MIP-Based Fault Location Identification Using MicroPMUs

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***Abstract*—In this paper, a mixed integer programming (MIP)-based method is proposed to identify not only fault locations and types in unbalanced three-phase distribution networks, but also the magnitude and the angle of the fault currents. Moreover, the minimum microPMU placement for faults detection is determined based on the impedance matrix characteristics. Different type of faults (symmetrical and unsymmetrical) are preformed on the IEEE 37-bus feeder to test the effectiveness of the proposed algorithm in fault location and current identification. Further tests are presented to examine the performance of the proposed approach against different substation transformer configurations. The suggested algorithm shows high success rate in identifying fault locations, types and currents.**

***Index Terms*—Radial distribution network; least squares estimation, mixed integer programming; fault location. mirco-PMUs, distribution feeder monitoring.**

I. INTRODUCTION

Improving reliability indices of electrical power distribution systems and providing high quality service for customers have been set as the main objectives to many electric utilities. The ability to detect fault locations immediately, and then restore service to customers would ensure reduction in the interruption time and improvements in these indices. Many methods have been proposed to identify fault location in distribution networks. These methods are generally classified into two major categories: local impedance-based and wide-area monitoring-based. Impedance-based methods identify the fault location using voltage and current measurements from a sensor. The line impedance between the fault and the sensor can then be found using the voltage and current measurements [1][4]. The performance of ten impedance-based fault detection methods for distribution feeders is compared and analyzed in [1]. According to [1], the main drawback of Impedance-based methods is that they may yield to multiple fault locations when applying it to distribution networks with many laterals.

Wide-area monitoring methods rely on synchronized phasor measurements at different locations. Changes in voltage and current due to a fault usually give information regarding fault location [5], [6]. MicroPMUs are synchrophasor measurement units for distribution systems. A new impedance-based technique which utilizes synchronized and non-synchronized measurements to located faults in distributions network using least-squares technique is presented in [7]. The proposed method in [7] utilizes Linear least-squares estimator if synchronized measurements are installed in the feeder, and utilizes nonlinear least-squares estimator if the measurements used are non-synchronized (smart meters).

Using microPMU for state estimation and fault location has been examined in [6], [8]. While microPMUs are assumed to be installed at every bus in [6] for state estimation and fault location, [8] indicates that only two microPMUs are needed for a single branch radial network. In this paper, a mixed integer programming (MIP) formulation is offered to identify fault locations and fault currents. This formulation is then extended for application in unbalanced distribution systems. The results show that the proposed algorithm can accurately identify fault locations and fault types.

The rest of the paper is organized as follows. Section II presents a mixed-integer programming (MIP) formulation for fault location and fault current identification in a simple tree network, then proceeds to examine the impedance matrix that relates the fault current injection and the node voltage deviation. Section III examines microPMU placement for the tree network. Section IV extends the The mixed integer programming formulation to three-phase unbalanced distribution system. Section VI shows numerical example on three-phase unbalance network. The conclusion is presented in section VII.

II. FAULT LOCATION IN A TREE NETWORK

An example distribution feeder with one main source (the substation) is shown in Fig. 1. When a fault occurs at Bus 4, the system can be viewed as the superposition of two circuits. The first circuit is essentially the pre-fault original circuit and the voltage phasors are the pre-fault voltage phasors. Since the sum of Circuit 1 and Circuit 2 is equivalent to the post-fault system, the node voltage phasors in Circuit 2 are $\Delta\bar{V} = \bar{V}^{\rm post} - \bar{V}^{\rm pre}$.

The two circuits have exactly the same topology. For the first circuit, we may express the voltage and current relationship by $\bar{I} = Y\bar{V}$. Further, we will differentiate the nodes by the substation bus (notated by the subscript S) and the rest of the buses (notated by the subscript N).

