Mixed Integer Programming (MIP)-Based Fault Location Identification Using MicroPMUs

Presentation at 50th North American Power Symposium

Mohammed Alqahtani
Department of Electrical Engineering
University of South Florida
Prince Sattam bin Abdulaziz University

Dr. Zhixin Miao & Dr. Lingling Fan
Department of Electrical Engineering
University of South Florida

September 10, 2018
Outline

Introduction
   Utility performance evaluation
   A common used method
   Our objective

The Advantages of the MIP-Based Proposed method

MIP Formulation for a Simple Single Phase Network

MIP formulation for Three-Phase Distribution Systems

Numerical example
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Utility performance evaluation

- Nowadays, the electric utility industry is a competitive field. Each utility wants to increase productivity and reduce economic losses.
- Reliability indices are a way of evaluating the performance of the utilities. To be a competitive in the utilities market, improve these indices.
  - The System Average Interruption Duration Index (SAIDI)=
    $$\frac{\sum \text{Customer Minutes of Interruption}}{\text{Total Number of Served Customers}}$$
  - The Customer Average Interruption Duration Index (CAIDI)=
    $$\frac{\sum \text{Customer Minutes of Interruption}}{\text{Total Number of Served Customers}}$$
- Reducing the interruption duration will ensure improvement in these indices.
A Common Used Method

Impedance Based Method

- Uses voltage and current measurements at the substation to estimate the impedance and then the distance to the fault point.
- A standard feature in most microprocessor-based relays.
- Adopted from the transmission system.
- In Distribution systems, the method suffers from multi-estimation due to the radial topology of the system (multiple points may have the same distance.)

Common practices to improve it: to use external information to narrow the searching area
- History of events
- Weather status
- Construction areas
- Customers calls!! (Do we really want to wait for an angry customer call?!)
Example

Figure: Fault event screen capture

Figure: Spreadsheet for the line parameters

Two Possible locations

Figure: Distribution Feeder
How many points will have the same distance to the substation?

Figure: IEEE-123 test feeder
Our objective

- To reduce the outage time in order to improve the reliability indices and reduce revenue loss caused by outages.
- To achieve that goal: a fast fault location method is required to speed up the restoration process.
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The Advantages of the MIP-Based Proposed method

- Does not yield to multiple locations.
- Only requires the pre and during fault voltages at the end of the branches in addition to the impedance bus matrix. (no need for current measurements)
- Applicable for grounded and ungrounded systems.
- The implementation of the proposed method is simple and does not require multiple stages or iterations.
- It determines the fault type, location and magnitude. While other methods rely on protective devises in substations to classify the fault type or determine the fault magnitude.
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MIP Formulation for a Simple Single Phase Network

Only one voltage phasor is required to find $\bar{I}_{f4}$, if more than one voltage phasors are measured, it will be an overdetermined problem.

\[
\begin{bmatrix}
0 & 0 & -\bar{I}_{f4} & 0 & 0
\end{bmatrix}^T = Y_{NN} \Delta \bar{V}_N
\tag{1}
\]

\[
\Delta \bar{V} = \bar{V}_{\text{post}} - \bar{V}_{\text{pre}}
\]

\[
\Delta \bar{V}_N = Z_{NN} \begin{bmatrix}
0 & 0 & -\bar{I}_{f4} & 0 & 0
\end{bmatrix}^T
\tag{2}
\]

\[
Z_{NN} = \begin{bmatrix}
z_{12} & z_{12} & z_{12} & z_{12} & z_{12} \\
z_{12} & z_{12} + z_{23} & z_{12} + z_{23} & z_{12} + z_{23} & z_{12} \\
z_{12} & z_{12} + z_{23} & z_{12} + z_{23} + z_{34} & z_{12} + z_{23} + z_{34} & z_{12} \\
z_{12} & z_{12} + z_{23} & z_{12} + z_{23} + z_{34} & z_{12} + z_{23} + z_{34} + z_{45} & z_{12} \\
z_{12} & z_{12} & z_{12} & z_{12} & z_{26}
\end{bmatrix}
\]
MIP Formulation For a Simple Single Phase Network

- Now we will assume that the fault location and magnitude are unknown.
- Hence a binary variable is introduced for each bus $u_i$ as shown in (3). If there a fault on bus $i$ then $u_i = 1$, if not $u_i = 0$

$$
\Delta \bar{V}_N = Z_{NN} \begin{bmatrix}
-u_2 \bar{I}_{f2} & -u_3 \bar{I}_{f3} & -u_4 \bar{I}_{f4} & -u_5 \bar{I}_{f5} & -u_6 \bar{I}_{f6}
\end{bmatrix}^T
$$  (3)
MIP Formulation For a Simple Single Phase Network

