Day-Ahead Distribution Market Analysis Via Convex Bilevel Programming

Abdullah Alassaf
Electrical Engineering Department
University of South Florida,
Tampa, FL 33620, USA
University of Hail, Hail, KSA
Email: alassaf@mail.usf.edu

Lingling Fan
Electrical Engineering Department
University of South Florida,
Tampa, FL 33620, USA
Email: linglingfan@usf.edu

Ibrahim Alsaleh
Electrical Engineering Department
University of South Florida,
Tampa, FL 33620, USA
University of Hail, Hail, KSA
Email: ialsaleh@usf.edu

***Abstract*—The proliferation of renewables in the distribution system has called for new constructs to manage the dispersed distributed energy resources (DERs). An effective approach is to employ a market-based dispatch in which private owners of DERs are financially motivated to participate in serving the local system. This paper conducts a day-ahead economic dispatch of a distribution system market where nodal prices are quantified using distribution locational marginal pricing (DLMP) to account for the system's spatial and temporal variations. This is formulated as a bilevel problem such that the upper level considers the on/off statuses of DERs, whereas the lower level is the second-order conic optimal power flow (SOCP-OPF) with an objective of minimizing the total generation cost of the active and reactive powers from DERs and the substation. Utilizing the SOCP duality, the overall problem is then recasted as a single-equivalent primal-dual mixed-integer SOCP (MISOCP) problem. A detailed presentation of the dual variables is provided. The effectiveness of the proposed formulation is validated on the 69-bus feeder, where DLMPs of both active and reactive powers are provided and analyzed.**

***Index Terms*—Unit commitment, distributed generators, bilevel programming, DER, DLMP, SOCP-OPF, MISOCP.**

I. Introduction

The electric distribution system is facing tremendous changes. In the past, for decades, it was quite passive that is limited to serving the load. Recently, the system characteristics have begun to vary with the advent of distributed energy resource (DER) technologies, such as microgrids (MGs), residential consumers with rooftop photovoltaic panels (PVs), distributed storage (DS), or even fleets of electric vehicles (EVs). These developments contribute to the distribution level great benefits including load profile peak shaving and valley filling, ancillary services, voltage support, and investment deferral. Moreover, in general, DERs are closer to the loads that lead to avoiding transmission line losses and congestions. For example, in New York State, the estimated rooftop photovoltaic panel production is 2,615 MW [1], [2]. This considerable amount of power could have been a burden on the transmission network.

Despite all the advantages offered by DERs, the increased penetration could raise some operation challenges e.g. voltage fluctuation and supply-demand imbalance. A futuristic

978-1-7281-0407-2/19/$31.00 © 2019 IEEE

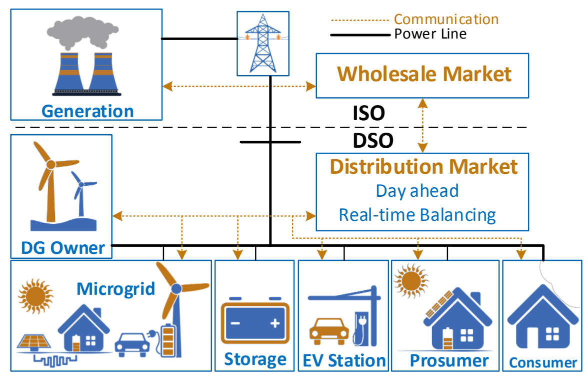

Fig. 1. Power flow and data exchange between DSO and DERs.

alternative to manage these DERs is to employ the recently-arisen concept of transactive energy, which is first defined by GridWise Architecture Council [3] as "techniques for managing the generation, consumption or flow of electric power within an electric power system through the use of economic or market-based constructs while considering grid reliability constraints". This enables the distribution system operator (DSO) to perform a local market with prosumers as market participants, while interacting and coordinating with the independent system operator (ISO) and abiding by the system's physical and security constraints. Proposed functionalities of the DSO are found in [4], prepared by CAISO in cooperation with Future Electric Company. In the US, distribution market policies are under reforming [5]. California state has already initiated plans regarding DER participation [6], and other states as Massachusetts, Hawaii and Minnesota are preparing the layout [7].

Locally, the DSO assumes the role of the ISO as in Fig. 1, by receiving the prosumer's bids, and clearing the market. [8] contends that the increased DER adoption justifies the use of unit commitment for the economic dispatch.

The locational marginal pricing is an effective pricing scheme as being used by all ISOs in the US [9], and can therefore be extended to the distribution market. In the day-ahead market of the transmission level, the locational marginal price (LMP) is attained from DCOPF, which approximates the voltage and ignores the reactive power. This approximation is somehow acceptable in the transmission level since the X/R ratio is high. In contrast, the X/R ratio of the distribution level is low, and so such approximation is not viable for the distribution system. Distribution locational marginal price (DLMP) was developed for distribution systems with the presence of DERs [10], [11], [12].

