

Data-Driven Dynamic Model Identification for Synchronous Generators

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Abstract—The objective of this paper is to find parameters of dynamic synchronous generator models through phasor measurement unit (PMU) data. Two estimation models are assumed. The first is an input/output model with electric power measurement as input and frequency as output. The second model assumes a single machine infinite bus system and the measurements are treated as outputs only. For these two models, various system identification methods are applied to achieve the goal, including autoregression exogenous (ARX)-based least squares estimation (LSE), subspace identification through `n4sid`, and linear grey-box model identification. Numerical examples are given to show the effectiveness of dynamic parameter estimation.

Index Terms—Data-driven, ARX, reduced-order model, parameter identification, PMU measurement.

I. INTRODUCTION

With a large amount of Phasor Measurement Units (PMU) installed in the power grid, the measurement data are easier than ever to be collected and analyzed. Data-driven system identification methods may be applied for dynamic model parameter estimation.

Synchronous generator is the main workhorse for electricity generation. Accurate estimation of the generator parameters is important and of practical interest. For example, from PMU data to find frequency response model has been investigated for the ERCOT system in [1], where a simplified dynamic model of ERCOT is validated and tuned for frequency response with PMUs data. The dynamic model includes steam turbine dynamic, gas turbine dynamic, and swing dynamic. The system inertia, governor time constants are estimated based PMU measurements for different contingencies.

In the literature, various approaches have been proposed to estimate synchronous generator parameters, including Kalman Filter approach [2]–[4] and least-square estimation [5]–[7].

Kalman Filter based estimation is done by each time step, and [2] proposed a Kalman Filter approach which aims to estimate the electro-mechanical parameters and states of synchronous machines. Different from Kalman Filter-based estimation, least-square estimation (LSE) are usually performed with a time-window of measurement data. In our previous work [6], LSE is applied to estimate the ARX model's

parameters. Further, from the ARX parameters, synchronous generator's parameters with physical meanings, e.g., inertia constant H , damping D_1 , droop parameter R and turbine-governor time constant T_g , are found. The ARX models in [6] are derived based on a synchronous generator's state-space model with input and output defined.

A challenge faced in [6] is that some of the ARX model parameters are not close to the real values. The current paper tackles the issue by applying scale factors on different measurements. Further, the current paper examines two types of estimation models: the input/output state-space model (Model 1) and the output only state-space model (Model 2). Based on both state-space models, black-box (ARX-based LSE and subspace `n4sid` of MATLAB system identification toolbox [8]) and grey-box model identification methods in [8] are applied.

In this paper, the measurement data for estimation is generated from dynamic model simulation in Power System Toolbox [9] via MATLAB. For the input/output model (Model 1), the ARX model coefficients are estimated via least-square estimation and subspace estimation (e.g., `n4sid` function of MATLAB System Identification toolbox). These two estimation methods can be viewed as black-box identification methods, since the structure of the estimation model is unknown. In the next stage, synchronous generator parameters are recovered from estimated coefficients.

For the output only estimation model (Model 2), a synchronous generator connects to an infinite bus (SMIB) system is investigated. Model 2 is estimated from the bus voltage magnitude measurement only. The discrete state-space model of Model 2 is derived first. With this step, the state-space model structure is known. It is then estimated via the Linear Grey-box Estimation from System Identification Toolbox in MATLAB [8].

The rest of the paper is organized as follows. Section II presents the derivation, system identification, parameter recovery, and case study of the input/output estimation model (Model 1). Section III presents the derivation, parameter recovery, and case study of the output-only estimation model (Model 2). Section IV presents the conclusion.

II. MODEL 1: INPUT/OUTPUT MODEL

Model 1 is the input/output model derived for synchronous generator parameter identification. The estimation model of

a synchronous generator with the primary frequency control (droop control) has been discussed in details in [6], and the ARX model has been derived from the discrete state-space model. Fig. 1 shows the block diagram of the small-signal model of the synchronous generator, where H stands for the machine inertia time constant, D_1 is the damping coefficient, T_g is the turbine governor time constant, and R is the primary frequency control droop parameter. In power grid, droop parameter is usually set at 5%, which means that the change of frequency will be 5% or 3 Hz if the nominal MW of the generator is lost.

