Distribution Locational Marginal Pricing (DLMP) for Multiphase Systems

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Motivation

- The unprecedented reformation that the modern distribution network is undergoing (will undergo) because of the increased deployment of distributed energy resources (DERs).
- Although DERs are immensely beneficial to the system, higher penetrations with ill-managed operation could fail to deliver the desired outcome, causing sharp voltage fluctuations, power flow congestion and supply-demand imbalance.
- The lack of adopting a convex multiphase power-flow distribution model that exhibits the practical system structure in the literature [**4**]-[**10**].
- The fact that DERs are currently seen as if they were connected to the T-D interface, disregarding the distribution.
- The need to incentivize the various resources to respond according to: i) loading at the phase of installation ii) their location (loss compensation), and iii) their capacity.

Objectives

- Design an electricity market for distribution that adopts locational marginal pricing to quantify and analyze signals sent to future-market for each phase.
- The optimization problem is centrally performed by a non-profit entity -distribution system operator (DSO), that to collect bids from participants including ISO at the wholesale level, control volt-var and DER assets, and settle the market, while abiding by the system physical and security constraints. .
- Clear the market with a day-ahead point forecast of loading and substation/DER supply.
- Leverage a convex multiphase distribution model to guarantee a global optimum solution.

Distribution Electricity Market

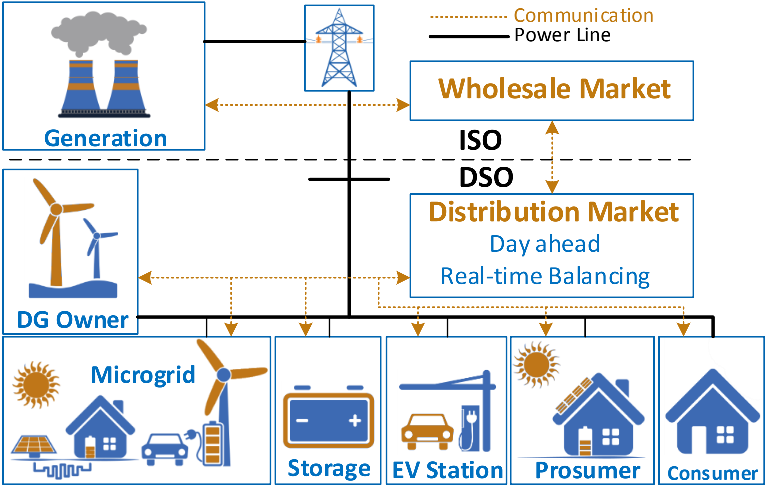


Figure: Distribution electricity market

Outline

Variables 3-ϕ Branch Flow Model

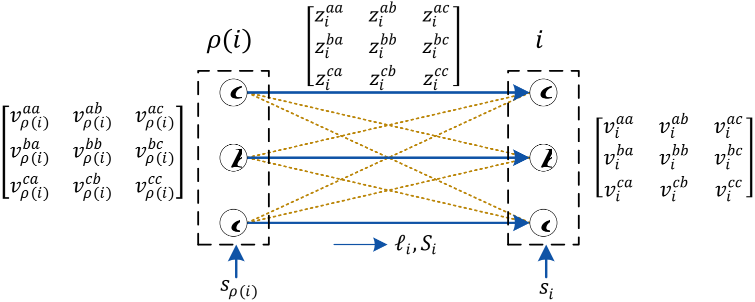


Figure: Variables in SDP relaxation of three-phase OPF.

Ohm's Law

The following equations are defined for branch $\rho(i) \to i$, assuming $\Phi_i = \Phi_{\rho(i)}$ **[1]**

$$V_i = V_{\rho(i)} - z_i I_i \qquad (1)$$

By multiplying both sides by their Hermitian transposes, and defining $v_i = V_i V_i^H$, $v_{\rho(i)} = V_{\rho(i)} V_{\rho(i)}^H$, $S_i = V_{\rho(i)} I_i^H$ and $\ell_i = I_i I_i^H$, then (1) can be re-written as

$$v_i = v_{\rho(i)} - (S_i z_i^H + z_i S_i^H) + z_i \ell_i z_i^H \quad \forall \ i \in \mathcal{N}^+ \qquad (2)$$

V_i & $I_i \in \mathbb{C}^{|\Phi_i|}$.