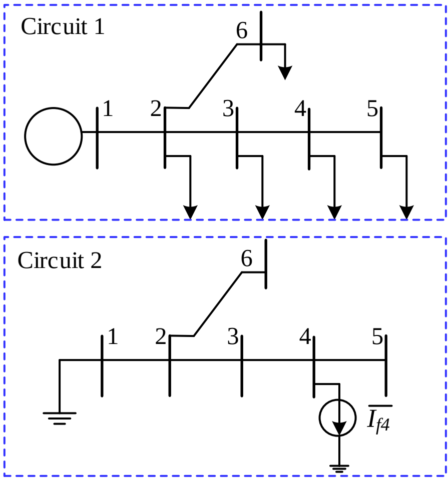

Fig. 1. A fault scenario can be viewed as the superposition of two circuits.

$$\begin{bmatrix}\bar{I}_S^{\text{pre}}\\ \bar{I}_N^{\text{pre}}\end{bmatrix} = \begin{bmatrix}Y_{SS} & Y_{SN}\\ Y_{NS} & Y_{NN}\end{bmatrix}\begin{bmatrix}\bar{V}_S^{\text{pre}}\\ \bar{V}_N^{\text{pre}}\end{bmatrix} \tag{1}$$

For the second circuit, we can establish the following relationship for its node voltage phasors and current inject phasors.

$$\begin{bmatrix}0 & 0 & -\bar{I}_{f4} & 0 & 0\end{bmatrix}^T = Y_{NN}\Delta\bar{V}_N \tag{2}$$

Using the impedance matrix $Z_{NN} = Y_{NN}^{-1}$, we have the following relationship:

$$\begin{aligned}\Delta\bar{V}_N &= Z_{NN}\begin{bmatrix}0 & 0 & -\bar{I}_{f4} & 0 & 0\end{bmatrix}^T\\ &= \begin{bmatrix}Z_{NN,24} & Z_{NN,34} & Z_{NN,44} & Z_{NN,54} & Z_{NN,64}\end{bmatrix}^T(-\bar{I}_{f4})\end{aligned} \tag{3}$$

To find $\bar{I}_{f4}$ and have a unique solution, we only need any one voltage phasor. If we have more than one measurement phasors, then we to have an overdetermined problem or an estimation problem.

We now extend this problem to fault location and fault current identification. Neither the location nor the size of the fault current is known. Hence we introduce a binary variable u_i for Node i to indicate if there is a fault ($u_i = 1$) or no fault ($u_i = 0$). Thus, Circuit 2's voltage and current relationship will be modified as follows.

$$\Delta\bar{V}_N = Z_{NN}\begin{bmatrix}-u_2\bar{I}_{f2} & -u_3\bar{I}_{f3} & -u_4\bar{I}_{f4} & -u_5\bar{I}_{f5} & -u_6\bar{I}_{f6}\end{bmatrix}^T \tag{4}$$

We will replace $u_i\bar{I}_{fi}$ by $\bar{I}_{xi}$. Note that $u_i\bar{I}_{fi}$ is a bilinear expression and the current off-shelf MIP solvers such as Mosek [12] cannot handle bilinear MIP problems. Hence, the big-M technique is employed to replace this equality constraint $\bar{I}_{xi} = u_i\bar{I}_{fi}$. Since u_i is a binary variable, then the above equation can be viewed as the following logic statement.

$$\bar{I}_{xi} = \begin{cases}\bar{I}_{fi}, & \text{if } u_i = 1\\ 0, & \text{if } u_i = 0\end{cases} \tag{5}$$

A big number $M = 10^6$ is introduced. Since phasors are complex variables, in computing, their real and imaginary parts will be considered seperately. The following inequality constraints are the equivalent of the logic statement in (5).

$$\begin{aligned}&-(1-u_i)M + \text{Re}(\bar{I}_{fi}) \le \text{Re}(\bar{I}_{xi}) \le \text{Re}(\bar{I}_{fi}) + (1-u_i)M\\ &-(1-u_i)M + \text{Im}(\bar{I}_{fi}) \le \text{Im}(\bar{I}_{xi}) \le \text{Im}(\bar{I}_{fi}) + (1-u_i)M\\ &-u_iM \le \text{Re}(\bar{I}_{xi}) \le +u_iM\\ &-u_iM \le \text{Im}(\bar{I}_{xi}) \le +u_iM\end{aligned} \tag{6}$$

If there is prior knowledge that the system has just one fault, then this inequality constraint will be imposed: $\sum_{i=2}^{6} u_i \le 1$.

