DC State Estimation Model-Based Mixed Integer Semidefinite Programming for Optimal PMU Placement

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Outline

1. Introduction of Optimal PMU Placement
2. Measurement Model
3. Mixed Integer Linear Programming Formulation Overview
4. Mixed Integer Semidefinite Programming Formulation
5. Four Case Studies
6. Conclusion

Introduction

- The power system security needs to have a detailed monitor to the operating conditions of the system.

- In the control center, state estimator has the ability to deal with the measurements that received form the remote terminal units (RTUs) at the substations.

- Phasor measurements units (PMUs) can provide synchronized phasor measurements of voltages and currents.

- PMUs will make the performance of the state estimator improved due to the synchronized measurements during the dynamic event which make the state estimation more accurate.

(Xu & Abur, 2004 – Hurtgen & Maun, 2008 – Gou, 2008)

Introduction

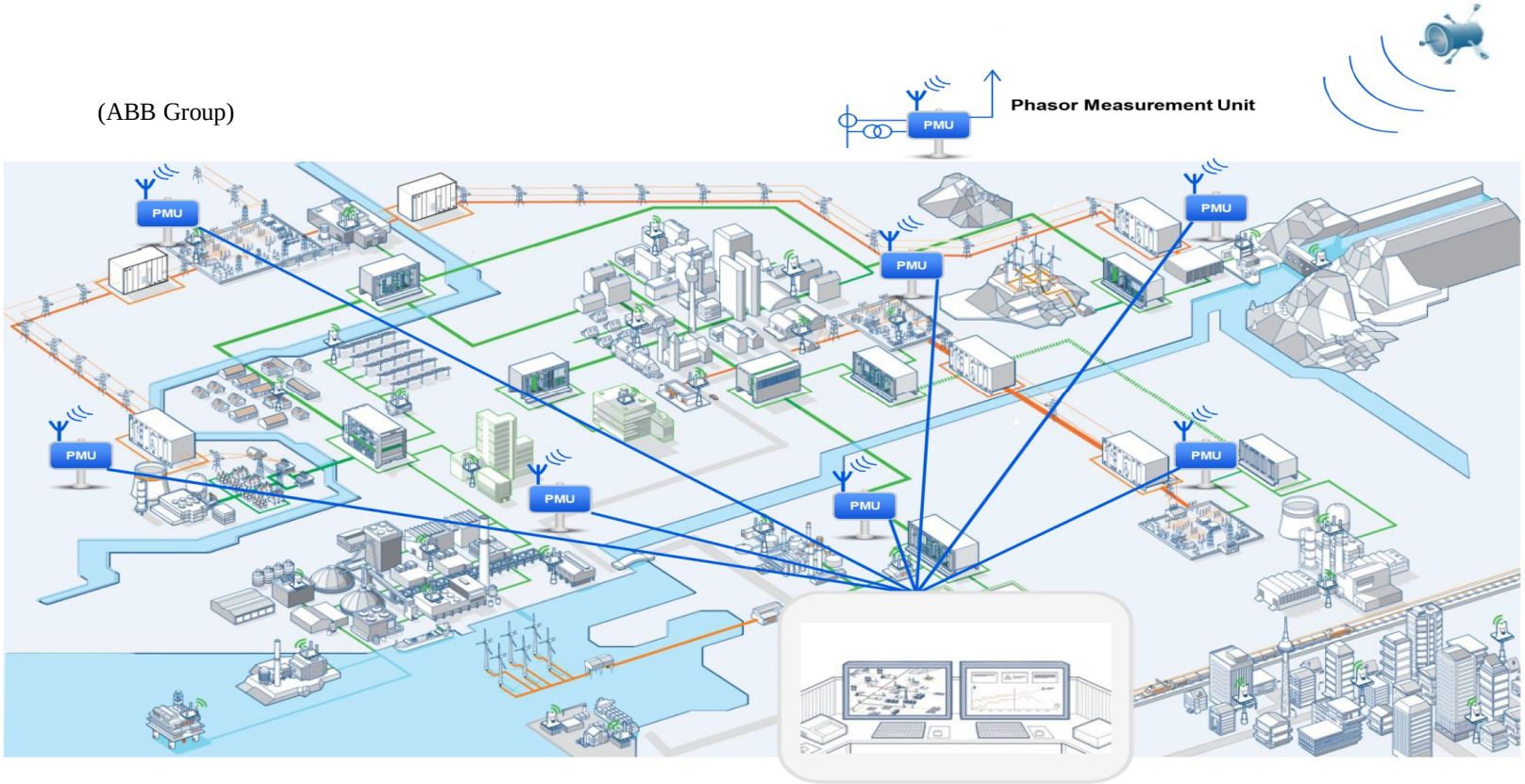
- When a PMU is placed at one bus, it can measure the bus phasor voltage and phasor currents of all lines connected to that bus making the system observable.