The references [13] and [14] proposed DLMP to overcome congestions with high penetration of EV loads. In [5], multi-period DLMPs are decomposed relying on a linearized branch flow model (BFM) with solutions of volt/var devices from an MISOCP BFM. Reference [15] solves DLMPs for three-phase radial distribution system using the multiphase BFM based on semidefinite programming (SDP).

The objective of this paper is to determine the multi-period on/off status of DERs and the DLMPs for distribution systems. A bilevel programming-based unit commitment problem for transmission grid LMP computing relying on DC-OPF has been formulated by the authors in [16]. In this paper, the bilevel programming-based approach is extended to distribution networks with reactive power and voltage fully considered.

In this paper, the distribution system is assumed to be balanced three phase and per-phase analysis is adopted. A unit commitment problem is formulated with DERs on/off statuses as binary variables while the DERs real and reactive power dispatch levels as continuous variables. The distribution system network constraints are considered. SOCP convex relaxation proposed for radial networks in [17] is adopted so the problem with DER on/off status given is indeed a SOCP optimal power flow (SOCP-OPF) problem. The unit commitment problem is thus a MISOCP problem. The unit commitment problem does not give DLMP directly. In order to find DLMP, we re-formulate the unit commitment problem as a bilevel problem with DLMP as decision variables.

The proposed formulation consists of two problems: the upper level and the lower level. The upper level problem has binary decision variables that represent the DERs on/off statuses. The lower level problem is the SOCP-OPF, which finds the optimal power flow of the distribution system with respects the physical constraints. The upper level problem sends the DER status information to the lower level problem. With DER statuses known, the lower level SOCP-OPF decides DER dispatch levels. In addition, DLMP can be determined. DLMP information is passed to the upper level problem and shall be utilized in the objective function.

With the bilevel problem adequately formulated, we further investigate the solving strategy. A single optimization problem is formulated by converting the lower-level optimization problem as constraints. The duality of an SOCP is utilized to merge the lower level with the upper level. Nonetheless, this procedure introduces bilinear constraints. Hence, linearization techniques are employed to make the optimization problem fit to the category of MISOCP that can be solved using commercial solvers such as Gurobi [18].

The rest of the paper is organized as follows. Section II introduces the bi-level problem formulation and solving strategy. Case studies are presented in Section III. Section IV concludes the paper.

II. Bilevel Programming Problem Formulation and Solving Strategy

We consider a multi-period unit commitment problem for distribution systems. A few notations are introduced as follows.

Notation: i is index of system nodes set $\mathcal{N}$. $\mathcal{N}$ contains all the system nodes except the substation node that connects the distribution level with the transmission system. k is index of generator set $\mathcal{N}^g$. $\mathcal{N}^g$ is set of the nodes that have a DER, $\mathcal{N}^g \subset \mathcal{N}$. (i,j) is the index of line i-j in the line set $\mathcal{L}$. t is the time horizon index of set $\mathcal{T}$.

The demand active and reactive node load at time t are P^d_{it} and Q^d_{it}, respectively. B_{ij} and G_{ij} are the shunt susceptance and conductance, respectively, of line $i-j$. The unit maximum and minimum active power limit are $\overline{P}$ and $\underline{P}$, respectively, whereas the unit maximum and minimum reactive power limit are $\overline{Q}$ and $\underline{Q}$, respectively.

A. The bilevel problem formulation

The following model minimizes the total operation cost of a distribution system.

$$\min_{u\in\mathcal{U}} \sum_{k\in J}\sum_{t\in T} C^g_{\text{fixed},k} u^g_{kt} + \min_{w\in\mathcal{W}} \sum_{k\in J}\sum_{t\in T} C^g_{pk} p^g_{kt} + C^g_{qk} q^g_{kt} \quad (1a)$$

subject to

$$\mathcal{U} = \Big\{ u^g_{kt} \in \{0,1\}; \forall k \in \mathcal{N}^g \Big\} \quad (1b)$$

$$\mathcal{W} = \Big\{ \underline{P}^g_k u^g_{kt} \le p^g_{kt} \le \overline{P}^g_k u^g_{kt} : (A^{\min}_{kt}, A^{\max}_{kt}); \forall k \in \mathcal{N}^g \quad (1c)$$

$$\underline{Q}^g_k p^g_{kt} \le q^g_{kt} \le \overline{Q}^g_k p^g_{kt} : (R^{\min}_{kt}, R^{\max}_{kt}); \forall k \in \mathcal{N}^g \quad (1d)$$

$$p^g_{kt} - P^d_{it} = G_{ii} e_{it} + \sum_{j=1, j\ne i} (G_{ij} c_{ijt} - B_{ij} s_{ijt}) : (\lambda^p_{it}); \forall i \in \mathcal{N} \quad (1e)$$