III. Minimum Placement of MicroPMUs

We now consider how many microPMUs are needed to accurately identify fault current at any location. If the location is known, then any one measurement will indicate the fault current. However, if the location is unknown, the identification needs more deliberation to generate accurate results.

Using the above example test feeder, we will examine the Z_{NN} matrix characteristics. Assume that all shunts are ignored and the branch impedance between a branch connecting Node i and Node j is z_{ij}. The impedance matrix Z_{NN} for the 6-node distribution feeder can be found as:

$$Z_{NN} = \begin{bmatrix}z_{12} & z_{12} & z_{12} & z_{12} & z_{12}\\ z_{12} & z_{12}+z_{23} & z_{12}+z_{23} & z_{12}+z_{23} & z_{12}\\ z_{12} & z_{12}+z_{23} & z_{12}+z_{23}+z_{34} & z_{12}+z_{23}+z_{34} & z_{12}\\ z_{12} & z_{12}+z_{23} & z_{12}+z_{23}+z_{34} & z_{12}+z_{23}+z_{34}+z_{45} & z_{12}\\ z_{12} & z_{12} & z_{12} & z_{12} & z_{26}\end{bmatrix}$$

The first column has the same z_{12}. This indicates that the effect of a fault current at any bus (2 to 6) is the same from Bus 2 voltage's perspective.

$$\Delta\bar{V}_2 = -z_{12}\bar{I}_{fi}, \text{ for all } i \ge 2. \tag{7}$$

The second column indicates that Bus 3's voltage phasor will show difference if the fault current is located at Buses 2, 6 versus at Buses 3, 4, 5. Bus 3's voltage cannot differentiate Bus 2 and Bus 6, or Buses 3, 4, and 5. Column 4 indicates that Bus 5's voltage can differentiate all locations except when the fault is located at Bus 2 or Bus 6. Column 5 indicates that the voltage at Bus 6 can differentiate fault located at the main feeder branch versus the fault located at another branch.

If we place two microPMUs at Bus 5 and Bus 6, then we can accurately find the fault location if the fault current phasor is known. These two buses are the end buses of branches. As an extension, the analysis shows that if we have all the end buses equipped with microPMUs, we can identify fault location if we know the fault current phasor.

If the fault current is not fixed, then it can very well happen for a fault at Bus 3 or a fault at Bus 4 resulting in the same $\Delta\bar{V}_5$. The fault current phasor can be found if we have a microPMU installed at Bus 2. Thus, $\Delta\bar{V}_2$ can be measured and the fault current phasor can be found using (7). The assumption is that the entire distribution feeder has only one fault. When there are multiple faults, $\Delta\bar{V}_2$ will no longer give the information of fault currents.

As a *summary*, the minimum number of microPMU needed equals the number of tree branches added by 1.

The overall mixed integer least norm 2 problem can now be formulated (shown in (8)) and solved by Mosek. The objective function will be the sum of the norm 2 of the error between measurements and their estimates.

$$\begin{aligned} \text{minimize} \sum_{i\in\mathcal{E}} \|\Delta\bar{V}_i^{\text{meas}} - \Delta\bar{V}_i\| \quad &(8a)\\ \text{subject to } \sum_{i\in\mathcal{N}} u_i \leq 1 \quad &(8b)\\ \Delta\bar{V}_N = -Z_{NN}\bar{I}_x \quad &(8c)\\ (6) \text{ for all } i\in\mathcal{N} & \end{aligned}$$

where $\mathcal{E}$ is the set of the buses located at the end of each branch and the bus closest to the substation bus, $\mathcal{N}$ is the set of the buses except the substation bus.

