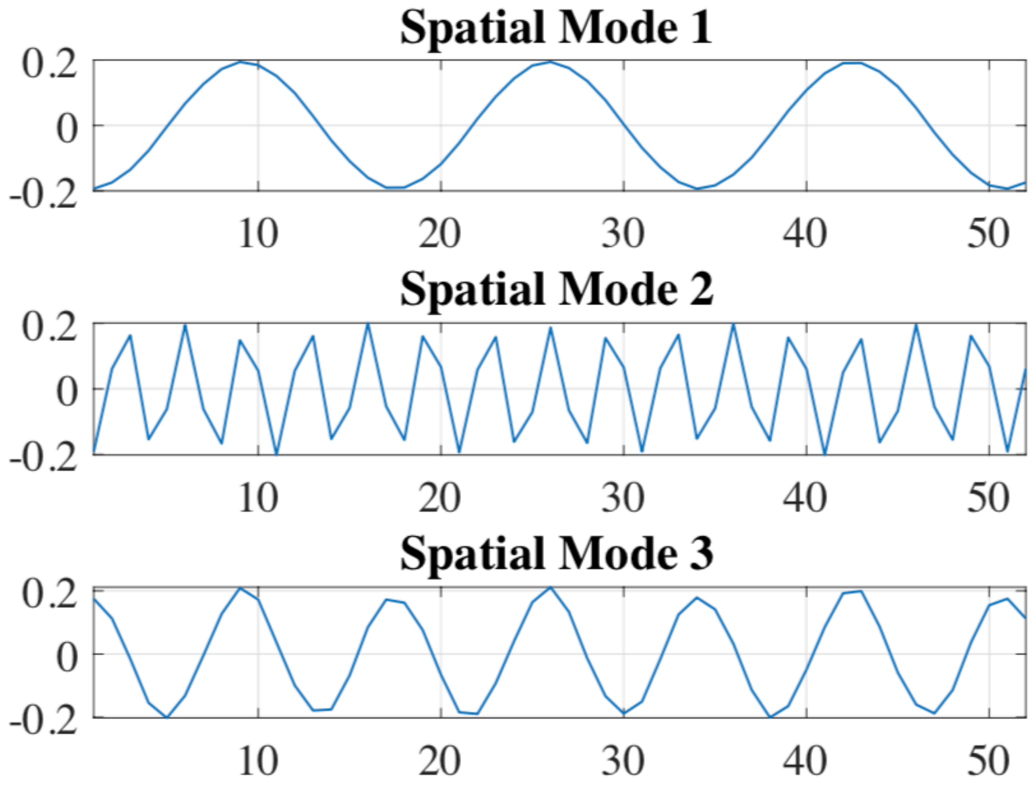
- The DMD algorithm can identified all the signal components.

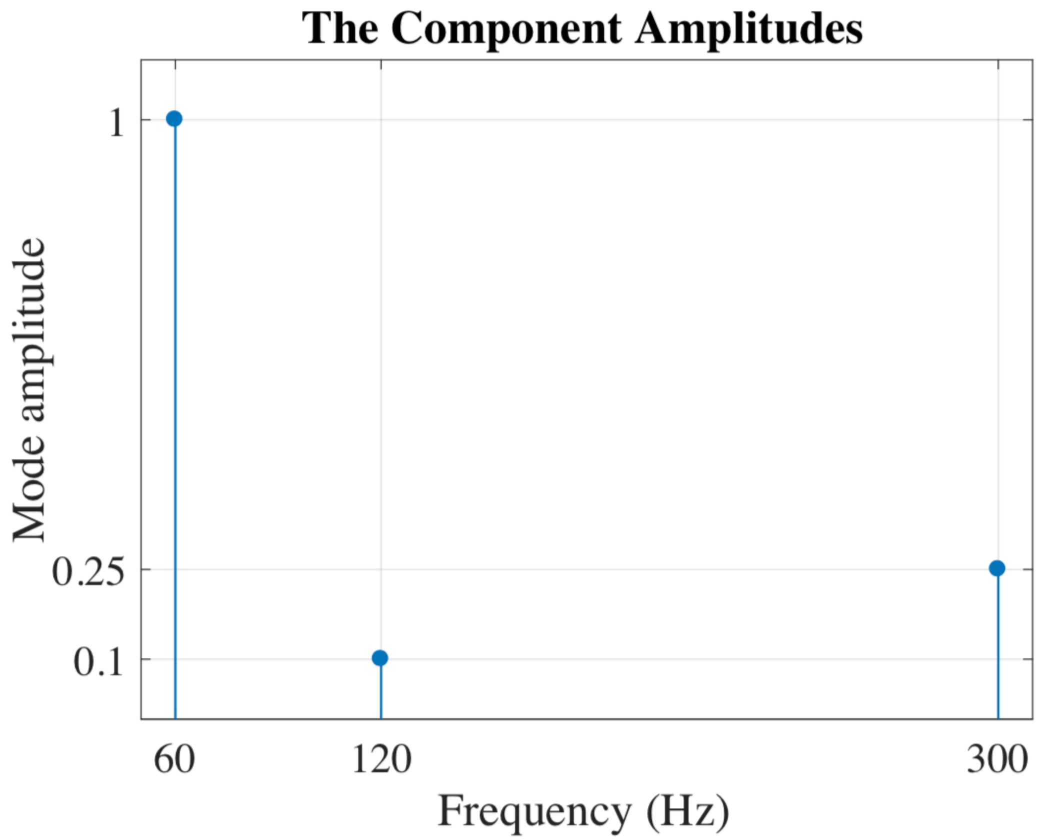
Abnormal Operation Analysis





Abnormal Operation Analysis





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Conclusion

- The dynamic mode decomposition algorithm has been found very useful in many engineering areas.
- Our work motivates the use of the DMD in power system applications.
- Besides of the DMD high accuracy of identifying a model, it has the capability to decompose the system dynamic modes.
- The DMD is capable to identify all the measurement component details.

Thank you.

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