$$q^g_{kt} - Q^d_{it} = -B_{ii} e_{it} - \sum_{j=1, j\ne i} (B_{ij} c_{ijt} + G_{ij} s_{ijt}) : (\lambda^q_{it}); \forall i \in \mathcal{N} \quad (1f)$$

$$\underline{V}^2_i \le e_{it} \le \overline{V}^2_i : (X^{\min}_{it}, X^{\max}_{it}); \forall i \in \mathcal{N} \quad (1g)$$

$$D^1_{ijt} = 2c_{ijt} : (\alpha_{ijt}); \forall (i,j) \in \mathcal{L}, \quad (1h)$$

$$D^2_{ijt} = 2s_{ijt} : (\beta_{ijt}); \forall (i,j) \in \mathcal{L} \quad (1i)$$

$$D^3_{ijt} = e_{it} - e_{jt} : (\phi_{ijt}); \forall (i,j) \in \mathcal{L} \quad (1j)$$

$$D^4_{ijt} = e_{it} + e_{jt} : (\theta_{ijt}); \forall (i,j) \in \mathcal{L} \quad (1k)$$

$$(D^1_{ijt})^2 + (D^2_{ijt})^2 + (D^3_{ijt})^2 \le (D^4_{ijt})^2; \forall (i,j) \in \mathcal{L} \Big\} \quad (1l)$$

where $\mathcal{U}$ represents the feasible region or constraints of the upper-level problem, and $\mathcal{W}$ represents the feasible region of the lower-level problem. The decision variable vector w of the lower-level problem is as follows: w= $\{p^g_{kt}, q^g_{kt}, e_{it}, c_{ijt}, s_{ijt}\}$. p^g_{kt}, q^g_{kt} are real and reactive power dispatch levels of ith DER at time t. c_{ijt}, s_{ijt} are introduced to replace voltage phasors at bus i and bus j: $\overline{V}_{it}$ and $\overline{V}_{jt}$:

$$\begin{aligned} e_{it} &= V_{it}^2, \\ c_{ijt} &= V_{it}V_{jt}\cos \angle \overline{V}_{it} - \overline{V}_{jt}, \\ s_{ijt} &= V_{it}V_{jt}\sin \angle \overline{V}_{it} - \overline{V}_{jt}. \end{aligned}$$

Hence, it is obvious that $c^2_{ijt} + s^2_{ijt} = e_{it}e_{jt}$. This equality constraint is nonconvex and it will be relaxed by changing the equality sign to inequality sign: $c^2_{ijt} + s^2_{ijt} \le e_{it}e_{jt}$. The resulting constraint is an SOCP convex constraint. (1l) indeed is equivalent to the inequality constraint. The format of (1l) better reflects second order cone definition, i.e., norm of affine expression of variables is less than a linear combination of variables.

The objective function represented in (1a) is minimization of overall operation cost. It consists of two part: the first one, which considers the fixed operation cost, is for the upper level problem. The second part of the objective function is related to minimizing the active and reactive generation level of the lower level problem.

In the upper-level problem, the decision variables are DER on/off statuses (u^g_{kt}). $C^g_{\text{fixed},k}$ is the no-load cost of the DER. This price covers the operation, maintenance, losses of the DERs.

In the lower-level problem, the decision variables are the active and reactive power dispatch levels of DERs (p^g_{kt} and q^g_{kt}). The active and reactive generation level costs are C^g_{pk} and C^g_{qk}, respectively.

The Var capability is constrained by a constant power factor (PF), thus relating the reactive power supply/absorption to the active power. Thereby, $\overline{Q}^g_k = -\underline{Q}^g_k = \sqrt{1-\text{PF}^2}/\text{PF}$. The distribution constraints are set in the lower level problem. The physical limits of the DERs are (1c)-(1d). The active and reactive power balance equations are (1e)-(1f), respectively. The voltage constraints are relaxed and given in (1g)-(1l). The dual variables of the constraints of the lower level problem are shown in parentheses. The dual variable of the active and reactive power balance equations are λ^p_{nt} and λ^q_{nt}, respectively. They state the active and reactive bus DLMPs for time slot t. The other dual variables are the constraint shadow prices. In general, they describe the cost of violating the constraints.

B. The solution strategy

The above bilevel problem is solved by merging the lower-level problem with the upper level one in order to have a single-equivalent model. This is approached by replacing the lower level problem by its primal and dual constraints with an additional constraint ensuring an equal primal-dual objective. The following shows the dual problem of lower level player and the single-equivalent model.

Note that in (1), for each constraint of the lower-level problem, dual variables have been defined. For example, $A^{\max}_{kt}$ and $A^{\min}_{kt}$ are dual variables for the real power maximum and minimum limits binding constraints for kth DER, while $R^{\max}_{kt}$ and $R^{\min}_{kt}$ are dual variables for the reactive power maximum and minimum limits binding constraints for kth DER. Dual variables λ^p_{it} and λ^q_{it} are related to the nodal real power and reactive power balance equality constraints. They are termed as DLMPs. $X^{\min}_{it}$ and $X^{\max}_{it}$ are dual variables related to voltage limit binding constraints. The rest four dual variables α_{ijt}, β_{ijt}, ϕ_{ijt} and θ_{ijt} are related to the second-order cone constraint for each line $i-j$ at period t.

