Bilevel Programming-Based Unit Commitment for Locational Marginal Price Computation

Abdullah Alassaf
Electrical Engineering Department
University of South Florida,
Tampa, FL 33620, USA
Email: alassaf@mail.usf.edu

Lingling Fan
Electrical Engineering Department
University of South Florida,
Tampa, FL 33620, USA
Email: linglingfan@usf.edu

***Abstract*—In the current practice, locational marginal prices (LMPs) are computed every few minutes by solving an optimization problem with continuous variables only. Generator on/off statuses are decided by the day-ahead unit commitment scheduling procedure. Thus, unit commitment and locational marginal price computing are two separate steps. This paper presents a model that substitutes the traditional two-step method. It is based on bilevel programming. The proposed problem determines generator on/off status and locational marginal prices simultaneously. The efficiency of the model is demonstrated on the 5- and 30-bus systems.**

***Index Terms*—Unit commitment, bilevel programming, mathematical program with primal and dual constraints, electric energy market, locational marginal price, transmission network, shadow price.**

I. Introduction

Locational marginal price (LMP) is the cost of the next increased demand unit at a bus. The values of LMP change spatially for different factors such as line losses and congestions [1]. If these aspects are not considered, the LMP value becomes uniform at all bus locations. In the US, locational marginal pricing system is applied in all ISOs, independent system operators [2]. The Federal Energy Regulatory Commission agency (FERC), ensures that the electricity rates and terms are fair and unbiased [3]. It follows that ISOs secure unduly discriminatory commerce in the wholesale electricity market.

The electric energy market rules and regulations change from one to another. For instance, in California ISO energy market, there are two types of bidding that are Economic bid and Self-schedule bid [4]. The process of the economic bid is that the buyer or seller submits both bid's price and quantity, the trading occurs when the two conditions meet. On the other hand, in the self-schedule bid, the buyer or seller gives the market operator only the bid's quantity, and the production revenue depends on the market cleared price. In practice, generation companies forecast the electricity market auction price. Based on this estimated price, they optimize their participations in the market so that they gain the highest possible profit.

The foremost step for generation unit owners before participating in the electric energy market auction is that they determine unit on/off status for the next 24 hours (or unit commitment). Reference [5] provides the mixed-integer linear programming formulation of unit commitment problem. [6] refines the same formulation by reducing the number of binary variables, which conveys to more efficient computation.

Exploiting the fact that transmission lines have relatively high X/R ratios, the optimal power flow (OPF) problem is commonly linearized using DC approximation, known as DCOPF, which simplifies the problem to a linear programming (LP) model. Besides its efficient computational performance, DCOPF is preferred because of its simplicity to be integrated in a mixed integer optimization problem.

In this paper, an efficient LMP method is computed, where we aim to solve a unit commitment (UC) problem and to find LMP at the same time. This is not possible by solving a mixed-integer programming (MIP) problem, since the current MIP solvers do not give the ability to reach the problem dual variables. Our philosophy is to formulate the UC problem as a bi-level problem. In the upper level, the binary variables related to unit on/off are the decision variables. The low-level problem receives the unit on/off status information from the upper-level problem. Thus the low-level problem is a continuous optimization problem and it is possible to find dual variables related to LMPs.

Our contribution is two-fold. A bilevel programming problem is formulated to replace a UC problem with the capability of LMP computation. The low-level and upper-level problems are suitably defined with their interactions correctly configured. The proposed problem is equivalent to a UC problem. Our second contribution lies in solving the bi-level problem. In the literature, there are two methods solving bi-level problems. The first method replaces the low-level problem by Karush-Kuhn-Tucker (KKT) conditions [7]. The second method explores primal-dual relationship [8]. We adopt the second method to explicitly express LMPs in the low-level problem as decision variables.

The rest of paper is organized as follows. Section II provides the model formulation. The analysis and case studies are covered in Section III. Section IV concludes the paper.