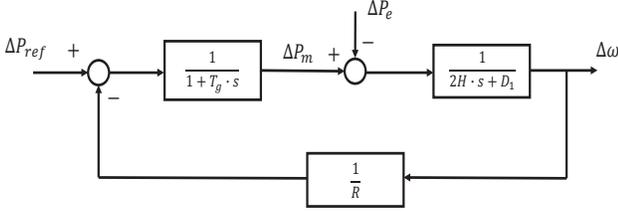


Fig. 1: Block diagram of the simplified synchronous generator small-signal model with primary frequency control.

A. State-space model to ARX model

The dynamic model of a synchronous generator with primary frequency control has been discussed in many textbooks, e.g., [10]. The small-signal model of the swing dynamic and the turbine governor dynamic for mechanical power control are presented in (1).

$$\begin{cases} \Delta \dot{\delta} &= \omega_0 \Delta \omega \\ \Delta \dot{\omega} &= \frac{1}{2H} (\Delta P_m - \Delta P_e - D_1 (\Delta \omega - 1)) \\ \Delta \dot{P}_m &= \frac{1}{T_g} (\Delta P_{ref} - \frac{1}{R} \Delta \omega - \Delta P_m) \end{cases} \quad (1)$$

where δ is the machine rotor angle, ω is the machine speed, P_m is the mechanical power, P_e is the electric power, and P_{ref} is the reference power and kept constant during primary frequency response period.

For Model 1, the input of the estimation model is selected as the machine electric power deviation (ΔP_e), and the machine speed deviation is set as the output. The input/output may come from PMU measurements. The continuous state-space model is as follows.

$$\begin{bmatrix} \Delta \dot{\delta} \\ \Delta \dot{\omega} \\ \Delta \dot{P}_m \end{bmatrix} = \begin{bmatrix} 0 & \omega_0 & 0 \\ 0 & -\frac{D_1}{2H} & \frac{1}{2H} \\ 0 & -\frac{1}{T_g R} & -\frac{1}{T_g} \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta \omega \\ \Delta P_m \end{bmatrix} + \begin{bmatrix} 0 \\ -\frac{1}{2H} \\ 0 \end{bmatrix} \Delta P_e \quad (2a)$$

$$\Delta \omega = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta \omega \\ \Delta P_m \end{bmatrix} \quad (2b)$$

The continuous model is discretized for time interval h .

$$\begin{bmatrix} \Delta \delta_{k+1} \\ \Delta \omega_{k+1} \\ \Delta P_{m_{k+1}} \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & \omega_0 h & 0 \\ 0 & 1 - \frac{D_1 h}{2H} & \frac{h}{2H} \\ 0 & -\frac{h}{T_g R} & 1 - \frac{h}{T_g} \end{bmatrix}}_A \begin{bmatrix} \Delta \delta_k \\ \Delta \omega_k \\ \Delta P_{m_k} \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ -\frac{h}{2H} \\ 0 \end{bmatrix}}_B \Delta P_{e_k} \quad (3a)$$

$$\Delta \omega_k = \underbrace{\begin{bmatrix} 0 & 1 & 0 \end{bmatrix}}_C \begin{bmatrix} \Delta \delta_k \\ \Delta \omega_k \\ \Delta P_{m_k} \end{bmatrix} \quad (3b)$$

The input/output relationship of the state-space model may be expressed using z -transformation.

$$\frac{y}{u} = C(zI - A)^{-1}B, \quad (4)$$

$$\Rightarrow A(z)y = B(z)u \quad (5)$$

where I should be a 3×3 identity matrix.

Expanding (4), the denominator and numerator are expressed as polynomials.

$$A(z) = 1 + a_1 z^{-1} + a_2 z^{-2} \quad (6)$$

$$B(z) = b_1 z^{-1} + b_2 z^{-2}, \quad (7)$$

where a_1 , a_2 , b_1 , and b_2 are the ARX model coefficients.