1) The Lower Level Dual Formulation:

$$\begin{aligned} \max_{z\,\in \mathcal{Z}} \sum_{i\,\in \mathcal{N}} \sum_{t\,\in T} & \lambda^p_{it} P^d_{kt} + \lambda^q_{it} Q^d_{kt} + \underline{V}^2_i X^{\min}_{it} - \overline{V}^2_i X^{\max}_{it} \\ + \sum_{k\,\in \mathcal{N}^g} \sum_{t\,\in T} & (\underline{P}^g_k A^{\min}_{kt} u^g_{kt} - \overline{P}^g_k A^{\max}_{kt} u^g_{kt} + \underline{Q}^g_k R^{\min}_{kt} \\ & - \overline{Q}^g_k R^{\max}_{kt}) \end{aligned} \tag{2a}$$

subject to

$$A^{\min}_{kt} - A^{\max}_{kt} + \lambda^p_{it} = C^g_k; \forall k \in \mathcal{N}^g \tag{2b}$$

$$R^{\min}_{kt} - R^{\max}_{kt} + \lambda^q_{it} = C^g_k; \forall k \in \mathcal{N}^g \tag{2c}$$

$$X^{\min}_{it} - X^{\max}_{it} - G_{ii}\lambda^p_{it} + B_{ii}\lambda^q_{it} + \phi_{ijt} + \theta_{ijt} = 0; \forall i \in \mathcal{N} \tag{2d}$$

$$-G_{ij}\lambda^p_{it} + B_{ij}\lambda^q_{it} + 2\alpha_{ijt} = 0; \forall ij \in \mathcal{L} \tag{2e}$$

$$B_{ij}\lambda^p_{it} + G_{ij}\lambda^q_{it} + 2\beta_{ijt} = 0; \forall ij \in \mathcal{L} \tag{2f}$$

$$(\alpha_{ijt})^2 + (\beta_{ijt})^2 + (\phi_{ijt})^2 \le (\theta_{ijt})^2; \forall ij \in \mathcal{L} \tag{2g}$$

$$\lambda^p_{it}, \lambda^q_{it}, X^{\min,\max}_{it} \ge 0; \forall i \in \mathcal{N} \tag{2h}$$

$$A^{\min}_{kt}, A^{\max}_{kt}, R^{\min}_{kt}, R^{\max}_{kt} \ge 0; \forall k \in \mathcal{N}^g \tag{2i}$$

$$\alpha_{ijt}, \beta_{ijt}, \theta_{ijt} \ge 0; \forall ij \in \mathcal{L} \tag{2j}$$

where z is the decision variable vector and includes dual variables in the following:

$$\{\lambda^p_{it}, \lambda^q_{it}, A^{\min}_{kt}, A^{\max}_{kt}, R^{\min}_{kt}, R^{\max}_{kt}, X^{\min}_{it}, X^{\max}_{it}, \alpha_{ijt}, \beta_{ijt}, \theta_{ijt}\}.$$

Note that cones are self-dual, and so (2g) is an SOCP constraint with variables associated with the primal constraint's terms (1h)-(1k).

2) The Single-Equivalent Model:

$$\begin{aligned} \max_{j\,\in \mathcal{J}} \sum_{i\,\in \mathcal{N}} \sum_{t\,\in T} & \lambda^p_{it} P^d_{kt} + \lambda^q_{it} Q^d_{kt} + \underline{V}^2_i X^{\min}_{it} - \overline{V}^2_i X^{\max}_{it} \\ + \sum_{k\,\in \mathcal{N}^g} \sum_{t\,\in T} & (\underline{P}^g_k A^{\min}_{kt} u^g_{kt} - \overline{P}^g_k A^{\max}_{kt} u^g_{kt} + \underline{Q}^g_k R^{\min}_{kt} \\ & - \overline{Q}^g_k R^{\max}_{kt}) - \sum_{k\,\in J} \sum_{t\,\in T} C^g_{\text{fixed},k} u^g_{kt} \end{aligned} \tag{3a}$$

subject to

$$\text{Constraints} \quad (1b) \tag{3b}$$

$$\text{Constraints} \quad (1c)-(1l) \tag{3c}$$

$$\text{Constraints} \quad (2b)-(2j) \tag{3d}$$

$$\sum_{i\in\mathcal{N}}\sum_{t\in T}\lambda^p_{it}P^d_{kt}+\lambda^q_{it}Q^d_{kt}+\underline{V}_i^2X^{\min}_{it}-\overline{V}_i^2X^{\max}_{it}$$
$$+\sum_{k\in\mathcal{N}^g}\sum_{t\in T}(\underline{P}^g_kA^{\min}_{kt}u^g_{kt}-\overline{P}^g_kA^{\max}_{kt}u^g_{kt}+\underline{Q}^g_kR^{\min}_{kt} \quad (3e)$$
$$-\overline{Q}^g_kR^{\max}_{kt})=\sum_{k\in J}\sum_{t\in T}C^g_{pk}p^g_{kt}+C^g_{qk}q^g_{kt}$$

The equality constraint (3e) ensures a strong duality, where both sides are the objective of (1) and (2). That is, the value of the original lower-level problem should equal to the value of its dual.

