Bilevel Programming-Based Unit Commitment for Locational Marginal Price Computation

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Outline

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Introduction

- In the US, locational marginal pricing system is applied in all ISOs, independent system operators [B. Eldridge, 2017].
- Locational marginal price (LMP) is the cost of the next increased demand unit at a bus.
- Generation company bidding is based on the LMP.
- The LMP is the dual variable of the power balance equation.
- In the day-ahead market, generation companies determine on/off status of each unit for the next 24 hours.
- Unit status causes nonconvexity in formulating unit commitment participating in the day-ahead market.

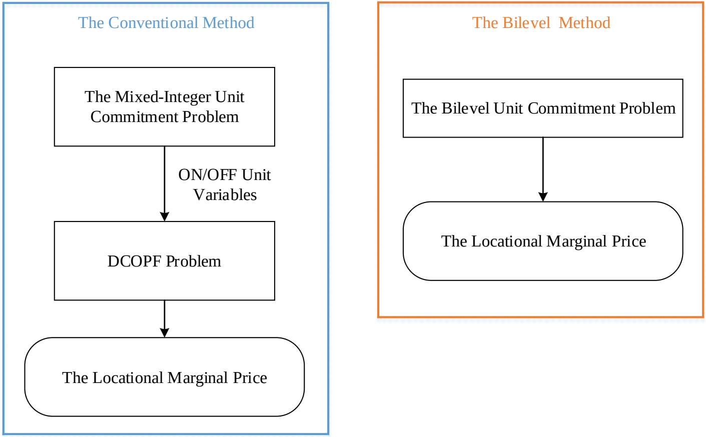
Introduction

- Issues
  - Commercial solvers cannot reach the problem dual variables.
  - Traditionally, the LMP is obtained by solving two problems. The first one is unit commitment. The second problem is DCOPF, where units' statuses are known.
- Solution
  - We propose a bilevel model that solves the unit commitment problem and attains the LMP and the other dual variables.
  - We present solving the bilevel problem using primal-dual relationship.
- The solution of the bilevel model and the conventional model exactly match.

Outline

Model Formulation

The comparison diagram of the two models



The Conventional Unit Commitment Model

$$\min_{p_l^f, p_j^g, \delta_n} \quad \sum_{j \in J} K_j^g p_j^g + \sum_{j \in J} c_j^u \tag{1a}$$

subject to

$$c_j^u \geq K_j^u (v_j - v_{j0}); \forall j \in J \tag{1b}$$

$$c_j^u \geq 0; \forall j \in J \tag{1c}$$

$$v_j \in \{0, 1\}; \forall j \in J \tag{1d}$$

$$\sum_{j \in J} p_j^g + \sum_{l|d(l)=n} p_l^f - \sum_{l|o(l)=n} p_l^f = P_n^d; \forall n \in N \tag{1e}$$

$$p_l^f = \frac{1}{x_l} (\delta_{o(l)} - \delta_{d(l)}); \forall l \in L \tag{1f}$$

$$-\overline{P}_l^f \leq p_l^f \leq \overline{P}_l^f; \forall l \in L \tag{1g}$$

$$\underline{P}_j^g v_j \leq p_j^g \leq \overline{P}_j^g v_j; \forall j \in J \tag{1h}$$

The Bilevel Unit Commitment Model

The Bilevel Unit Commitment Model

- The Upper-Level Problem

$$\max_{v_j} \sum_{n \in N} P_n^d \lambda_n^* - \sum_{j \in J} c_j^u \tag{2a}$$

subject to

$$c_j^u \geq K_j^u (v_j - v_{j0}); \forall j \in J \tag{2b}$$

$$c_j^u \geq 0; \forall j \in J \tag{2c}$$

$$v_j \in \{0, 1\}; \forall j \in J \tag{2d}$$

The Bilevel Unit Commitment Model

- The Lower-Level Problem

$$\min_{p_l^f, p_j^g, \delta_n} \quad \sum_{j \in J} K_j^g p_j^g \tag{3a}$$

subject to

$$p_l^f = \frac{1}{x_l}(\delta_{o(l)} - \delta_{d(l)}); (\nu_l); \forall l \in L \tag{3b}$$