B. ARX approach & n4sid sub-space method

To solve the coefficients, the ARX model is transferred into an over-determined problem ($Y = \lambda \cdot X$) with discrete input and output data and shown in (8).

$$\underbrace{\begin{bmatrix} y(k+2) \\ \vdots \\ y(i) \\ \vdots \\ y(N) \end{bmatrix}}_Y = \underbrace{\begin{bmatrix} y(k+1) & y(k) & u(k+1) & u(k) \\ \vdots & \vdots & \vdots & \vdots \\ y(i-1) & y(i-2) & u(i-1) & u(i-2) \\ \vdots & \vdots & \vdots & \vdots \\ y(N-1) & y(N-2) & u(N-1) & u(N-2) \end{bmatrix}}_\lambda \cdot \underbrace{\begin{bmatrix} -a_1 \\ -a_2 \\ b_1 \\ b_2 \end{bmatrix}}_X \quad (8)$$

Y is the output of the over-determined problem, λ is the Hankel matrix formed by measurement data, and X is the ARX model coefficients vector.

By applying (9), the over-determined problem is solved and ARX model coefficients are estimated.

$$X = (\lambda^T \cdot \lambda)^{-1} \cdot \lambda^T \cdot Y \quad (9)$$

Alternatively, we may use *n4sid*, a subspace method-based black-box estimation tool in MATLAB system identification toolbox to find the ARX parameters.

This approach assumes the estimation model structure is a black-box and requires the information of the input/output measurements, time interval, and the desired model order to process estimation. An example of operating *n4sid* estimation is shown below.

```
data = iddata(y, u, h);
nx = 2;
sys = n4sid(data, nx),
```

where *iddata* is the command to manage input/output (y/u) data and fixed step time, nx is the given estimation model order, and *sys* is the output estimated model in state-space format. From the state-space model, we may further find its z transfer function or ARX model parameters.

The *n4sid* command output is shown as follows when nx is given at 2.

```
Discrete-time identified state-space model:
x(t+Ts) = A x(t) + B u(t) + K e(t)
y(t) = C x(t) + D u(t) + e(t)
```

```

A =
      x1      x2
x1  0.9962  0.002401
x2 -0.00286  0.9993
B =
      u1
x1 -0.000794
x2 -0.0006823
C =
      x1      x2
y1  0.0164 -0.006509
D =
      u1
y1  0
K =
      y1
x1  7436
x2  1.853e+04
Sample time: 0.001 seconds
Status:
Estimated using N4SID on time domain data
"data".
Fit to estimation data: 100%
FPE: 8.334e-23, MSE: 8.311e-23

```

Then, the estimated state-space model is transferred to transfer function form that is shown in (4) as $C(zI - A)^{-1}B$, so that the polynomials of the denominator and numerator are obtained. This gives the estimation of the ARX model coefficients. The operation and output transfer function (ARX model) are shown as follows.

```

[NUM, DEM] = ss2tf(A, B, C, D);
sys = tf(NUM, DEM, h);
sys =
-----
-8.579e-06 z + 8.545e-06
-----
z^2 - 1.996 z + 0.9956

```

From the transfer function generated from the estimated state-space model, the coefficients of the ARX model are found.

C. Parameter Recovery

The polynomials for $A(z)$ and $B(z)$ can be expressed by generator parameters. The expressions of ARX model coefficients are shown in (10).

$$a_1 = \frac{2H \cdot h - 4H \cdot T_g + D_1 \cdot T_g \cdot h}{2 \cdot H \cdot T_g} \quad (10a)$$

$$a_2 = \frac{h^2 + R \cdot (2H \cdot T_g - 2H \cdot h + D_1 \cdot h^2 - D_1 \cdot T_g \cdot h)}{2H \cdot R \cdot T_g} \quad (10b)$$

$$b_1 = -\frac{h}{2H} \quad (10c)$$

$$b_2 = \frac{h \cdot (T_g - h)}{2H \cdot T_g} \quad (10d)$$

Based on (10), the parameters of physical meaning can be recovered.

$$H = -\frac{h}{2 \cdot b_1} \quad (11a)$$

$$T_g = \frac{b_1 \cdot h}{b_1 + b_2} \quad (11b)$$

$$D_1 = -\frac{b_1 - b_2 + a_1 \cdot b_1}{b_1^2} \quad (11c)$$

$$R = -\frac{b_1^2 \cdot (b_1 + b_2)}{a_2 \cdot b_1^2 - a_1 \cdot b_1 \cdot b_2 + b_2^2} \quad (11d)$$

Remarks: The synchronous generator's parameters can be estimated in two stages. In the first stage, a black-box model is estimated. In the second stage, the physical parameters are recovered from the black-box model parameters.