$z_i \in \mathbb{C}^{|\Phi_i| \times |\Phi_i|}$.

Diagonal entree of $v_i \in \mathbb{C}^{|\Phi_i| \times |\Phi_i|}$ and $\ell_i \in \mathbb{C}^{|\Phi_i| \times |\Phi_i|}$ are squared magnitudes and off-diagonal are complex mutual elements.

Power Balance

For each $\rho(i) \rightarrow i \rightarrow k$ where $k \in \delta(i)$, to interpret the power balance at i, (1) is multiplied by I_i^H

$$V_i I_i^H = V_{\rho(i)} I_i^H - z_i I_i I_i^H \quad (3)$$

$$V_i \Big(\sum_{k \in \delta(i)} I_k^H - I_{i\,\mathsf{inj}}^H \Big) = S_i - z_i \ell_i \quad (4)$$

As a result, the power balance at bus i is the diagonal of (4)

$$\sum_{k \in \delta(i)} \mathsf{diag}(S_k) - s_i = \mathsf{diag}(S_i - z_i \ell_i) \quad \forall \; i \in \mathcal{N}^+ \quad (5)$$

where

$$s_i = s_i^g + s_i^{\mathsf{svc}} - s_i^L \quad (6)$$

$I_{i\,\mathsf{inj}}$: Current injection.

$s_i \in \mathbb{C}^{|\Phi_i|}$: Net power injection .

L: Load.

g: PV/wind DG.

svc: Static-Var compensator.

PSD and Rank-1 Matrix

To set the relationship among the variables, the following positive and rank one matrix is defined

$$X_i = \begin{bmatrix} V_{\rho(i)} \\ I_i \end{bmatrix} \begin{bmatrix} V_{\rho(i)} \\ I_i \end{bmatrix}^H = \begin{bmatrix} v_{\rho(i)} & S_i \\ S_i^H & \ell_i \end{bmatrix}$$

$$X_i \succeq 0 \qquad \forall \; i \in \mathcal{N}^+ \tag{7}$$

$$\text{rank}(X_i) = 1 \quad \forall \; i \in \mathcal{N}^+ \tag{8}$$

Distributed Generators

It is essential that inverters of both PVs and wind be equipped with curtailment capability so as to dispatch an appropriate amount of generated active power for market clearing. The set of inequality limits constraints, $\mathcal{DG} = \mathcal{PV} + \mathcal{WT}$, is as follows

$$\begin{aligned} \mathcal{DG} =& \{ s_i^g \in \mathbb{C}^{|\Phi_i|} \mid 0 \leq \text{real}(s_i^g) \leq P_i^g \\ & - a\,\text{real}(s_i^g) \leq \text{imag}(s_i^g) \leq a\,\text{real}(s_i^g) \} \quad \forall\ i \in \mathcal{N}_g \end{aligned} \tag{9}$$

$$a = \sqrt{1 - \text{PF}^2} / \text{PF} \tag{10}$$

where

$$P_i^{g\phi} = \omega^{\phi} \Big(\sum_{i \in \mathcal{N}^+} \sum_{\phi \in |\Phi_i|} \text{real}(s_i^{L\phi}) / |\mathcal{PV}| \times |\Phi_i| \Big) \quad \forall\ i \in \mathcal{DG} \tag{11}$$

P_i^g: The maximum DG power generation.

a: Fixes inverter's power factor

$\omega \in \mathbb{R}^{|\Phi_i|}$: Penetration level.