- Now will replace \( u_i \bar{I}_{fi} \) with \( \bar{I}_{xi} \)

\[
\Delta \bar{V}_N = Z_{NN} \begin{bmatrix} -\bar{I}_{x2} & -\bar{I}_{x3} & -\bar{I}_{x4} & \bar{I}_{x5} & \bar{I}_{x6} \end{bmatrix}^T
\]

(4)

\[
\bar{I}_{xi} = \begin{cases} \bar{I}_{fi}, & \text{if } u_i = 1 \\ 0, & \text{if } u_i = 0 \end{cases}
\]

(5)

- If the system has only one fault, then the inequality constrains is imposed

\[
\sum_{i=2}^{6} u_i \leq 1
\]

(6)
MIP formulation for a simple single phase network

- Note that $I_f$ is a phasor so its real and imaginary parts will be considered separately using the big-M technique as shown in (6)

\[
\begin{align*}
-(1 - u_i)M + \text{Re}(\bar{I}_{fi}) & \leq \text{Re}(\bar{I}_{xi}) \leq \text{Re}(\bar{I}_{fi}) + (1 - u_i)M \\
-(1 - u_i)M + \text{Im}(\bar{I}_{fi}) & \leq \text{Im}(\bar{I}_{xi}) \leq \text{Im}(\bar{I}_{fi}) + (1 - u_i)M \\
-u_iM & \leq \text{Re}(\bar{I}_{xi}) \leq +u_iM \\
-u_iM & \leq \text{Im}(\bar{I}_{xi}) \leq +u_iM \\
\end{align*}
\]

\(7\)

\[\begin{align*}
\text{if } u_i = 1 & \quad \text{if } u_i = 0 \\
\text{Re}(\bar{I}_{xi}) &= \text{Re}(\bar{I}_{fi}) & \text{Re}(\bar{I}_{xi}) &= 0 \\
\text{Im}(\bar{I}_{xi}) &= \text{Im}(\bar{I}_{fi}) & \text{Im}(\bar{I}_{xi}) &= 0 \\
-M \leq \text{Re}(\bar{I}_{xi}) & \leq M & -M \leq \text{Re}(\bar{I}_{xi}) & \leq M \\
-M \leq \text{Im}(\bar{I}_{xi}) & \leq M & -M \leq \text{Im}(\bar{I}_{xi}) & \leq M \\
\end{align*}\]
The Overall MIP Problem Formulation

- The objective function is the sum of the norm 2 of the error between the measured and estimated voltage deviation values.
- The measured voltage values are collected using MicroPMUs located at the end of the branched + the first bus after the source bus.

\[
\begin{align*}
\text{minimize} & \quad \sum_{i \in \mathcal{E}} \| \Delta \bar{V}_i^{\text{meas}} - \Delta \bar{V}_i \| \\
\text{subject to} & \quad \sum_{i \in \mathcal{N}} u_i \leq 1 \\
& \quad \Delta \bar{V}_N = -Z_{NN} \bar{I}_x \\
& \quad -u_i M \leq \text{Re}(\bar{I}_{xi}) \leq +u_i M \\
& \quad -u_i M \leq \text{Im}(\bar{I}_{xi}) \leq +u_i M \\
& \quad -(1 - u_i) M + \text{Re}(\bar{I}_{fi}) \leq \text{Re}(\bar{I}_{xi}) \leq \text{Re}(\bar{I}_{fi}) + (1 - u_i) M \\
& \quad -(1 - u_i) M + \text{Im}(\bar{I}_{fi}) \leq \text{Im}(\bar{I}_{xi}) \leq \text{Im}(\bar{I}_{fi}) + (1 - u_i) M
\end{align*}
\]

where $\mathcal{E}$ is the set of the buses located at the end of each branch and the bus closest to the substation bus, $\mathcal{N}$ is the set of the buses except the substation bus.
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Numerical example
MIP Formulation for Three-Phase Distribution Systems

- If three phase fault occurs at Bus4, the faults currents can be calculated as shown in (8).

$$\begin{bmatrix}
0 \\
... \\
0 \\
-\bar{I}^a_{f4} \\
-\bar{I}^b_{f4} \\
-\bar{I}^c_{f4} \\
0 \\
...
\end{bmatrix} = Y_{abc}^{NN} \begin{bmatrix}
\Delta \bar{V}^a_2 \\
\Delta \bar{V}^b_2 \\
\Delta \bar{V}^c_2 \\
... \\
\Delta \bar{V}^a_6 \\
\Delta \bar{V}^b_6 \\
\Delta \bar{V}^c_6
\end{bmatrix}$$

(9)

$$\Delta \bar{V}^{abc}_N = Z^{abc}_{NN} \begin{bmatrix}
-u^a_{2f2} \\
-u^b_{3f2} \\
-u^c_{4f2} \\
... \\
-u^a_{14f6} \\
-u^b_{15f6} \\
-u^c_{16f6}
\end{bmatrix}$$

(10)

A fault location binary variable $u_i$ is designated to each phase in each bus except substation bus.
MIP Formulation For Three-Phase Distribution Systems

- To exclude the possibility of having faults between phases from different buses, another location binary variable $k_i$ is designated to each bus except the substation bus.