D. Case Study

A 2-area 4-machine dynamic model is simulated in Power System Toolbox via MATLAB and provides the measurement data for Model 1 estimation. The synchronous machine type is selected as classical model with swing dynamics only. Fig. 2 shows the single-line view system diagram of the 2-area 4-machine system.

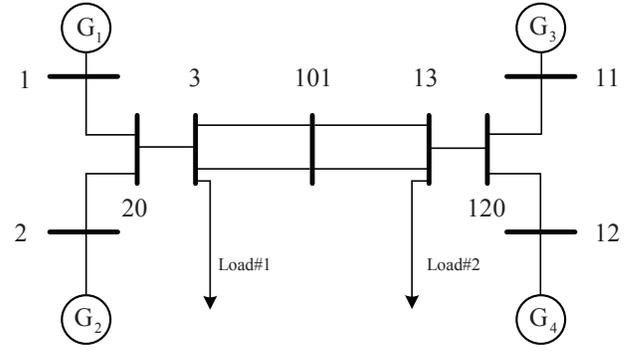


Fig. 2: The single-line view system diagram of 2-area 4-machine testbed.

A three-phase fault is designed to create a dynamic event on the transmission line between Bus 3 and Bus 101 at 1.1 second. At 1.15 second, the near end fault is cleared, and the remote end fault is cleared at 1.2 second. For Model 1 estimation, the input signal is ΔP_e and the output signal is $\Delta \omega$. For all four synchronous generators, the input/output measurement signals are plotted in Fig. 3.

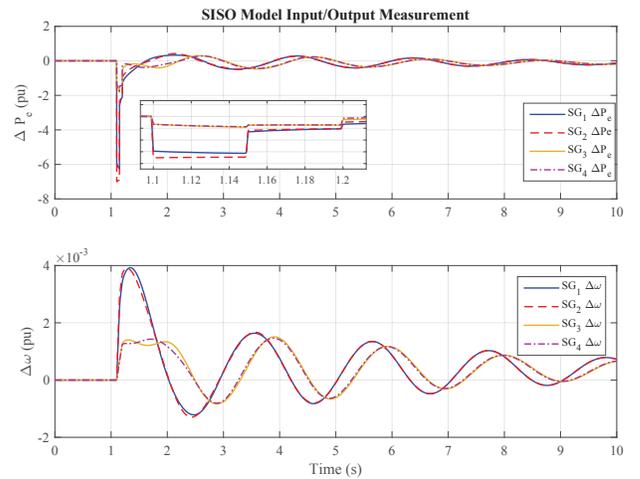


Fig. 3: Input/output measurements for Model 1 Estimation.

The parameter setup for each synchronous generators is the same. The reference values of parameters and computed ARX model coefficients are presented in Table.I.

In this case study, both ARX-based LSE approach and *n4sid* method are examined. Speed deviation dynamic responses

TABLE I: Model 1 Parameters & Coefficients Reference Values

Parameter	Reference Value	Parameter	Reference Value
H	58.5	D_1	54
T_g	0.25	R	0.004444
Coefficient	Reference Value	Coefficient	Reference Value
a_1	-1.995538	a_2	0.995548
b_1	-8.547009E-6	b_2	8.512821E-6

after the last event will be used for estimation. It is found that the best estimations of four generators are around 1.33 second. The numerical results from the estimations of Synchronous Generator 1 (Area 1) and Synchronous Generator 3 (Area 2) are compared in Tables II and III. *Note, it is important to ensure the scales of input/output signals are at the same level to have accurate estimation.* The scale of $\Delta\omega$ is enlarged by 1000 times for estimation to have the same level variance as ΔP_e .