3) The Linearized Single-Equivalent Model: The single-equivalent problem in (3) is nonlinear. The nonlinearity stems from the multiplication of continuous and binary variables in (3a) and (3e). This is overcome by defining $AD^{\min}_{kt}=A^{\min}_{kt}u^g_{kt}$ and $AD^{\max}_{kt}=A^{\max}_{kt}u^g_{kt}$, and applying the big-$M$ method. The overall linearized problem is presented in (4) with (4c)-(4f) as additional constraints related to the big-M method.

$$\max_{j\in\mathcal{J}}\sum_{i\in\mathcal{N}}\sum_{t\in T}\lambda^p_{it}P^d_{kt}+\lambda^q_{it}Q^d_{kt}+\underline{V}_i^2X^{\min}_{it}-\overline{V}_i^2X^{\max}_{it}$$
$$+\sum_{k\in\mathcal{N}^g}\sum_{t\in T}(\underline{P}^g_kAD^{\min}_{kt}-\overline{P}^g_kAD^{\max}_{kt}+\underline{Q}^g_kR^{\min}_{kt}-\overline{Q}^g_kR^{\max}_{kt})$$
$$-\sum_{k\in J}\sum_{t\in T}C^g_{\text{fixed},k}u^g_{kt} \quad (4a)$$

subject to

$$\text{Constraints} \quad (3b)-(3d) \quad (4b)$$
$$0\le AD^{\min}_{kt}\le Mu^g_{kt};\forall k\in\mathcal{N}^g \quad (4c)$$
$$0\le A^{\min}_{kt}-AD^{\min}_{kt}\le M(1-u^g_{kt});\forall k\in\mathcal{N}^g \quad (4d)$$
$$0\le AD^{\max}_{kt}\le Mu^g_{kt};\forall k\in\mathcal{N}^g \quad (4e)$$
$$0\le A^{\max}_{kt}-AD^{\max}_{kt}\le M(1-u^g_{kt});\forall k\in\mathcal{N}^g \quad (4f)$$

$$\sum_{i\in\mathcal{N}}\sum_{t\in T}\lambda^p_{it}P^d_{kt}+\lambda^q_{it}Q^d_{kt}+\underline{V}_i^2X^{\min}_{it}-\overline{V}_i^2X^{\max}_{it}$$
$$+\sum_{k\in\mathcal{N}^g}\sum_{t\in T}(\underline{P}^g_kAD^{\min}_{kt}-\overline{P}^g_kAD^{\max}_{kt}+\underline{Q}^g_kR^{\min}_{kt} \quad (4g)$$
$$-\overline{Q}^g_kR^{\max}_{kt})=\sum_{k\in J}\sum_{t\in T}C^g_{pk}p^g_{kt}+C^g_{qk}q^g_{kt}$$

(4) is a MISOCP problem and can be solved by commercial off-shelf solvers such as Gurobi.

III. Case Study

The proposed model has been tested on several distribution systems, this paper shows the implementation of the model on 69-bus system. The model is implemented using CVX toolbox [19] with MATLAB R2015a, and solved by Gurobi solver [18].

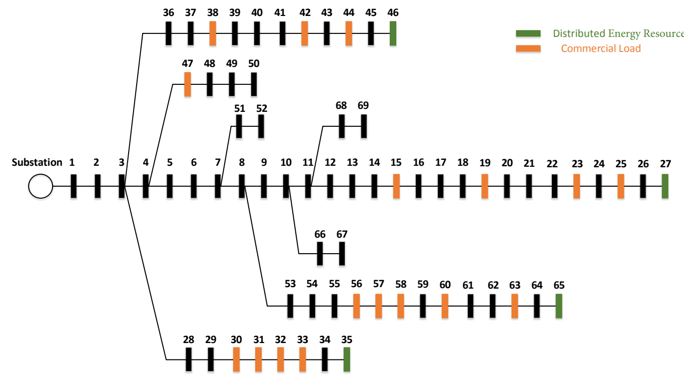

Fig. 2. 69-node distribution system [20].

A. The Tested System

The system is taken from [20], it is shown on Fig. 2. The following modifications have been applied on the system.