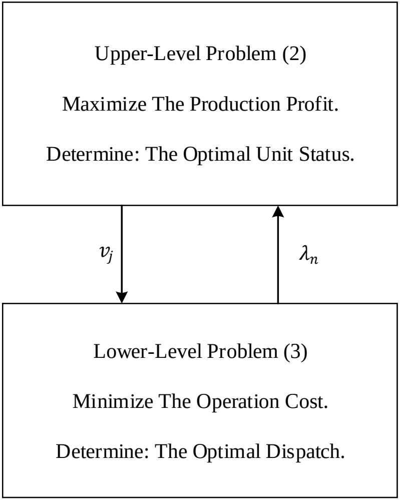
$$-\overline{P}_l^f \leq p_l^f \leq \overline{P}_l^f; (\phi_l^{min}, \phi_l^{max}); \forall l \in L \tag{3c}$$

$$\underline{P}_j^g v_j^* \leq p_j^g \leq \overline{P}_j^g v_j^*; (\theta_j^{min}, \theta_j^{max}); \forall j \in J \tag{3d}$$

$$\sum_{j \in J} p_j^g + \sum_{l|d(l)=n} p_l^f - \sum_{l|o(l)=n} p_l^f = P_n^d; (\lambda_n); \forall n \in N \tag{3e}$$

The Bilevel Unit Commitment Model

The structure of the bilevel model



The Bilevel Unit Commitment Model

The lower-level dual problem

$$\max_{\lambda,\nu,\phi,\theta} Z_D = \sum_{n\in N} P_n^d \lambda_n - \sum_{l\in L} \overline{P}_l^f (\phi_l^{max} + \phi_l^{min}) + \sum_{j\in J} v_j^* (\theta_j^{min} \underline{P}_j^g - \theta_j^{max} \overline{P}_j^g) \quad (4)$$

subject to

$$K_j^g - \lambda_n + \theta_j^{max} - \theta_j^{min} = 0; \forall j \in J \quad (5a)$$

$$\lambda_{o(l)=n} - \lambda_{d(l)=n} - \nu_l + \phi_l^{max} - \phi_l^{min} = 0; \forall l \in L \quad (5b)$$

$$\sum_{l|o(l)=n} \frac{1}{x_l}\nu_l - \sum_{l|d(l)=n} \frac{1}{x_l}\nu_l = 0; \forall l \in L \quad (5c)$$

$$\theta_j^{max}, \theta_j^{min} \geq 0; \forall j \in J \quad (5d)$$

$$\phi_l^{max}, \phi_l^{min} \geq 0; \forall l \in L \quad (5e)$$

The Bilevel Unit Commitment Model

The single-level equivalent

$$\max_{p_l^f, p_j^g, \delta_n, \lambda_n, \nu_l, \phi_l, \theta_j} \sum_{n \in N} P_n^d \lambda_n - \sum_{j \in J} c_j^u \tag{6a}$$

subject to

$$\text{Constraints} \quad (2b) - (2d) \tag{6b}$$

$$\text{Constraints} \quad (3b) - (3e) \tag{6c}$$

$$\text{Constraints} \quad (5a) - (5e) \tag{6d}$$

$$\sum_{j \in J} P_n^d \lambda_n + \sum_{j \in J} (\underline{P}_j^g \theta_j^{min} v_j - \overline{P}_j^g \theta_j^{max} v_j) - \sum_{l \in L} \overline{P}_l^f (\phi_l^{max} + \phi_l^{min}) = \sum_{n \in N} K_j^g p_j^g \tag{6e}$$

The Bilevel Unit Commitment Model

Constraint (6e) is linearized

$$\sum_{j \in J} P_n^d \lambda_n + \sum_{j \in J} (\underline{P}_j^g b_j - \overline{P}_j^g a_j) - \sum_{l \in L} \overline{P}_l^f (\phi_l^{max} + \phi_l^{min}) = \sum_{n \in N} K_j^g p_j^g \quad (7a)$$

$$0 \leq a_j \leq \overline{\theta_j^{max}} v_j \quad (7b)$$

$$0 \leq \theta_j^{max} - a_j \leq \overline{\theta_j^{max}} (1 - v_j) \quad (7c)$$

$$0 \leq b_j \leq \overline{\theta_j^{min}} v_j \quad (7d)$$

$$0 \leq \theta_j^{min} - b_j \leq \overline{\theta_j^{min}} (1 - v_j) \quad (7e)$$