- Then the problem that needs to be solved is to find the optimal placement for PMUs that can make the system observable using PMUs at certain buses.

(Xu & Abur, 2004 – Hurtgen & Maun, 2008 – Gou, 2008)

Introduction

(ABB Group)



The overall cost of the PMU installation is ranged from $40,000 - $180,000. (U.S. Department of Energy, 2014)

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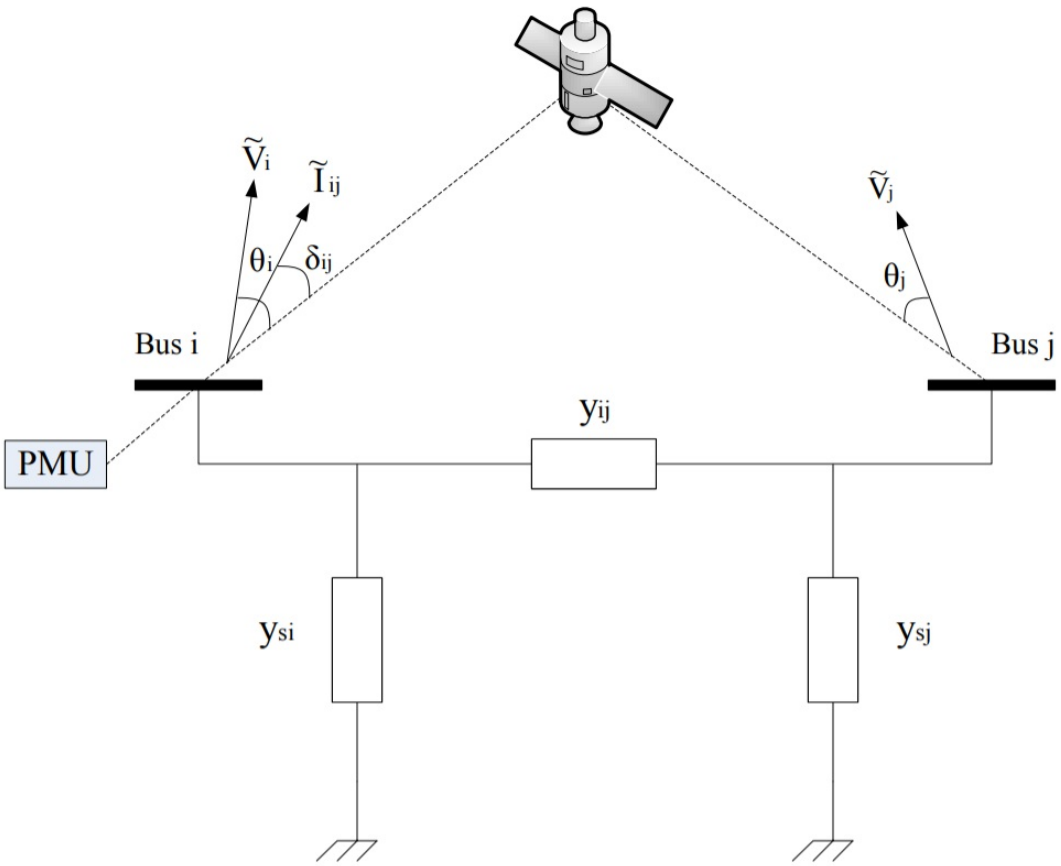
Measurement Model



Measurement Model

$$P_{ij}^{\text{meas}} = (E_i^2 + F_i^2)(g_{si} + g_{ij}) + E_i(-g_{ij}E_j + b_{ij}F_j) - F_i(b_{ij}E_j + g_{ij}F_j) + e_{P_{ij}^{\text{meas}}} \quad (1)$$

$$Q_{ij}^{\text{meas}} = -(E_i^2 + F_i^2)(b_{si} + b_{ij}) + E_i(b_{ij}E_j + g_{ij}F_j) + F_i(-g_{ij}E_j + b_{ij}F_j) + e_{Q_{ij}^{\text{meas}}} \quad (2)$$