- For the 6-node network, the dimension of $u$ is 15, and the dimension of $k$ is 5.

\[
\begin{align*}
    u_2 + u_3 + u_4 &= 3(k_2) \\
    u_5 + u_6 + u_7 &= 3(k_3) \\
    u_8 + u_9 + u_{10} &= 3(k_4) \\
    u_{11} + u_{12} + u_{13} &= 3(k_5) \\
    u_{14} + u_{15} + u_{16} &= 3(k_6) \\
    \sum k_i &= 1
\end{align*}
\] (11)

- When a fault happened at Bus 2, $k_2 = 1$ and all the phase binaries at that bus are equal to 1, however that does not mean all the phases are under fault.

- The magnitude of the current will determine the faulted phase.
The Overall Three-Phase MIP Problem Formulation

\[
\text{minimize } \sum_{i \in \mathcal{E}} \| \Delta \tilde{V}_{i}^{abc,\text{meas}} - \Delta \tilde{V}_{i}^{abc} \|
\]

subject to

\[
\Delta \tilde{V}_{N}^{abc} = -Z_{NN}^{abc} \tilde{I}_{x}^{abc}
\]

\[
-u_{i}M \leq \text{Re}(\tilde{I}_{xi}) \leq +u_{i}M
\]

\[
-u_{i}M \leq \text{Im}(\tilde{I}_{xi}) \leq +u_{i}M
\]

\[
-(1 - u_{i})M + \text{Re}(\tilde{I}_{fi}) \leq \text{Re}(\tilde{I}_{xi}) \leq \text{Re}(\tilde{I}_{fi}) + (1 - u_{i})M
\]

\[
-(1 - u_{i})M + \text{Im}(\tilde{I}_{fi}) \leq \text{Im}(\tilde{I}_{xi}) \leq \text{Im}(\tilde{I}_{fi}) + (1 - u_{i})M
\]

for all \(i \in \mathcal{B}\)

\[
u_{a} + u_{b} + u_{c} = 3(k_{i})
\]

\[
\sum_{i \in \mathcal{E}} k_{i} = 1
\]

for all \(i \in \mathcal{N}\)

where \(\mathcal{E}\) is the set of the microPMU phases, \(\mathcal{N}\) is the set of the buses except the substation bus. \(\mathcal{B}\) is the set of the phases except the substation bus’s phases.
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- Very unbalanced feeder.
- (PQ,I and Z) loads.
- Ungrounded
- micro-PMUs are placed at the end of the branches as shown in the figure.

Figure: Modified version IEEE-37 Test Feeder
Numerical example

- All the single phase faults were identified correctly. (108 tests for each transformer configuration)

<table>
<thead>
<tr>
<th>Location</th>
<th>Identification Results</th>
<th>OpenDSS Measurements</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Obj</td>
<td>$I_F$</td>
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<tr>
<td>729.b</td>
<td>0.070</td>
<td>2.111</td>
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<tr>
<td>711.c</td>
<td>0.170</td>
<td>2.064</td>
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<td>702.b</td>
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<tr>
<td>775.a</td>
<td>0.002</td>
<td>$4.9e10^{-4}$</td>
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<tr>
<td>702.a</td>
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<td>710.c</td>
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</table>
Double and Three Phase faults

- More than 97% of the double and Three-phase faults were identified correctly.
- Only 2-3p Faults at 707 were identified as faults at Bus 722, which is adjacent bus to Bus 707. By excluding Bus 722 from the searching space, the proposed method correctly identified the faulted bus, which means it was at the second least objective function value.
### Double and Three Phase Faults

#### Table: Double-Phase to Ground faults (\(\Delta - \Delta\))

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<tr>
<td></td>
<td>(I_a^a)</td>
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<tr>
<td>775.(ab)</td>
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<td>10630.7</td>
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#### Table: Double-Phase Faults (\(\Delta - \Delta\))

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<td></td>
<td>(I_a^a)</td>
<td>(I_b^a)</td>
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<tr>
<td>727.(abc)</td>
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<tr>
<td>708.(abc)</td>
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<td>720.(abc)</td>
<td>257</td>
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#### Table: Double-Phase Faults (\(\Delta - Y\))

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<td>(I_b^a)</td>
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<td>704.(ab)</td>
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<td>731.(ab)</td>
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#### Table: Three-phase faults (\(\Delta - Y\))

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<td>(I_a^a)</td>
<td>(I_b^a)</td>
</tr>
<tr>
<td>725.(abc)</td>
<td>265</td>
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<td>744.(abc)</td>
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<tr>
<td>737.(abc)</td>
<td>204</td>
<td>1631.1</td>
</tr>
</tbody>
</table>
THANK YOU