Static-Var Compensators and Voltage Limits

Voltage profile can be improved by dispatching SVCs to either generate or absorb reactive powers.

$$\begin{aligned} \mathcal{SVC} =& \{ s_i^{\text{svc}} \in \mathbb{C}^{|\Phi_i|} \ \mid \ \text{real}(s_i^{\text{svc}}) = 0 \\ & - \underline{q} \leq \text{imag}(s_i^{\text{svc}}) \leq \overline{q} \} \quad \forall \ i \in \mathcal{N}_{\text{svc}} \end{aligned} \tag{12}$$

Except for the substation node $(v_0 = V_0 V_0^H)$, $\pm 5\%$ of the nominal voltage are enforced as bounds on each element of the diagonal voltage squares.

$$\underline{V}^2 \leq \text{diag}(v_i) \leq \overline{V}^2 \quad \forall \ i \in \mathcal{N}^+ \tag{13}$$

DSO Optimization Problem

The objective is to minimize market participants' generation costs ahead of a day ($|T| = 24$)

$$
\begin{aligned}
\min_{v_i, \ell_i, S_i, s_i^g, s_i^{\text{svc}}} \quad & f = \sum_{t \in T} \sum_{\phi \in \Phi_i} \Big(\sigma_s^P \text{real}(S_{t,1}^{\phi\phi}) + \sigma_s^Q \text{imag}(S_{t,1}^{\phi\phi}) \\
& + \sum_{i \in \mathcal{N}_{\text{g}}} \sigma_g^P \text{real}(s_{t,i}^{g\phi}) + \sigma_g^Q \text{imag}(s_{t,i}^{g\phi}) \\
& + \sum_{i \in \mathcal{N}_{\text{svc}}} \sigma_{\text{svc}} \text{imag}(s_{t,i}^{\text{svc}\phi}) \Big) \qquad (14) \\
\text{s. t.} \quad & v_0 = V_0 V_0^H \\
& (2), (5), (7), (8), (9), (12), (13)
\end{aligned}
$$

where σ denotes the generation bidding price with superscripts P and Q for active and reactive power, while $S_{t,1}$ is the power flow from the substation.

Convexification

- The non-convex optimization problem in (14) is convexified by removing (relaxing) the rank constraint (8). Thus, an SDP-relaxed problem is obtained.
- In [**1**], it has been shown that a tight relaxation holds for most IEEE distribution feeders. For validation, a tightness check will be conducted for the case studies.

Outline

Distribution Locational Marginal Pricing (DLMP)

The Lagrangian function of the general form [2] for the overall single-period problem, with emphasis on the power balance equation, is

$$\begin{aligned}\mathcal{L} = f(x) + \sum_{i\in\mathcal{F}} \lambda_i^f f_i(x) + \sum_{i\in\mathcal{H}} \mu_i^h h_i(x) \\ + \sum_{i\in\mathcal{N}^+} \sum_{\phi\in|\Phi_i|} \lambda_i^{\mathsf{p}\phi} \mathsf{real}\Big(\sum_{k\in\delta(i)} S_k^{\phi\phi} - s_i^\phi - S_i^{\phi\phi} + (z_i \ell_i)^{\phi\phi}\Big) \\ + \sum_{i\in\mathcal{N}^+} \sum_{\phi\in|\Phi_i|} \lambda_i^{\mathsf{q}\phi} \mathsf{imag}\Big(\sum_{k\in\delta(i)} S_k^{\phi\phi} - s_i^\phi - S_i^{\phi\phi} + (z_i \ell_i)^{\phi\phi}\Big)\end{aligned} \tag{15}$$

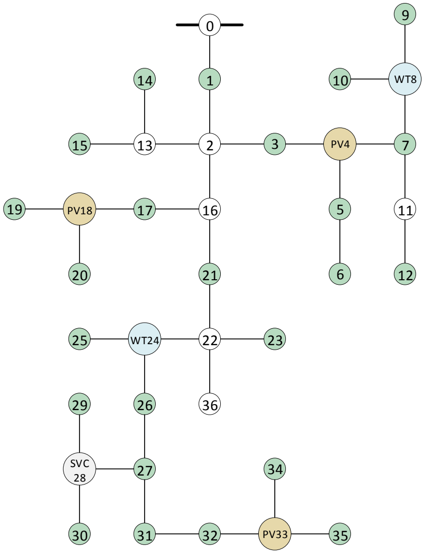
The partial derivative of (15) $w.r.t$ real$(s_i^{L\phi})$ and imag$(s_i^{L\phi})$ result in