II. Model Formulation

This section introduces unit commitment problem of a constrained network participating in the electric energy market

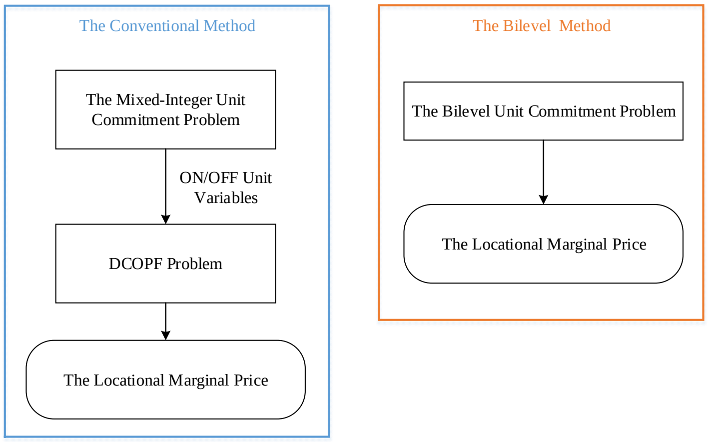

Fig. 1. The process of conventional and bilevel models.

in two models. First, the conventional model that is based on mixed-integer programing. Second, the bilevel model, which consists of two problem that are simplified to a single-equivalent problem. Both approaches yield the optimal operation and the LMP, nonetheless, the latter has a higher efficiency in terms of simplicity and performance. The comparison diagram of the two models is shown in Fig. 1.

A. The Conventional Unit Commitment Problem

The following model is a unit commitment problem of a constrained system [8]:

$$\min_{p_l^f, p_j^g, \delta_n} \quad \sum_{j\in J} K_j^g p_j^g + \sum_{j\in J} c_j^u \tag{1a}$$

$$\text{subject to} \quad c_j^u \geq K_j^u (v_j - v_{j0}); \forall j \in J \tag{1b}$$

$$c_j^u \geq 0; \forall j \in J \tag{1c}$$

$$v_j \in \{0,1\}; \forall j \in J \tag{1d}$$

$$\sum_{j\in J} p_j^g + \sum_{l|d(l)=n} p_l^f - \sum_{l|o(l)=n} p_l^f = P_n^d; \forall n \in N \tag{1e}$$

$$p_l^f = \frac{1}{x_l}(\delta_{o(l)} - \delta_{d(l)}); \forall l \in L \tag{1f}$$

$$-\overline{P}_l^f \leq p_l^f \leq \overline{P}_l^f; \forall l \in L \tag{1g}$$

$$\underline{P}_j^g v_j \leq p_j^g \leq \overline{P}_j^g v_j; \forall j \in J \tag{1h}$$

The objective function (1a) is to minimize the total operational cost that is the production cost, $K_j^g p_j^g$ along with the start-up cost, c_j^u. K_j^g and K_j^u are the price of power production and the price unit start-up, respectively. J is the set of generators, indexed by j. Constraints (1b) and (1c) include the start-up cost of a unit that depends on v_j, ON/OFF unit binary variable, defined in (1d). v_{j0} is the unit initial status. The power balance equation is enforced in (1e), while (1f) determines the power flow direction. N is the buses set indexed by n, whereas L is the lines set indexed by l. The subscripts $o(l)$ and $d(l)$ refer to the line origin and destination, respectively. The upper and lower limits of the line and generator capacities are bounded in (1g) and (1h), respectively.

The nonconvexity of this problem is caused by the binary variable for each unit. It obstructs obtaining the dual variables. These values are necessary to be calculated by generation companies prior to offering energy to the pool. With this traditional formulation, the dual variables can only be obtained if the binary variables are given. Therefore, this entails solving the problem twice. The first time is the original MIP problem and the second time is a linear problem in which the binary variables are plugged. This difficulty can be overcome in our approach that reaches all the optimal primal and dual solution in one step.