- The original voltage tolerance of the system is 10%. To be consistent with the ANSI C84.1 standard, the voltage range is adjusted to 5%. Since this step leads to infeasibility, 90% of the load is dropped.
- In the original system, several nodes do not have loads. In order to analyze the operation under heavier loading, which is reflected on the voltage and active and reactive DLMPs, active and reactive commercial loads are inserted to the empty nodes. Each commercial load ranges 0.10-0.30 MW and 0.01-0.10 MVar for the active and the reactive power, respectively.
- The demand load at each bus is considered to be inelastic. As a result, in unbiased markets, the offer cost minimization is equivalent to the social welfare maximization. The system total active and reactive demand load is 0.2765 MW and 0.0555 MVar, respectively.
- While unit 1 represents the substation, 4 DERs are added to the system. The DER units 2, 3, 4, and 5 are located at the nodes 27, 35, 46, and 65, respectively. The buses that includes DERS are represented as green nodes in Fig. 2.
- The total DER capacity is defined by the MW penetration. We choose 100% penetration of the total load, with a curtailment functionality. That is, each DER has maximum active power amounting to 25% of the MW load, and 0MW as the minimum. The participation of the reactive power is based on the PF that is set to 0.9.
- The substation's and all the DER's costs are set to be equal in this case study. [16] shows the effect of the objective function weights on the shadow prices. The linear active power cost, C^g_{pk}, is \$15, whereas the linear reactive power cost, C^g_{qk}, is \$3. The fixed cost, $C^g_{fixed,k}$, is \$1.

B. The Load Profile

As the procedure of the classical day-ahead transmission market, the model takes the estimated to achieve the optimal

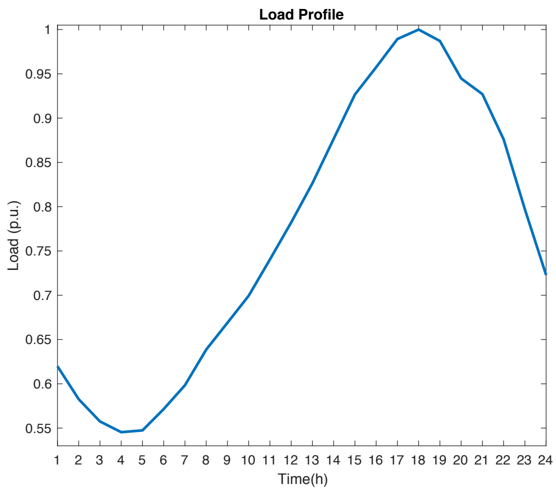

Fig. 3. The day-ahead estimated load [21].

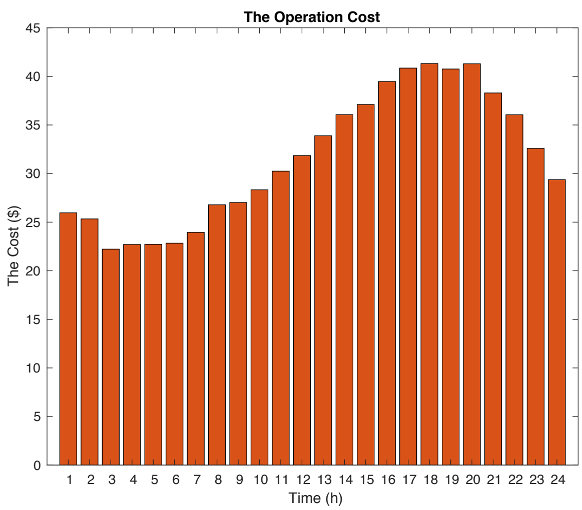

Fig. 4. The operation cost.

operation. The load profile used in the case study is taken from CAISO, [21], which appears in Fig. 3.

C. The System Operation and Participation

The main technical factors that affect the day-ahead unit commitment are the load location and amount, voltage limits, and the line capacity. The optimal solution of our model seeks the lowest operation with the aforementioned elements. The operation cost for the given estimated load is shown in Fig. 4.

The unit commitment is shown in Fig. 5. The first plot shows the unit on/off statuses. The substation, unit 1, is set on all day, whereas the DERs commit based on the location. The active and reactive produced power of the substation and the DERs are shown in the second and the third plots.

D. The Active and Reactive DLMPs Analysis

The day-ahead active and reactive DLMPs are shown in Fig. 6. Since bus 1 is the slack bus, supplied by almost unlimited active and reactive power, its DLMPs remain constant with all