IV. MIP FORMULATION FOR THREE-PHASE DISTRIBUTION SYSTEMS

The mixed integer programming formulation is now extended to three-phase unbalanced distribution system. A fault location binary variable u is designated to each phase at each bus except substation bus. Another location binary variable k is designated to each bus except the substation bus. For example, if the 6-node feeder in Fig1 is assumed to be three-phase unbalanced network and all the lines are three-phase, the dimension of u is 15, and the dimension of k is 5. If there is prior knowledge that the system has just one fault, then this equality constraint will be imposed: $\sum_{i=2}^{6} k_i = 1$

For each node, there are abc-phase voltage phasors and abc-phase injected current phasors. If a three-phase fault occurs at Bus 4, the relationship in (2) can be written as the following:

$$\begin{bmatrix} 0\\ \vdots\\ 0\\ -\bar{I}_{f4}^a\\ -\bar{I}_{f4}^b\\ -\bar{I}_{f4}^c\\ 0\\ \vdots\\ 0 \end{bmatrix} = Y_{NN}^{abc} \begin{bmatrix} \Delta\bar{V}_2^a\\ \Delta\bar{V}_2^b\\ \Delta\bar{V}_2^c\\ \vdots\\ \Delta\bar{V}_6^a\\ \Delta\bar{V}_6^b\\ \Delta\bar{V}_6^c \end{bmatrix} \quad (9)$$

The relationship in (4) can be extended to the three-phase format and written as the following:

$$\Delta\bar{V}_N^{abc} = Z_{NN}^{abc} \begin{bmatrix} -u_{2a}\bar{I}_{f2}^a\\ -u_{2b}\bar{I}_{f2}^b\\ -u_{2c}\bar{I}_{f2}^c\\ \vdots\\ -u_{6a}\bar{I}_{f6}^a\\ -u_{6b}\bar{I}_{f6}^b\\ -u_{6c}\bar{I}_{f6}^c \end{bmatrix} \quad (10)$$

The overall mixed integer least norm 2 problem in (8) can now be extended to the three-phase format (shown in (11)).

$$\begin{aligned} \text{minimize} \sum_{i\in\mathcal{E}} \|\Delta\bar{V}_i^{\text{meas}} - \Delta\bar{V}_i\| & \\ \text{subject to } \quad \Delta\bar{V}_N^{abc} = -Z_{NN}^{abc}\bar{I}_x^{abc} \quad &(11a)\\ (6) \text{ for all } i\in\mathcal{B} & \\ u_{ia} + u_{ib} + u_{ic} = 3(k_i) \quad &(11b)\\ \sum_{i\in\mathcal{E}} k_i = 1 \quad &(11c)\\ \text{for all } i\in\mathcal{N} & \end{aligned}$$

where $\mathcal{E}$ is the set of the microPMU phases, $\mathcal{N}$ is the set of the buses except the substation bus. $\mathcal{B}$ is the set of the phases except the substation bus's phases. The impedance matrix Z_{NN}^{abc} was found by removing the first three columns and rows in the Y_{bus}^{abc} obtained from OpenDSS[12], and then inverting it. u_{ia},u_{ib} and u_{ic} are the binary variables designated to phase a, phase b and phase c at Bus i. For example, for the 6-node distribution feeder. the constrain in (11b) is the following:

$$\begin{aligned} u_{2a} + u_{2b} + u_{2c} &= 3(k_2)\\ u_{3a} + u_{3b} + u_{3c} &= 3(k_3)\\ u_{4a} + u_{4b} + u_{4c} &= 3(k_4)\\ u_{5a} + u_{5b} + u_{5c} &= 3(k_5)\\ u_{6a} + u_{6b} + u_{6c} &= 3(k_6) \end{aligned} \quad (12)$$

A. FAULTED BUS AND PHASE DETERMINATION

Determining the faulted bus is simply by looking to the bus that its binary variable k is equal to 1. The equality constrain in (11c) ensures that there is only one faulted bus.

Due to the equality constrain in (11b), when the bus-binary variable k_i is equal to 1, the phase-binary variables u_a,u_b and u_c must equal to 1 even for a single or a double phase fault. As a result, the faulted phase(or phases) is distinguished by its high magnitude compared to the unfaulted phases (the currents assign to the unfaulted phases are random and neglected).