The Bilevel Unit Commitment Model

The linearized single-level equivalent

$$\max_{p_l^f, p_j^g, \delta_n, \lambda_n, \nu_l, \phi_l, \theta_j} \sum_{n \in N} P_n^d \lambda_n - \sum_{j \in J} c_j^u \tag{8a}$$

$$\text{subject to} \quad \text{Constraints} \quad (2b) - (2d) \tag{8b}$$

$$\text{Constraints} \quad (3b) - (3e) \tag{8c}$$

$$\text{Constraints} \quad (5a) - (5e) \tag{8d}$$

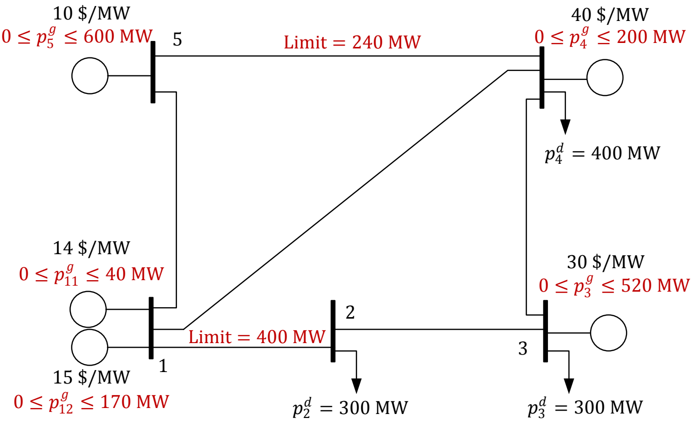
$$\text{Constraints} \quad (7a) - (7e) \tag{8e}$$

Outline

Case Studies

- The model (8) has been tested on different scale systems.
- The tested cases are nesta_case5_pjm and nesta_case30_ieee, taken from [C. Coffrin, 2014].
- The model has been implemented in Matlab with CVX using solver Gurobi 7.51.

5-Bus PJM System Case Study



The System Normal Operation

Bus	1	2	3	4	5
LMP ($/MWh)	16.9774	26.3845	30	39.9427	10
Total Revenue = $32,892					

Table: 5-Bus System LMPs of The Buses and The Revenue

	ON/OFF	Output (MW)	θ^{max}	θ^{min}
G11	1	40	2.9774	0
G12	1	170	1.9774	0
G3	1	323.4948	0	0
G4	0	0	0	0
G5	1	466.5052	0	0
Total Generation Cost = $17,480				

Table: 5-Bus System Generator Primal and Dual Components

The System Normal Operation

	p_{12}^f	p_{14}^f	p_{15}^f	p_{23}^f	p_{34}^f	p_{45}^f
Flow (MW)	249.7	186.8	-226.5	-50.3	-26.8	-240
ϕ^{max} ($/MWh)	0	0	0	0	0	62.32
ϕ^{min} ($/MWh)	0	0	0	0	0	0

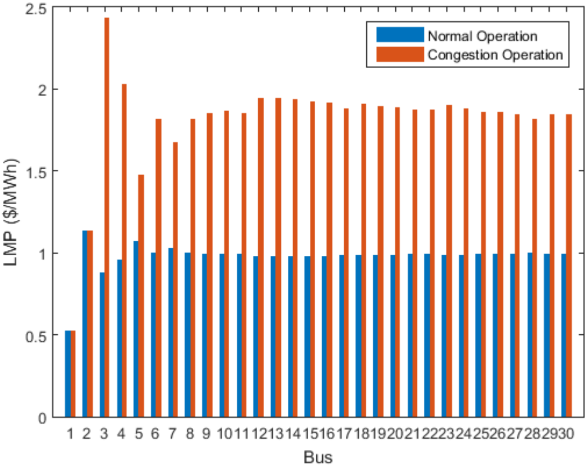
Table: 5-Bus System Primal and Dual Components of The Lines

IEEE 30-Bus System Case Study

- The system contains 6 generator units and 41 transmission lines.
- The system is tested under normal operation.
- The system is examined under congestion operation. The maximum capacity limit of line 1-3 is set 72.5 MW.

IEEE 30-Bus System Case Study

The LMPs of the system buses in both normal and congested operations.



IEEE 30-Bus System Case Study

	ON/OFF	Output (MW)	θ^{max}	θ^{min}
G1	1	215.7540	0	0
G2	1	67.6460	0	0
Total Generation Cost = $189.2789				

Table: Generator Primal and Dual Components

	ON/OFF	Output (MW)	θ^{max}	θ^{min}
G1	1	184.3122	0	0
G2	1	99.0878	0	0
Total Generation Cost = $208.5774				

Table: Generator Primal and Dual Components (Congested)

Outline

Conclusion

- The nonconvexity of unit commitment problem causes difficulty of achieving the problem dual variables that play an important rule in market participation.
- We develop a bilevel model that consists of two level problems, namely, the upper-level and the lower-level.
  - The upper-level has binary decision variables.
  - The lower-level has only continuous decision variables.
- The bilevel problems are transformed to a single-level problem and solved efficiently using Gurobi.