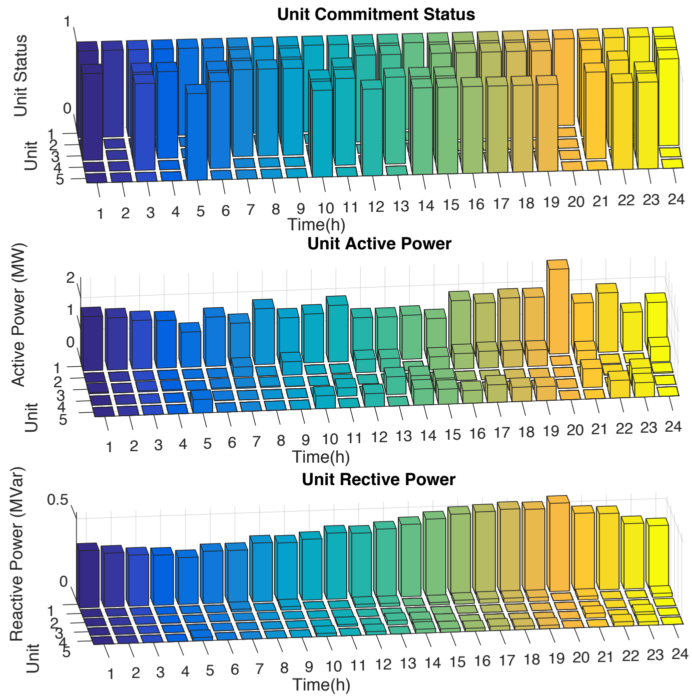

Fig. 5. The operation cost.

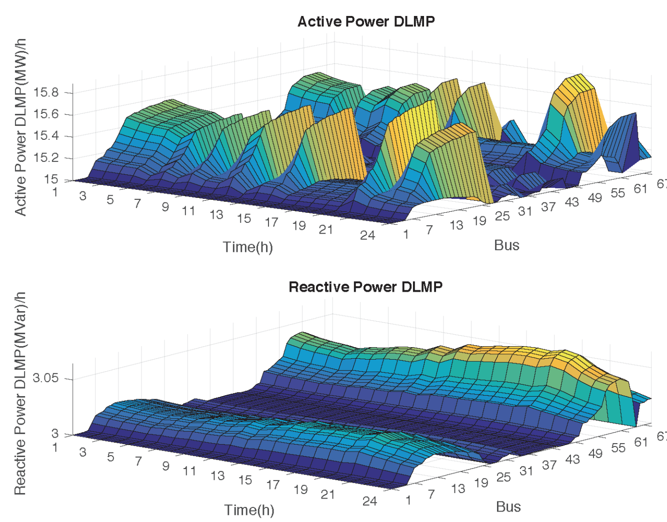

Fig. 6. The Active and Reactive DLMPs.

the time slots. As the nodes get farther from bus 1, the DLMPs start to vary in accordance with the system constraints and load variations. For instance, the active DLMP of node 19, which has commercial load and is located far from the substation, is high, and it drops when the DER at node 27, the closest DER, is turned on. It can be seen that the DLMP at t= 20, while the 27 node DER is on, is $ 15.17, and at t= 21, the 27 node DER turns off, the DLMP rises to $ 15.8. Because of the fact that the voltage magnitude is confined within a tight tolerance, the reactive DLMP variations are less conspicuous than those of the active power DLMPs.

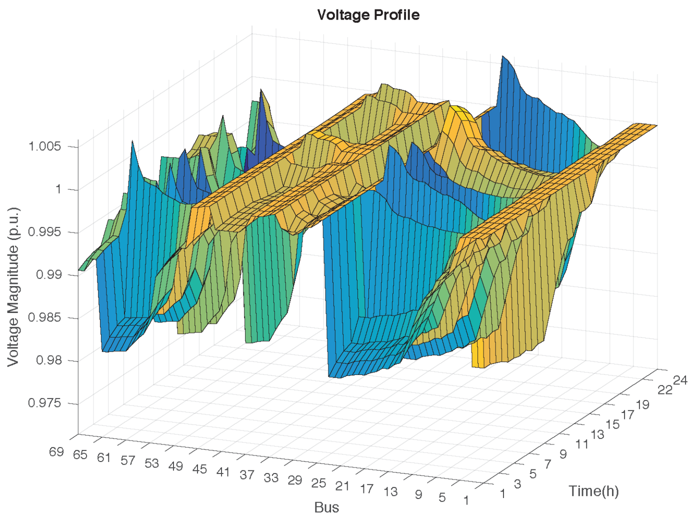

Fig. 7. The voltage profile.

As for the reactive power DLMPs, we observe that they follow a similar pattern as active power the DLMPs, yet with lower fluctuation since the reactive power supply/absorption is further constrained by the PF. It is obvious that bus 60 constitutes the utmost reactive power DLMPs because its respective inductive load is the largest in the system.

E. The System Voltage

The voltages of all the buses at all time horizons are shown in Fig. 7. It is observed from Fig. 6 that DERs mostly managed to keep the magnitude within a close proximity to unity. This shows the advantage of the generation minimization over loss minimization which is reported to yield increased voltages to their upper bound [22], [23].