B. Bilevel Unit Commitment Problem

Our proposed model is based on bilevel programming. The problem is constructed of two players, one acting as a leader and the other acting as a follower. The leader player is the upper-level problem, and exchanges the optimal variables with the follower. The upper-level problem contains the binary decision variables that determine the optimal ON/OFF unit statuses. The lower-level problem is essentially a DCOPF problem. The structure of the bilevel model is shown in Fig. 2. The upper-level problem is given by:

$$\max_{v_j} \quad \sum_{n\in N} P_n^d \lambda_n^* - \sum_{j\in J} c_j^u \tag{2a}$$

$$\text{subject to} \quad \text{Constraints} \quad (1b) - (1d) \tag{2b}$$

The objective function (2a) maximizes the revenue and minimizes unit start-up cost. P_n^d and λ_n^* are the demand load and the LMP, respectively, of bus n. λ_n^* is the optimal dual variable of the power balance equation, from the lower-level problem. The constraints of this problem are represented in (2b), which are related to the optimal unit ON/OFF variables. The lower-level problem is formulated as follows:

$$\min_{p_l^f, p_j^g, \delta_n} \quad \sum_{n\in N} K_j^g p_j^g \tag{3a}$$

$$\text{subject to} \quad p_l^f = \frac{1}{x_l}(\delta_{o(l)} - \delta_{d(l)}) : (\nu_l); \forall l \in L \tag{3b}$$

$$-\overline{P}_l^f \leq p_l^f \leq \overline{P}_l^f : (\phi_l^{min}, \phi_l^{max}); \forall l \in L \tag{3c}$$

$$\underline{P}_j^g v_j^* \leq p_j^g \leq \overline{P}_j^g v_j^* : (\theta_j^{min}, \theta_j^{max}); \forall j \in J \tag{3d}$$

$$\sum_{j\in J} p_j^g + \sum_{l|d(l)=n} p_l^f - \sum_{l|o(l)=n} p_l^f = P_n^d : (\lambda_n); \forall n \in N \tag{3e}$$

The problem (3) represents optimal DC power flow of a constrained network system. Although constraint (3d) has ON/OFF component, its optimal value is received from the upper-level problem. The exchanged variables between the two level problems are v_j and λ_n. The dual variables of the

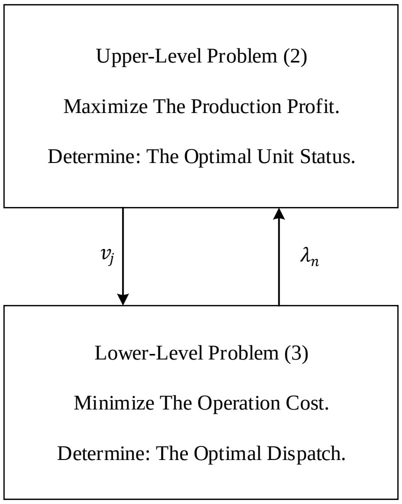

Fig. 2. The Bilevel Model.

constraints are shown in parentheses, which are the shadow prices representing the cost of the constraint violation [9]. For instance, an increase of 1 unit of the load demand is worth λ_n (\$ / unit), which is the LMP. ν_l is the dual variable of the power flow equation, while ϕ_l^{min} and ϕ_l^{max} are the dual variables related to minimum and maximum line capacity constraints, respectively. The dual variables associated with the minimum and maximum unit generation capacity constraints are θ_j^{min} and θ_j^{max}, respectively.

Since the lower-level problem is convex, the two problems can be transformed to a single-level problem. The new equivalent problem is constrained by primal feasibility constraints, lower-level problem dual feasibility constraints, and the equality of the lower-level problem's primal and dual objective functions [10], [8]. The equality constrains is based on the strong duality theorem. The Lagrangian dual objective function and constraints of the lower-level problem are presented as follows:

$$\begin{aligned}
&L(p_l^f, p_j^g, \delta_n, \lambda_n, \nu_l, \phi_l^{max}, \phi_l^{min}, \theta_j^{max}, \theta_j^{min}) \\
&= \sum_{j\in J} K_j^g p_j^g - \sum_{n\in N} \lambda_n\big(\sum_{j\in J} p_j^g + \sum_{l|d(l)=n} p_l^f \\
&\quad - \sum_{l|o(l)=n} p_l^f - P_n^d\big) - \sum_{l\in L} \nu_l\big(p_l^f \\
&\quad - \frac{1}{x_l}(\delta_{o(l)} - \delta_{d(l)})\big) + \sum_{l\in L} \phi_l^{max}(p_l^f - \overline{P}_l^f) - \sum_{l\in L} \phi_l^{min}(p_l^f \\
&\quad + \overline{P}_l^f) + \sum_{j\in J} \theta_j^{max}(p_j^g - \overline{P}_j^g v_j^*) + \sum_{j\in J} \theta_j^{min}(-p_j^g + \underline{P}_j^g v_j^*)
\end{aligned} \tag{4}$$

$$\begin{aligned}
&= \sum_{j\in J} p_j^g (K_j^g - \lambda_n + \theta_j^{max} - \theta_j^{min}) \\
&\quad + \sum_{l\in L} p_l^f \big(-\lambda_{d(l)=n} + \lambda_{o(l)=n} - \nu_l + \phi_l^{max} - \phi_l^{min}\big) \\
&\quad + \sum_{l\in L} \delta_l \big(\frac{1}{x_l}\nu_{o(l)} - \frac{1}{x_l}\nu_{d(l)}\big) \\
&\quad + \sum_{n\in N} P_n^d \lambda_n - \sum_{l\in L} \phi_l^{max} \overline{P}_l^f - \sum_{l\in L} \phi_l^{min} \overline{P}_l^f \\
&\quad - \sum_{j\in J} \theta_j^{max} v_j^* \overline{P}_j^g + \sum_{j\in J} \theta_j^{min} v_j^* \underline{P}_j^g
\end{aligned} \tag{5}$$

The lower-level dual problem:

$$\max_{\lambda,\nu,\phi,\theta} Z_D = \inf_{p_l^f, p_j^g, \delta} L(p_l^f, p_j^g, \delta_n, \lambda_n, \nu_l, \phi_l^{max,min}, \theta_j^{max,min})$$

$$\begin{aligned}
&= \sum_{n\in N} P_n^d \lambda_n - \sum_{l\in L} \overline{P}_l^f (\phi_l^{max} + \phi_l^{min}) \\
&\quad + \sum_{j\in J} v_j^* (\theta_j^{min} \underline{P}_j^g - \theta_j^{max} \overline{P}_j^g)
\end{aligned} \tag{6}$$

subject to

$$K_j^g - \lambda_n + \theta_j^{max} - \theta_j^{min} = 0; \forall j \in J \tag{7a}$$

$$\lambda_{o(l)=n} - \lambda_{d(l)=n} - \nu_l + \phi_l^{max} - \phi_l^{min} = 0; \forall l \in L \tag{7b}$$

$$\sum_{l|o(l)=n} \frac{1}{x_l}\nu_l - \sum_{l|d(l)=n} \frac{1}{x_l}\nu_l = 0; \forall l \in L \tag{7c}$$

$$\theta_j^{max}, \theta_j^{min} \geq 0; \forall j \in J \tag{7d}$$

$$\phi_l^{max}, \phi_l^{min} \geq 0; \forall l \in L \tag{7e}$$

The single-level equivalent problem becomes as follows:

$$\max_{p_l^f, p_j^g, \delta_n, \lambda_n, \nu_l, \phi_l, \theta_j} \sum_{n\in N} P_n^d \lambda_n - \sum_{j\in J} c_j^u \tag{8a}$$

$$\text{subject to} \quad \text{Constraints} \quad (1b) - (1d) \tag{8b}$$

$$\text{Constraints} \quad (3b) - (3e) \tag{8c}$$

$$\text{Constraints} \quad (7a) - (7e) \tag{8d}$$

$$\begin{aligned}
&\sum_{j\in J} P_n^d \lambda_n + \sum_{j\in J} (\underline{P}_j^g \theta_j^{min} v_j - \overline{P}_j^g \theta_j^{max} v_j) \\
&\quad - \sum_{l\in L} \overline{P}_l^f (\phi_l^{max} + \phi_l^{min}) = \sum_{n\in N} K_j^g p_j^g
\end{aligned} \tag{8e}$$