$$P_i^{\text{meas}} = (E_i^2 + F_i^2) \sum_{j \in Sad_i} (g_{si} + g_{ij}) + E_i \sum_{j \in Sad_i} (-g_{ij}E_j + b_{ij}F_j) - F_i \sum_{j \in Sad_i} (b_{ij}E_j + g_{ij}F_j) + e_{P_i^{\text{meas}}} \quad (3)$$

$$Q_i^{\text{meas}} = -(E_i^2 + F_i^2) \sum_{j \in Sad_i} (b_{si} + b_{ij}) + E_i \sum_{j \in Sad_i} (b_{ij}E_j + g_{ij}F_j) + F_i \sum_{j \in Sad_i} (-g_{ij}E_j + b_{ij}F_j) + e_{Q_i^{\text{meas}}} \quad (4)$$

(Abur & Exposito, 2004 – Korres & Manousakis, 2011)

Measurement Model

$$I_{ij,\text{real}}^{\text{meas}} = (g_{si}+g_{ij})E_i - (b_{si}+b_{ij})F_i - (g_{ij}E_j - b_{ij}F_j) + e_{I_{ij,\text{real}}^{\text{meas}}} \quad (5)$$

$$I_{ij,\text{imag}}^{\text{meas}} = (b_{si}+b_{ij})E_i + (g_{si}+g_{ij})F_i - (b_{ij}E_j + g_{ij}F_j) + e_{I_{ij,\text{imag}}^{\text{meas}}} \quad (6)$$

$$E_i^{\text{meas}} = E_i + e_{E_i^{\text{meas}}} \quad (7)$$

$$F_i^{\text{meas}} = F_i + e_{F_i^{\text{meas}}} \quad (8)$$

(Abur & Exposito, 2004 – Korres & Manousakis, 2011)

Measurement Model

Then consider the measurement model of a DC power system state estimation as the following:

$$z = Hx + e \qquad (9)$$

The linear weighted least squares (WLS) state estimation will be as follows.

$$G\hat{x} = H^T R^{-1} z \qquad (10)$$

where:

$$G = H^T R^{-1} H$$

(Abur & Exposito, 2004 – Korres & Manousakis, 2011)

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OPP Using Mixed Integer Linear Programming

- Integer linear programming formulation used to find the specific buses for installing PMUs and assure the system will be observable.

$$\min \sum_{k=1}^{N} x_k$$

$$\text{subject to: } Ax \geq B$$

$$x = \begin{bmatrix} x_1 & x_2 & \cdots & x_N \end{bmatrix}^T$$

$$x_i \in \{0, 1\}$$

$$B = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}$$

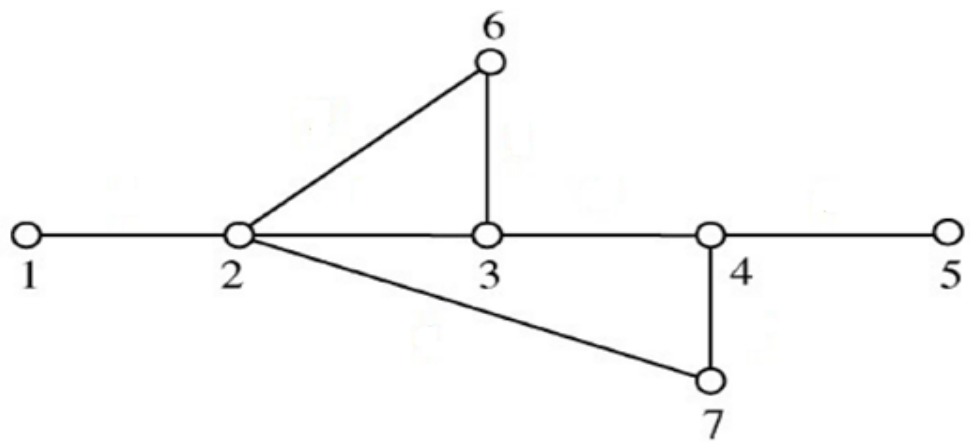
B is k × 1 matrix where k is the number of buses.