IV. Conclusion

This paper presents a bilevel model in which its upper level problem commits the optimal DER unit statuses with respect to the lower level problem that takes into account the AC optimal power flow. The proposed model exploits the duality theory and big-M method to arrive at a comprehensive primal-dual MISOCP formulation. The resultant optimization problem shows potential for eliciting both active- and reactive-power DLMPs. The validity of our approach is demonstrated on the 69-bus feeder. For future work, the model can be extended to incorporate demand elasticity with either price-based or incentive-based implementations.

References

[1] A. Hassan, R. Mieth, M. Chertkov, D. Deka, and Y. Dvorkin, "Optimal load ensemble control in chance-constrained optimal power flow," *IEEE Transactions on Smart Grid*, 2018.

[2] NYISO, "A review of distributed energy resources," 2015. [Online]. Available: https://goo.gl/tSHzKQ

[3] T. Council, "Gridwise transactive energy framework version 1.0," *The GridWise Architecture Council, Tech. Rep.*, 2015.

[4] P. De Martini, L. Kristov, and L. Schwartz, "Distribution systems in a high distributed energy resources future," Lawrence Berkeley National Lab.(LBNL), Berkeley, CA (United States), Tech. Rep., 2015.

[5] L. Bai, J. Wang, C. Wang, C. Chen, and F. Li, "Distribution locational marginal pricing (dlmp) for congestion management and voltage support," *IEEE Transactions on Power Systems*, vol. 33, no. 4, pp. 4061–4073, July 2018.

[6] "Distribution resources plan," california Public Utility.

[7] C. Gu, J. Wu, and F. Li, "Reliability-based distribution network pricing," *IEEE Transactions on Power Systems*, vol. 27, no. 3, pp. 1646–1655, Aug 2012.

[8] S. Yin, J. Wang, and F. Qiu, "Decentralized electricity market with transactive energy–a path forward," *The Electricity Journal*, vol. 32, no. 4, pp. 7–13, 2019.

[9] B. Eldridge, R. P. O'Neill, and A. Castillo, "Marginal loss calculations for the dcopf," *Federal Energy Regulatory Commission, Tech. Rep*, 2017.

[10] P. M. Sotkiewicz and J. M. Vignolo, "Nodal pricing for distribution networks: efficient pricing for efficiency enhancing dg," *IEEE transactions on power systems*, vol. 21, no. 2, pp. 1013–1014, 2006.

[11] R. K. Singh and S. Goswami, "Optimum allocation of distributed generations based on nodal pricing for profit, loss reduction, and voltage improvement including voltage rise issue," *International Journal of Electrical Power & Energy Systems*, vol. 32, no. 6, pp. 637–644, 2010.

[12] F. Meng and B. H. Chowdhury, "Distribution lmp-based economic operation for future smart grid," in *2011 IEEE Power and Energy Conference at Illinois*. IEEE, 2011, pp. 1–5.

[13] S. Huang, Q. Wu, S. S. Oren, R. Li, and Z. Liu, "Distribution locational marginal pricing through quadratic programming for congestion management in distribution networks," *IEEE Transactions on Power Systems*, vol. 30, no. 4, pp. 2170–2178, 2015.

[14] R. Li, Q. Wu, and S. S. Oren, "Distribution locational marginal pricing for optimal electric vehicle charging management," *IEEE Transactions on Power Systems*, vol. 29, no. 1, pp. 203–211, 2014.

[15] I. Alsaleh and L. Fan, "Distribution locational marginal pricing (dlmp) for multiphase systems," in *2018 North American Power Symposium (NAPS)*. IEEE, 2018, pp. 1–6.

[16] A. Alassaf and L. Fan, "Bilevel programming-based unit commitment for locational marginal price computation," in *2018 North American Power Symposium (NAPS)*. IEEE, 2018, pp. 1–6.

[17] R. A. Jabr, "Radial distribution load flow using conic programming," *IEEE transactions on power systems*, vol. 21, no. 3, pp. 1458–1459, 2006.

[18] L. Gurobi Optimization, "Gurobi optimizer reference manual," 2018. [Online]. Available: http://www.gurobi.com

[19] M. Grant and S. Boyd, "CVX: Matlab software for disciplined convex programming, version 2.1," http://cvxr.com/cvx, Mar. 2014.

[20] D. Das, "Optimal placement of capacitors in radial distribution system using a fuzzy-ga method," *International Journal of Electrical Power & Energy Systems*, vol. 30, no. 6-7, pp. 361–367, 2008.

[21] "California iso," (last accessed on April 15th, 2019). [Online]. Available: http://www.caiso.com

[22] M. Farivar, R. Neal, C. Clarke, and S. Low, "Optimal inverter var control in distribution systems with high pv penetration," in *2012 IEEE Power and Energy Society general meeting*. IEEE, 2012, pp. 1–7.

[23] I. Alsaleh, L. Fan, and H. G. Aghamolki, "Volt/var optimization with minimum equipment operation under high pv penetration," in *2018 North American Power Symposium (NAPS)*. IEEE, 2018, pp. 1–6.