TABLE II: Estimations of Synchronous Generator 1

Coefficient	Reference	ARX-base LSE		$n4sid$	
		Estimation	Error rate (%)	Estimation	Error rate (%)
a_1	-1.995538	-1.995535	0.00015	-1.995549	0.00055
a_2	0.995548	0.995545	0.00030	0.995559	0.00110
b_1	-8.5470E-6	-8.5879E-6	0.4774	-8.5792E-6	0.37674
b_2	8.5128E-6	8.5534E-6	0.4769	8.5452E-6	0.3806
Parameter	Reference	Estimation	Error rate (%)	Estimation	Error rate (%)
H	58.5	58.2212	0.4789	58.2805	0.3752
D_1	54	52.0740	3.5667	57.2394	5.9989
T_g	0.25	0.2489	0.4223	0.2520	0.8
R	0.004444	0.004535	1.9918	0.004578	3.0153

TABLE III: Estimations of Synchronous Generator 3

Coefficient	Reference	ARX-base LSE		$n4sid$	
		Estimation	Error rate (%)	Estimation	Error rate (%)
a_1	-1.995538	-1.995552	0.0007	-1.995549	0.0005
a_2	0.995548	0.995561	0.00131	0.995559	0.00111
b_1	-8.5470E-6	-8.5887E-6	0.4879	-8.5792E-6	0.3767
b_2	8.5128E-6	8.5547E-6	0.4922	8.5452E-6	0.3806
Parameter	Reference	Estimation	Error rate (%)	Estimation	Error rate (%)
H	58.5	58.2158	0.4858	58.2806	0.3750
D_1	54	56.9436	5.4881	56.4457	4.5291
T_g	0.25	0.2526	1.04	0.2521	0.84
R	0.004444	0.004525	1.8227	0.004502	1.3051

From the comparison of the numerical results shown in Table II, both methods can deliver precise estimation of the ARX model coefficients, and the generator parameter identification produced by both approaches show estimation close to the real values.

To evaluate the identified parameters, the dynamic behavior is replayed. The same input signals are fed into the estimated state-space models and the generated output signals are compared to the original signals from Power System Toolbox simulation. The replays start at 1.1 second to include the large disturbance due to the designed fault.

Bode diagram is employed to compare the estimated systems from both approaches based on the perspective of system stability. Fig. 5 shows the Bode diagrams of both estimated models. It can be seen that the proposed two-stage estimation approach is capable of accurate parameter identification.

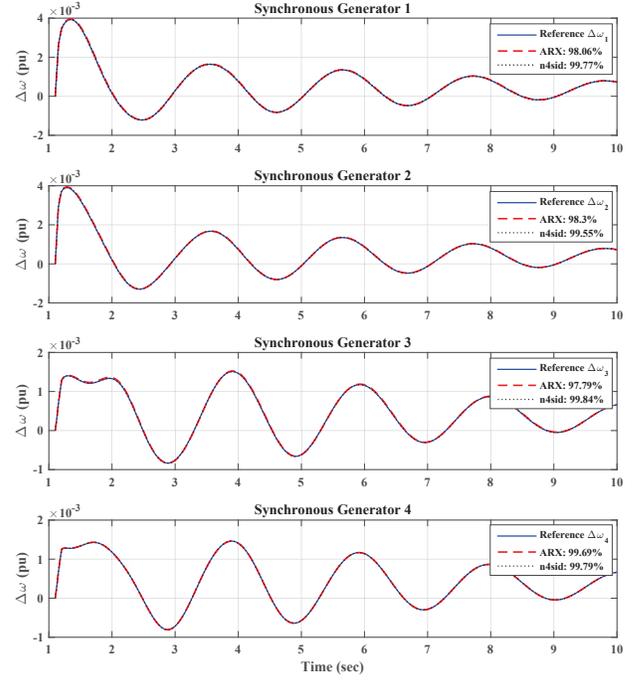


Fig. 4: Replays of dynamic behaviors for estimated models by ARX-based LSE approach and $n4sid$ method. Goodness of fit to the reference $\Delta\omega$ is shown in percentage.

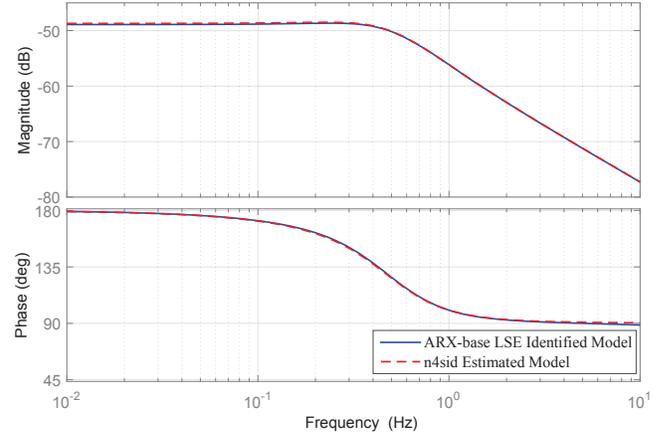


Fig. 5: Bode diagram comparison between ARX-based LSE estimated model and $n4sid$ estimated model.