(Gou, 2008)

OPP Using MILP-Example

$$\min \sum_{k=1}^{7} x_k$$

subject to: $Ax \geq B$

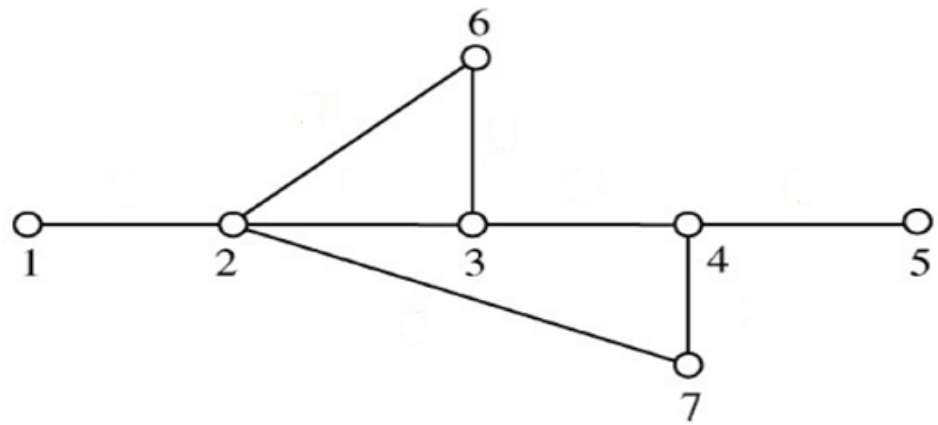


$$A = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \quad x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{bmatrix} \quad \geq \quad B = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

(Gou, 2008)

OPP Using MILP-Example

$$\min \sum_{k=1}^{7} x_k$$



Subject to :

$$C(x) = \begin{cases} x_1 + x_2 \geq 1 \\ x_1 + x_2 + x_3 + x_6 + x_7 \geq 1 \\ x_2 + x_3 + x_4 + x_6 \geq 1 \\ x_3 + x_4 + x_5 + x_7 \geq 1 \\ x_4 + x_5 \geq 1 \\ x_2 + x_3 + x_6 \geq 1 \\ x_2 + x_4 + x_7 \geq 1 \end{cases} \Rightarrow x = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

Placement at bus 2 and 5.

(Gou, 2008)

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OPP Using Mixed Integer Semidefinite Programming

• Optimal PMU placement can be formulated as mixed integer semidefinite programming subjected to linear matrix inequality.

$$\min_{x} \quad \sum_{i=1}^{N} w_i \; x_i$$

$$\text{subject to:} \quad G(x) = G_0 + \sum_{i=1}^{N} x_i G_i \succ 0$$

$$x_i \in \{0,1\}, i = 1, \cdots, N$$

where :

- $G(x)$ is a full rank matrix, and $G(x) \succ 0$ is the positive semidefinite constraint.
- $G_0(x)$= $H_0^T R_0^{-1} H_0$, and $G_i(x)$= $H_i^T R_i^{-1} H_i$

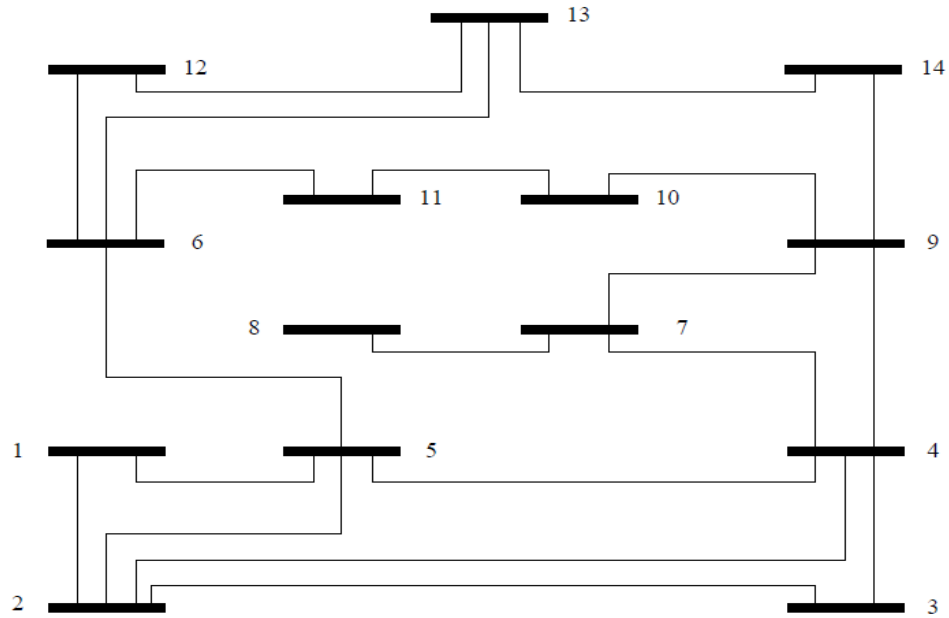
(Manousakis & Korres, 2017)

OPP Using MISDP-Example

$$\min_{x} \quad \sum_{i=1}^{14} w_i \; x_i$$

$$\text{subject to:} \quad G(x) = G_0 + \sum_{i=1}^{14} x_i G_i \succ 0$$

$$x_i \in \{0, 1\}, i = 1, 2, \cdots, 14$$



(Power System Case Archive by University of Washington.)