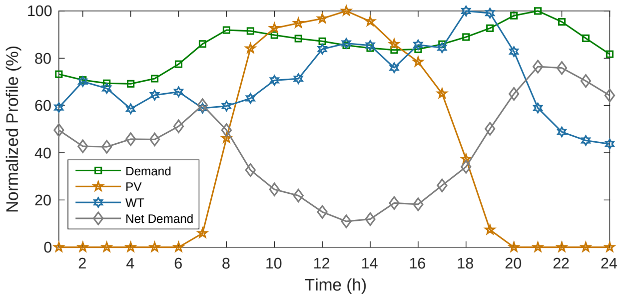
$$\text{A-DLMP} : \frac{\partial \mathcal{L}}{\partial\, \mathsf{real}(s_i^{L\phi})} = \lambda_i^{\mathsf{p}\phi}, \qquad \text{R-DLMP} : \frac{\partial \mathcal{L}}{\partial\, \mathsf{imag}(s_i^{L\phi})} = \lambda_i^{\mathsf{q}\phi} \tag{16}$$

Each A-DLMP and R-DLMP accumulates an energy price, a loss price, and a congestion price.

Outline

IEEE 37-bus Feeder and Profiles of Load,PV and wind from CAISO [3]





Passive Network

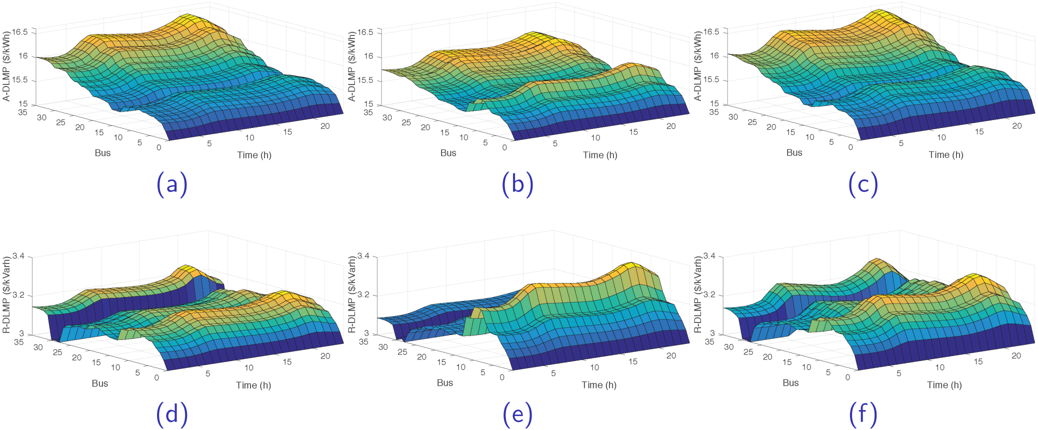


Figure: (a)-(c) Active-power DLMPs, and (d)-(f) reactive-power DLMPs for passive network.

Passive Network

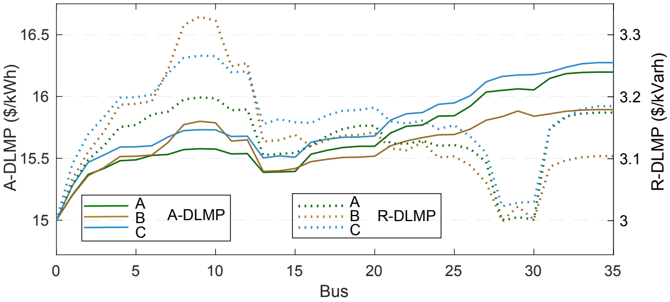


Figure: A-DLMPs and R-DLMPs at peak hour 13:00.