The objective function of the single-level equivalent problem is (8a), which is the same as the objective function of the upper-level problem. The constraints of the upper-level problem are represented in (8b). Constraints (8c) and (8d) are the primal and dual feasibility constraints of the lower-level problem; respectively. The equality of the primal and the dual objective functions is given in (8e).

The single-equivalent problem (8) is nonlinear because of constraint (8e) that contains multiplication of the dual variables with the binary variables. The problem is linearized using Fortuny-Amat McCarl method. (8e) is replaced by the following:

$$\sum_{j \in J} P_n^d \lambda_n + \sum_{j \in J} (\underline{P}_j^g b_j - \overline{P}_j^g a_j) - \sum_{l \in L} \overline{P}_l^f (\phi_l^{max} + \phi_l^{min}) = \sum_{n \in N} K_j^g p_j^g \tag{9a}$$

$$0 \le a_j \le \overline{\theta_j^{max}} v_j \tag{9b}$$

$$0 \le \theta_j^{max} - a_j \le \overline{\theta_j^{max}} (1 - v_j) \tag{9c}$$

$$0 \le b_j \le \overline{\theta_j^{min}} v_j \tag{9d}$$

$$0 \le \theta_j^{min} - b_j \le \overline{\theta_j^{min}} (1 - v_j) \tag{9e}$$

where $\overline{\theta_j^{max}}$ and $\overline{\theta_j^{min}}$ are the maximum limit for the generation constraint dual variables.

The linearized single-level equivalent:

$$\max_{p_l^f, p_j^g, \delta_n, \lambda_n, \nu_l, \phi_l, \theta_j} \sum_{n \in N} P_n^d \lambda_n - \sum_{j \in J} c_j^u \tag{10a}$$

subject to	Constraints	(1b) – (1d)	(10b)
	Constraints	(3b) – (3e)	(10c)
	Constraints	(7a) – (7e)	(10d)
	Constraints	(9a) – (9e)	(10e)

III. Case Studies

The formulation model (10) has been tested on different scale systems. Due to the paper size, only two systems are discussed in this paper. The tested cases are nesta_case5_pjm and nesta_case30_ieee, taken from [11]. More details of the test systems are found in [12]. The model solution is compared with the conventional solution using (1). The model has been implemented in Matlab with CVX [13] using solver Gurobi 7.51 [14]. Both MILP based UC and bi-level programming-based UC are solved. Their solutions on unit on/off status and generator power dispatch levels are exactly same.

A. 5-Bus System Case Study

Fig. 3 shows a system that has 5 generator units and 6 transmission lines two of which are limited. Different types of investigation are implemented on the system to assess the model ability. First, the system is solved under normal operation, without modifications. Second, some modifications are applied on the system physical constraints to highlight the effect of the shadow prices. Third, all the line limits are set high for the sake of analyzing the effect of uncongestion on the LMPs of the system buses.

1) The System Normal Operation: The exact results of the unit commitment conventional method are obtained by solving the bilevel model in 0.5332 second. The LMPs of the buses and the total revenue are shown in Table I. The LMP values change from a bus to another. The generator statuses, outputs, maximum dual variables, and minimum dual variables are listed in Table II, while the optimal primal and dual solution of the lines are given in Table III. All the generator units are ON except the generator at bus 4 because it is the most expensive one. Since G11 and G12 are cheap, they are fully utilized, which explains the non-zero dual variables of their capacity constraints. Even though G5 is the cheapest, it is not wholly exploited because it is surrounded by the limited transmission lines.