IEEE 14-bus system

OPP Using MISDP-Example

Then the Jacobian matrix H_0 is zero since it is assumed that there is no flow or injection measurements, and H_i can be found as follows.

For Bus 1: $H_1 =$

$$
\begin{array}{c}
 \\ F_1^{\text{meas}} \\ I_{1-2}^{\text{meas}} \\ I_{1-5}^{\text{meas}}
\end{array}
\begin{array}{c}
\begin{array}{cccccccccccccc} F_1 & F_2 & F_3 & F_4 & F_5 & F_6 & F_7 & F_8 & F_9 & F_{10} & F_{11} & F_{12} & F_{13} & F_{14} \end{array} \\
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
16.9 & -16.9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
4.5 & 0 & 0 & 0 & -4.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\end{array}
$$

For Bus 2: $H_2 =$

$$G_i(x) = H_i^T R_i^{-1} H_i$$

$$
\begin{array}{c}
 \\ F_2^{\text{meas}} \\ I_{2-1}^{\text{meas}} \\ I_{2-3}^{\text{meas}} \\ I_{2-4}^{\text{meas}} \\ I_{2-5}^{\text{meas}}
\end{array}
\begin{array}{c}
\begin{array}{cccccccccccccc} F_1 & F_2 & F_3 & F_4 & F_5 & F_6 & F_7 & F_8 & F_9 & F_{10} & F_{11} & F_{12} & F_{13} & F_{14} \end{array} \\
\begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-16.9 & 16.9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 5.1 & -5.1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 5.7 & 0 & -5.7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 5.8 & 0 & 0 & -5.8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\end{array}
$$

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OPP MISDP Formulation with Power Flow Measurements

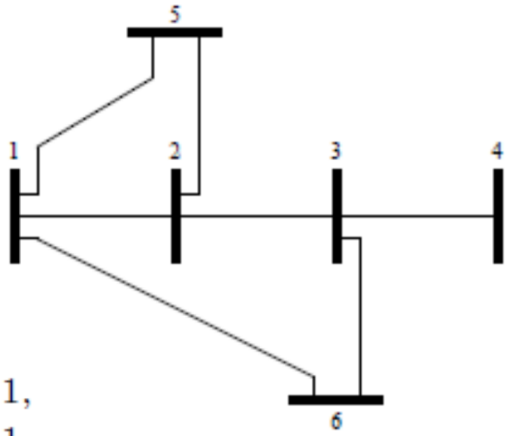
- The Jacobian matrix H_0 will consider the power flow measurements.
- Suppose that there are power flow measurements on lines 2-3, 3-4, 6-11, 6-12, and 7-8 in the IEEE 14-bus system. Then the Jacobian matrix H_0 will be as follows.

$$H_0 = \begin{array}{c} \\ P_{2-3}^{\text{meas}} \\ P_{3-4}^{\text{meas}} \\ P_{6-11}^{\text{meas}} \\ P_{6-12}^{\text{meas}} \\ P_{7-8}^{\text{meas}} \end{array} \begin{array}{c} \begin{array}{cccccccccccccc} F_1 & F_2 & F_3 & F_4 & F_5 & F_6 & F_7 & F_8 & F_9 & F_{10} & F_{11} & F_{12} & F_{13} & F_{14} \end{array} \\ \begin{bmatrix} 0 & 5.1 & -5.1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 5.8 & -5.8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 5 & 0 & 0 & 0 & 0 & -5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3.9 & 0 & 0 & 0 & 0 & 0 & -3.9 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 5.7 & -5.7 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

OPP MISDP Formulation with Injection Measurements

- The mixed integer semidefinite programming can solve the problem with a compact and simple formulation.
- The Jacobian matrix H_0 will consider the injection measurements.

$$H_0 = \begin{array}{c} \\ P_1^{\text{meas}} \\ P_3^{\text{meas}} \end{array} \begin{array}{c} \begin{array}{cccccc} F_1 & F_2 & F_3 & F_4 & F_5 & F_6 \end{array} \\ \begin{bmatrix} 3 & -1 & 0 & 0 & -1 & -1 \\ 0 & -1 & 3 & -1 & 0 & -1 \end{bmatrix} \end{array}$$