- The system runs as a passive distribution network, and only includes the SVC with bidding prices σ_s^P =15\$/kWh and $\sigma_s^Q = \sigma_{\text{svc}}$ =3\$/kVarh.
- A common trend of ADLMPs and RDLMPs is that they gradually increase as buses locate farther from the substation.
- A-DLMPs among the three phases differ notably because of the unbalanced loads and line impedances (losses).
- R-DLMPs increase in a similar trend except at buses near the SVC at bus 28.

Active Network

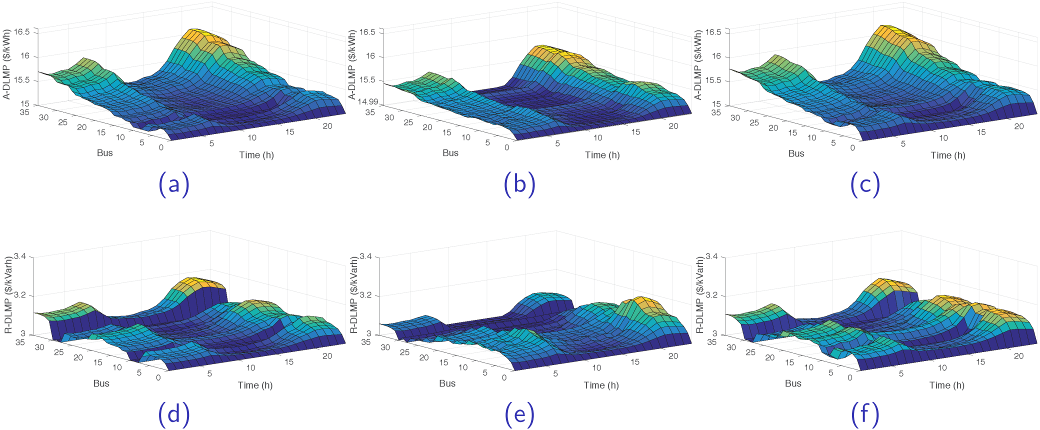


Figure: (a)-(c) Active-power DLMPs, and (d)-(f) reactive-power DLMPs for active network.

Active Network

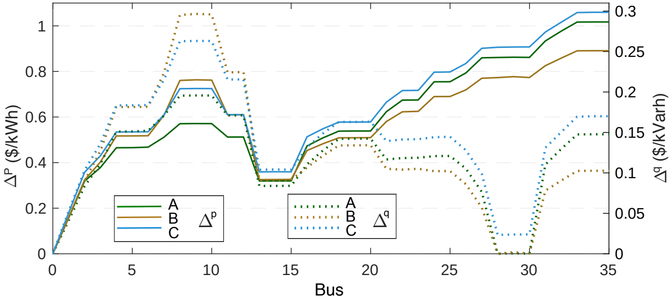


Figure: Nodal change in marginal prices at 13:00 between passive and active networks.

- PV- and wind-based DGs participate in the market with the same bidding prices as the substation and 40% penetration.
- DLMPs change with the participation of DGs, mostly at times when DGs produce excessive power (peak).
- A lower net demand is viewed as light loading, and thereby it cuts down on DLMPs.
- manifests the change in marginal prices at 13:00, where Δ^p and Δ^q are the difference of DLMPs between the cases and this case. The change is the DG contribution to reducing losses, especially those of remote lines, and alleviating the SVC binding cost.