2) Dual Variable Interpretations: As mentioned, the dual variables, shadow prices, portray the cost of violating the problem constraints. The shadow price of an inequality constraint equals zero as long as the constraint is not binding. If so, then the dual variable has a value indicating the objective functions

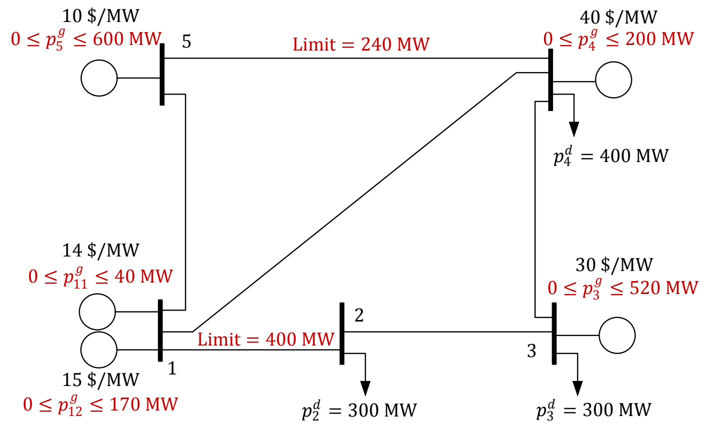

Fig. 3. PJM 5-bus system [12].

TABLE I
5-Bus System LMPs of The Buses and The Revenue

Bus	1	2	3	4	5
LMP ($/MWh)	16.9774	26.3845	30	39.9427	10
Total Revenue = 32892 $					

TABLE II
5-Bus System Generator Primal and Dual Components

	ON/OFF	Output (MW)	θ^{max} ($/MWh)	θ^{min} ($/MWh)
G11	1	40	2.9774	0
G12	1	170	1.9774	0
G3	1	323.4948	0	0
G4	0	0	0	0
G5	1	466.5052	0	0
Total Generation Cost = 17480 $				

TABLE III
5-Bus System Primal and Dual Components of The Lines

	p_{12}^f	p_{14}^f	p_{15}^f	p_{23}^f	p_{34}^f	p_{45}^f
Flow (MW)	249.7	186.8	-226.5	-50.3	-26.8	-240
ϕ^{max} ($/MWh)	0	0	0	0	0	62.32
ϕ^{min} ($/MWh)	0	0	0	0	0	0

TABLE IV
5-Bus System LMPs of The Buses and The Revenue (Uncongested)

Bus	1	2	3	4	5
LMP ($/MWh)	30	30	30	30	30
Total Revenue = 30000 $					

TABLE V
5-Bus System Generator Primal and Dual Components (Uncongested)

	ON/OFF	Output (MW)	θ^{max} ($/MWh)	θ^{min} ($/MWh)
G11	1	40	16	0
G12	1	170	15	0
G3	1	190	0	0
G4	0	0	0	0
G5	1	600	20	0
Total Generation Cost = 14810 $				

TABLE VI
5-Bus System Primal and Dual Components of The Lines (Uncongested)

	p^f_{12}	p^f_{14}	p^f_{15}	p^f_{23}	p^f_{34}	p^f_{45}
Flow (MW)	317.6	209.6	-317.2	17.6	-92.4	-282.8
ϕ^{max} ($/MWh)	0	0	0	0	0	0
ϕ^{min} ($/MWh)	0	0	0	0	0	0

is affected by this amount if that constraint changes one unit. The dual variables of the lines, in Table III, are all zeros except the line between 4 and 5 that has value of 62.32 $/MW. This implies only line 4-5 is congested since its upper bound is binding.

Under normal operation, the total revenue is 32892 $ and the total generation cost = 17480 $. The profit of the total operation is 15412 $. If line 4-5 capacity limit increases from 240 to 241 MW, the generation cost decreases by 62.32 $ and the profit raises to be the same value to be 15474 $. In the same manner, If the capacity line 4-5 decreases from 240 to 239 MW the generation cost increases 62.32 $. In case the line capacity of the same line is reduced to 100 MW, representing a higher congestion on the line, G4 is forced to run to supply the system with 160.1384 MW. In this situation the profit drops to 6722 $. This example shows that high congestion adversely affects on the profit.