Six-bus system

- MILP Formulation will be:
  (Khajeh, Bashar, Rad, & Gharehpetian, 2017)

$$f_{inj,1\&3} = \begin{cases} f_1 + f_2 + f_3 \geq 1, & f_1 + f_2 + f_4 \geq 1, & f_1 + f_2 + f_6 \geq 1, \\ f_1 + f_3 + f_6 \geq 1, & f_1 + f_4 + f_6 \geq 1, & f_2 + f_3 + f_5 \geq 1, \\ f_2 + f_4 + f_5 \geq 1, & f_2 + f_5 + f_6 \geq 1, & f_3 + f_5 + f_6 \geq 1, \\ f_4 + f_5 + f_6 \geq 1, & f_2 + f_3 + f_6 \geq 1, & f_2 + f_4 + f_6 \geq 1, \\ f_1 + f_2 + f_6 \geq 1, & f_1 + f_5 \geq 1, & f_3 + f_4 \geq 1, \quad f_2 + f_6 \geq 1. \end{cases}$$

OPP MISDP Formulation-Limited Communication Facility

- The limited communication facility in the substation can prevent the PMU installation due to the lack of data links required to enable the communication between PMUs and control center.

- This problem can affect the installation cost of the PMU to be much higher.

- Thus, a high installation cost w_i will be assigned to the bus that has a limited communication facility for both MILP and MISDP.

- Consequently, the high installation cost will exclude the limited communication facility buses from the optimal set.

(Almunif & Fan, 2017)

OPP MISDP Formulation with Single PMU Failure

- It is known that the PMUs are reliable devices, but there is a possibility that a single PMU fails.
- In order to protect the system from losing one PMU and leaving the system unobservable, the optimal PMU set is divided into two sets: (Almunif & Fan, 2017)

1. The main set which is the set that obtained without PMU failure .
2. The backup set which is the set that we are going to obtain by removing all of the terms x_i that related to the main set from the observability constraints.

$$\min_{x} \quad \sum_{i=1}^{N} w_i \; x_i$$

$$\text{subject to:} \quad G(x) = G_0 + \sum_{i=1}^{N} x_i G_i \succ 0$$

$$\boxed{x_i = 0, \forall i \in \mathbf{M}}$$

$$x_i \in \{0, 1\}, i = 1, \cdots, N$$

(Almunif & Fan, 2017)

OPP Problem Simulation Results

- The MILP optimization problem is solved by MATLAB *intlinprog* function.
- The MISDP optimization problem is solved using MATLAB toolbox YALMIP with an outer approximation solver SCIP.
- DC model is used for the MISDP formulation with standard deviations of 0.0076, 0.016, and 0.0001 for power flows, injections, and phasor measurements (voltage and current), respectively.
- A standard deviation of 0.00002 is taken for zero injections.

OPP Problem Simulation Results

TABLE I. COMPARISON RESULTS OF MILP AND MISDP FOR OPP

Case	Optimal Set	Power Flows	Injections	PMU Placement Bus	
				MILP	MISDP
1	4	—	—	2,6,7,9	2,6,7,9
2	3	2-3,3-4,6-11,6-12,7-8	—	2,9,12	5,9,14*
3	3	—	7	2,6,9	2,6,9
4	3	—	8,11,13	2,4,6	1,4,6*
5	2	2-3,3-4,6-11,6-12,7-8	8,11,13	5,9	5,9

* Although the two formulations give different results, both guarantee observability.

OPP Problem Simulation Results

TABLE II. COMPARISON RESULTS FOR OPP CONTINGENCIES

Case	Contingency & Location	Power Flows	Injections	PMU Placement Bus	
				MILP	MISDP
6	Limited Communication at 2,9	—	—	1,3,7,10,13	4,5,7,10,13
7		2-3,3-4,6-11,6-12,7-8	8,11,13	5,8,14	5,8,14
8	Single PMU Failure at 2,6,7,9	—	—	2,6,7,9 (Main) + 4,5,8,11,13 (Backup)	2,6,7,9 (Main) + 1,4,8,10,13 (Backup)
9	Single PMU Failure at 5,9	2-3,3-4,6-11,6-12,7-8	8,11,13	5,9 (Main) + 2,7,12 (Backup)	5,9 (Main) + 2,4,6 (Backup)

Thank you for your attention

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