III. MODEL 2: OUTPUT ONLY MODEL

Model 2 is the output only estimation model of a single-machine to infinite bus (SMIB) system. The single-line SMIB system is shown in Fig. 6, where E_a is the generator phase A internal voltage magnitude, δ is the machine's angle, $Ve^{j\theta}$ is the generator bus voltage phasor, X_s is the synchronous reactance, and X_L is the reactance of the transmission network. In this paper, the voltage on infinite bus is assumed as $1\angle 0^\circ$.


```

Par2 = 0.6847 | Par6 = 0.09491
Par3 = 450   | Par7 = 7.987
Par4 = 0     | Par8 = 1.489

Sample time: 0.001 seconds
Status:
Estimated using GREYEST on time domain
data "data".
Fit to estimation data: 94.49%
FPE: 3.884e-09, MSE: 3.877e-09

```

C. Case Study

The SMIB system is built in Power System Toolbox. The synchronous generator type is classic machine model. The infinite bus is modeled by a large synchronous generator with extremely high inertia time constant. The dynamic event is designed to be a three-phase fault on the transmission line at 0.1 second. The output signal is designed to be the generator bus voltage magnitude's square, V^2 . The deviation of ΔV^2 is shown in Fig. 7.

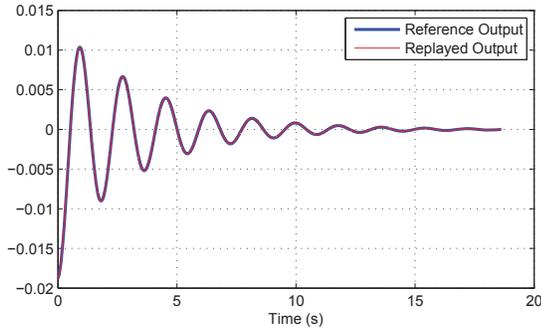


Fig. 7: The plot of measurement used for Model 2 estimation. The fixed step time is 0.001 second.

The parameters are identified using grey-box identification. Among the 8 parameters, 3 are chosen to be fixed to reduce the freedom. These three parameters are the initial rotor angle, line impedance, and synchronous reactance. True values are used as initial parameters except $\text{Par3} = 1/(T_g R)$. The true value is 450 while 400 is used as the initial value. Table IV shows the numerical results of the 8 parameters from Linear Grey-box Estimation.

TABLE IV: Reference Parameters of SMIB.

Parameter	Reference	Estimation	Parameter	Reference	Estimation
H_2	0.008547	0.008799	X_s	0.02778	fixed
DH_2	0.076923	0.1273	X_L	0.3	fixed
T_{gr1}	450.045	400	$E_a X$	3.1881	3.177
T_{g1}	2	1.95	δ^0	0.5029	fixed

The numerical results of estimation show that not all parameters can be accurately identified. $\text{Par1} = \frac{1}{2H}$ can be accurately identified. $\text{Par4} = 1/T_g$ and $\text{Par7} = E_a/\hat{X}$ can be identified with close-match values. On the other hand, $\text{Par2} = \frac{D_1}{2H}$ and $\text{Par3} = \frac{1}{T_g R}$ cannot be identified accurately.

Even though, the output from the estimation model can match well with the measurement data. The comparison is shown in Fig. 7. One reason is that both D_1 and R contribute to system damping. To have more accurate estimation, we may explore enhancement in future work, including have more

event data and put constraints on parameters to reduce the search region.

IV. CONCLUSION

In this paper, data-drive system identification approaches for dynamic models of synchronous generator are presented. Two types of estimation models (input/output model and output only model) are examined and two types of identification approaches (black-box and grey-box) are examined. The proposed approaches, along with specific data handling techniques (e.g., proper scaling), lead to reasonable parameter estimation.

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