Active Network

Table: DG Active-power Output and A-DLMPs at 13:00

	A		*B*		*C*	
	kW	\$/kWh	kW	\$/kWh	kW	\$/kWh
Sub.	144.9	15	71.3	15	371	15
PV4	109.2	15.0154	103	14.9972	109.2	15.0567
PV18	109.2	15.0494	76.7	14.9983	109.2	15.0925
PV33	109.2	15.1776	54.7	15.0000	109.2	15.2062
WT8	8.8	15.0000	141.3	15.0119	94.9	15.0000
WT24	141.3	15.0868	100	15.0000	141.3	15.1399

- DGs are burdened at phase A and C because of the phase heavy loading, increasingl congestion costs predominantly at distant buses.
- DGs at phase B (lightly loaded) mostly curtails its output so as to balance supply with demand, except at WT8 due to the large load at bus 10.

Lower DG Bidding Prices

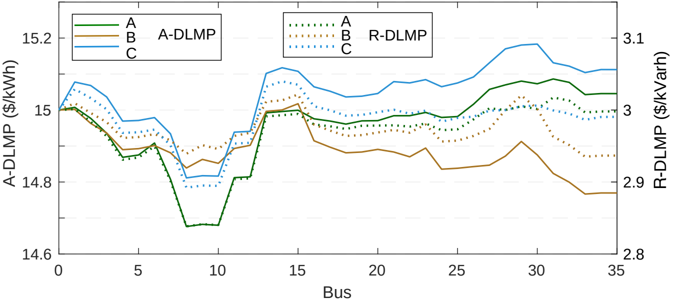


Figure: DLMPs at peak PV, 13:00.

- Setting σ_g^P =\$13/kWh and σ_g^Q =2\$/kVarh results in reduced DLMPs in general, and they are minimum at buses of DG installation.
- This incentivizes DG owners to increase their generation capacity to participate in imbalance and loss reduction for the next increment of load.

Total Daily Consumer Payment $w.r.t.$ Penetration Level

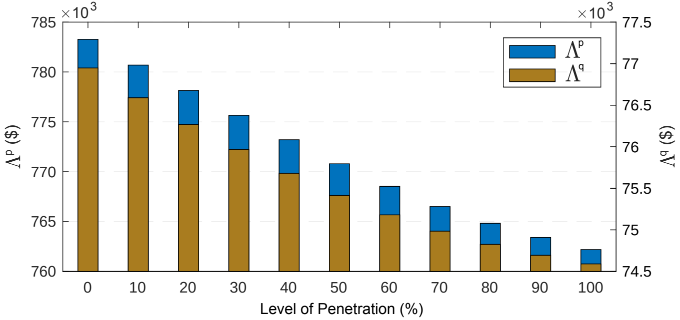


Figure: Change in total daily payment with DG penetrations.

The figure shows the decay of the total consumer?s payment for the entire day $w.r.t.$ incremental DG penetration.

$$\Lambda^{\mathsf{p}} = \sum_{t \in T} \sum_{i \in \mathcal{N}^+} \sum_{\phi \in \Phi} \lambda_{t,i}^{\mathsf{p}\phi} \mathsf{real}(s_{t,i}^{L\phi})$$

$$\Lambda^{\mathsf{q}} = \sum_{t \in T} \sum_{i \in \mathcal{N}^+} \sum_{\phi \in \Phi} \lambda_{t,i}^{\mathsf{q}\phi} \mathsf{imag}(s_{t,i}^{L\phi})$$

Outline

Tightness of SDP Relaxation

The rank is examined by computing the devision of the second largest eigenvalue by the largest eigenvalue, $|\text{eig}_2/\text{eig}_1|$, where $\text{eig}_1 > \text{eig}_2 > 0$. Smaller ratios indicate the solution proximity to being rank one. The maximum ratio is computed.

$$\text{Tightness} = \max\Big(\sum_{t \in T} \sum_{\text{eig} \in X^*_{t,i}} |\text{eig}_2/\text{eig}_1|\Big)$$

Table: Tightness of Numerical Solutions

Case	Tightness
1	5.1615e-07
2	6.1587e-07
3	4.9779e-07

Since the overall solution ratios satisfy sufficiently small values, $|\text{eig}_2/\text{eig}_1| \leq 6.1587 \times 10^{-7}$, the SDP relaxation is tight.

Outline

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