3) Uncongested System: Another type of examinations is carried out carried out by setting the line capacity two times higher than that of the first case. As a result, there is no congestions in the lines at all. The total revenue and the LMPs of the buses and are presented in Table IV. Table V and Table VI show the primal and dual components of the generators and lines, respectively.

Relaxing the line capacity limits lead to a reduction in both the revenue and the total generation cost to 30000 $ and 14810 $, respectively. The profit turns into 15190 $. This case is more beneficial to the consumers side than the generator owners side because the consumers pay 2892 $ less, while th
of generator owners reduces 222 $ comparing to the c
case. In this case, all the buses share the same value
LMP that is 30 $/MW. The shadow prices of all the li
zero since they are not congested. In contrast, the ge
shadow prices are relatively high because of the ma
limits of the generator units are binding. This is not th
in the normal operation in which one of the maximum
binding.

B. 30-Bus System Case Study

1) The System Normal Operation: The method is em
on the IEEE 30-bus test system that contains 6 ge
units and 41 transmission lines [12]. As in the previ
bus case study, the bilevel model solution is compare
and matches the conventional one. The solution is ac
in 6.2440 second. The total revenue 290.9752 $, and t
LMPs are shown in Fig. 4. Table VII lists the generator
and dual values. Note that G1 and G2 are the only active power generation units, and the rest of the units are synchronous condensers, which only supply reactive power to the system. The generation cost of G1 and G2 are 0.5213 and 1.1351 $/MWh, respectively. The LMPs of buses 1 and 2 have the exact value of the generation unit cost. The prices of the buses hinge on the bus's location and its demand load. Line 1-2 is binding, congested, whereas all the other lines are below the upper bounds. This means all line dual variables are zeros except one the two dual variable related to line 1-2. The value of ϕ^{min}_{1-2} is 0.7369 $/MWh.

2) The System Congestion Operation: To conduct congestion analysis, the capacity of one of the lines is decreased.Under normal operation, p^f_{1-3}, the power flow from bus 1 to bus 2, is 77.75 MW. The maximum capacity limit

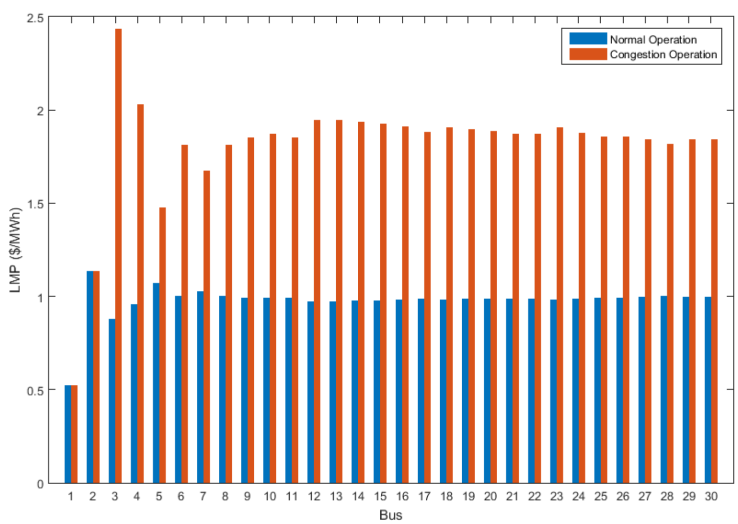

Fig. 4. Bus LMPs of IEEE 30-bus system.