V. NUMERICAL EXAMPLE

To test the proposed method on 3-phase system, a modified version of the IEEE-37 test feeder is used (only the substation voltage regulator is omitted). The IEEE-37 test feeder is very unbalanced feeder. All the line segments are 3-phase and underground. The substation's transformer is connected in Delta-Delta. The connected loads are a combination of three types of load; constant power, constant current and constant impedance loads. Around 50% of the connected loads are constant power loads [11]. A total of 16 microPMUs are used as shown in Fig. 2.

OpenDSS is used for load flow computing and faults simulating. The pre-fault and post-fault voltage phasors at the measurement buses are recorded after load flow computation. The admittance matrix is assumed to be known and not changing during the fault. The data collected from OpenDSS is fed to MATLAB and solved by Mosek, Gurobi [17] (using

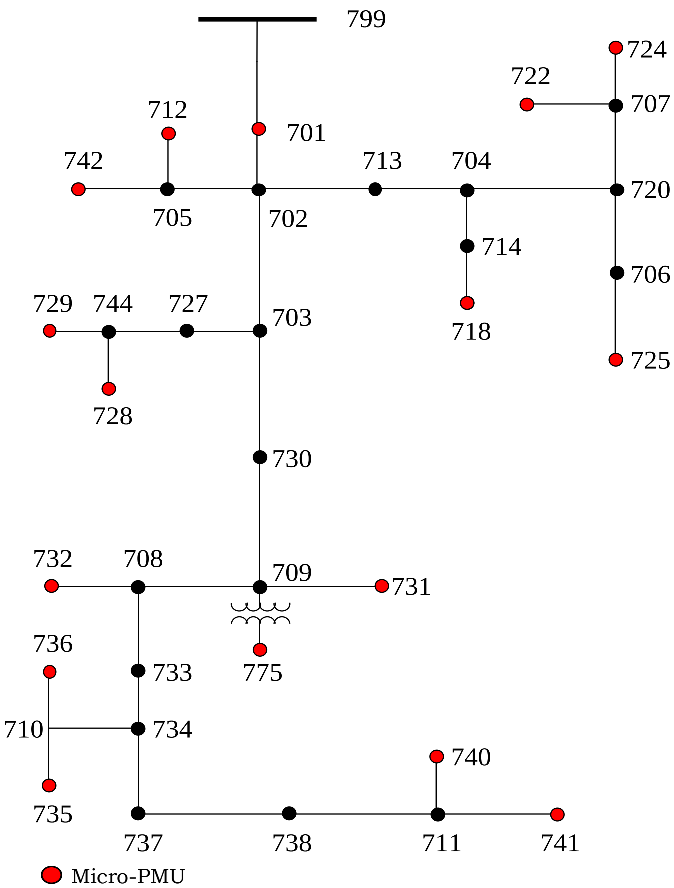

Fig. 2. Modified version of the IEEE- 37 test feeder.

CVX [14]) or BNB (using YALMIP [15]), which uses CPLEX [16] as a lower solver and Rounder as an upper solver.

During the tests, Mosek (CVX precision is set to its default) was the primary solver, and it was successful to identify the optimal solution for most of the cases. For the few cases where Mosek could not find the optimal solution using the default setting of CVX precision, optimal solutions were found by either changing the CVX precision or using Gurobi or BNB. The value of the big number M was set to 10^6 during the tests. However, for some cases (single phase faults on bus 775) where the fault current is small ($< 10^{-3}$ Ampere), the big number M was tighten up.

A. SINGLE PHASE FAULT

A single phase fault was applied at each phase in each bus in the feeder, one at a time (total of 108 faults). Even though in Delta (ungrounded) systems, the phase-to-ground fault is difficult to detect due to its small magnitude [18], the proposed method was successful in identifying all the faulted phases and the magnitude of the faults with high accuracy. Samples of the identification results are presented in Table I. The identified fault current (in Ampere) is compared with the actual fault currents from OpenDSS.