TABLE VII
30-BUS SYSTEM GENERATOR PRIMAL AND DUAL COMPONENTS

	ON/OFF	Output (MW)	θ^{max} ($/MWh)	θ^{min} ($/MWh)
G1	1	215.7540	0	0
G2	1	67.6460	0	0
Total Generation Cost = 189.2789 $				

TABLE VIII
30-BUS SYSTEM GENERATOR PRIMAL AND DUAL COMPONENTS (CONGESTED)

	ON/OFF	Output (MW)	θ^{max} ($/MWh)	θ^{min} ($/MWh)
G1	1	184.3122	0	0
G2	1	99.0878	0	0
Total Generation Cost = 208.5774 $				

of this line is set 72.5 MW. The system generation details are given in Table VIII, while the bus LMPs are found in Fig. 4. The LMPs raise in all buses except the ones that have active power generation units, bus 1 and 2. Due to the congestion, G2, the expensive unit, is forced to participate more in the system. In this study, only line 1-3 is the only congested line. So, all the dual variables associated to the line capacity constraints are zeros except ϕ_{1-3}^{min} that equals 3.6731 $/MWh.

In this procedure, the total revenue is 474.8805 $, and the total generation cost is 208.5774 $. This leads to a total profit of 266.3031 $. Although the congestion seems minor, it increases the total profit 105.0983 %.

IV. CONCLUSION

This paper presents a bilevel model that comprises two level problems, namely, the upper-level and the lower-level. The upper-level has binary decision variables while the low-level has only continuous decision variables. Further, we explore the primal-dual relationship and explicitly express LMP as decision variables in the low-level problem. The bilevel problems are transformed to a single-level problem and solved efficiently using Gurobi.

REFERENCES

[1] A. L. Motto, F. D. Galiana, A. J. Conejo, and J. M. Arroyo, "Network-constrained multiperiod auction for a pool-based electricity market," *IEEE Transactions on Power Systems*, vol. 17, no. 3, pp. 646–653, Aug 2002.

[2] B. Eldridge, R. P. ONeill, and A. Castillo, "Marginal loss calculations for the dcopf," *Federal Energy Regulatory Commission, Tech. Rep*, 2017.

[3] FERC, "Fy 2019 congressional budget request and fy 2017 annual performance report," 2018. [Online]. Available: https://www.ferc.gov/about/strat-docs.asp

[4] H. Mohsenian-Rad, "Optimal bidding, scheduling, and deployment of battery systems in california day-ahead energy market," *IEEE Transactions on Power Systems*, vol. 31, no. 1, pp. 442–453, 2016.

[5] J. M. Arroyo and A. J. Conejo, "Optimal response of a thermal unit to an electricity spot market," *IEEE Transactions on Power Systems*, vol. 15, no. 3, pp. 1098–1104, Aug 2000.

[6] M. Carrion and J. M. Arroyo, "A computationally efficient mixed-integer linear formulation for the thermal unit commitment problem," *IEEE Transactions on Power Systems*, vol. 21, no. 3, pp. 1371–1378, Aug 2006.

[7] S. A. Sadat and L. Fan, "Mixed integer linear programming formulation for chance constrained mathematical programs with equilibrium constraints," in *2017 IEEE Power Energy Society General Meeting*, July 2017, pp. 1–5.

[8] R. Fernndez-Blanco, J. M. Arroyo, and N. Alguacil, "Network-constrained day-ahead auction for consumer payment minimization," *IEEE Transactions on Power Systems*, vol. 29, no. 2, pp. 526–536, March 2014.

[9] S. Boyd and L. Vandenberghe, *Convex optimization*. Cambridge university press, 2004.

[10] J. M. Arroyo, "Bilevel programming applied to power system vulnerability analysis under multiple contingencies," *IET Generation, Transmission Distribution*, vol. 4, no. 2, pp. 178–190, February 2010.

[11] C. Coffrin, D. Gordon, and P. Scott, "Nesta, the NICTA energy system test case archive," *CoRR*, vol. abs/1411.0359, 2014. [Online]. Available: http://arxiv.org/abs/1411.0359

[12] F. Li and R. Bo, "Small test systems for power system economic studies," in *IEEE PES General Meeting*, July 2010, pp. 1–4.

[13] M. Grant, S. Boyd, and Y. Ye, "Cvx: Matlab software for disciplined convex programming," 2008.

[14] I. Gurobi Optimization, "Gurobi optimizer reference manual," 2016. [Online]. Available: http://www.gurobi.com