In order to test the performance of the proposed technique with different transformer configurations, the connection of the substation transformer was changed from Delta-Delta to Delta-Y with solidly grounded neutral. 108 single phase tests were run again one at a time. The success rate was 100%. Table II presents the identification results for phase-to-ground in Delta-Y configuration.

TABLE I
PHASE-TO-GROUND FAULTS (Δ - Δ)

	Identification Results			OpenDSS Measurements	
Location	Obj	I_F	$\angle \bar{I}_F$	I_{flow}	$\angle \bar{I}_{\text{flow}}$
729.b	0.070	2.111	-36	2.225	-33.5
711.c	0.170	2.064	-158.9	2.188	-156.1
702.b	0.053	2.124	-36	2.240	-33.5
775.a	0.002	$4.9e10^{-4}$	-97.5	$5.3e10^{-4}$	-94.2

TABLE II
PHASE-TO-GROUND FAULTS (Δ - Y)

	Identification Results			OpenDSS Measurements	
Location	Obj	I_F	$\angle \bar{I}_F$	I_{flow}	$\angle \bar{I}_{\text{flow}}$
702.a	89.6	2636.2	-100.4	2832.9	-97.3
741.c	100.2	1210	38.5	1285.4	41.5
707.b	94.2	1513	159.8	1591.9	162.2
710.c	99.88	1414.1	36.9	1503.3	39.9

B. PHASE-TO-PHASE FAULT

Phase-to phase (between phase a and b) faults were applied at each bus in the feeder, separately (total of 36 tests for each transformer configuration). The proposed method was successful in identifying all the faulted phases for the two transformer configurations except a phase-to-phase fault at bus 707. The identified fault was phase-to-phase fault (phase a and b) at Bus 722, which is adjacent bus to Bus 707. Table III and Table IV presents the identification results for phase-to-phase faults in Delta-Delta and Delta-Y transformer connection respectively.

TABLE III
PHASE-TO-PHASE FAULTS (Δ - Δ)

	Identification Results			OpenDSS Measurements	
Location	Obj	I_F^a	I_F^b	I_{flow}^a	I_{flow}^b
736.ab	155.1	1147.7	1168.5	1263.8	1262.9
714.ab	165.8	1992.8	2027.1	2191.5	2191
742.ab	153.3	1954.4	1990	2151.1	2150.4
775.ab	39.07	10630.7	10869.4	11796	11796

TABLE IV
PHASE-TO-PHASE FAULTS (Δ - Y)

	Identification Results			OpenDSS Measurements	
Location	Obj	I_F^a	I_F^b	I_{flow}^a	I_{flow}^b
735.ab	140.6	1581.4	1446.2	1706.4	1583.4
704.ab	172.7	2407.3	2172	2576.5	2375.5
728.ab	119	2150.6	1934.4	2309.4	2120.3
731.ab	84.5	2018.3	1788.3	2171.7	1962

C. THREE PHASE FAULT

Three-phase faults were set at each bus in the feeder, independently(total of 36 tests for each transformer configuration). Table III and Table IV presents the identification results for phase-to-phase faults in Delta-Delta and Delta-Y transformer connection respectively. The success rate of the proposed method is more than 97%. Only fault at Bus 707 was identified as a three-phase fault at Bus 722. By excluding Bus 722 from the searching space, the proposed method correctly identified the faulted bus.

TABLE V
THREE-PHASE FAULTS (Δ - Δ)

	Identification Results				OpenDSS Measurements		
Location	Obj	I_F^a	I_F^b	I_F^c	I_{flow}^a	I_{flow}^b	I_{flow}^c
727.abc	131.69	2231.8	2377.5	2242	2500.8	2573.9	2434.4
708.abc	140.2	1998.6	2159.2	2027.1	2242.2	2337.3	2197.2
738.abc	216.4	1545.8	1715.9	1606.5	1741.1	1859.7	1738.3
720.abc	257	2030.3	2188.6	2070.9	2261.3	2366.7	2253.6

TABLE VI
THREE-PHASE FAULTS (Δ - Y)

	Identification Results				OpenDSS Measurements		
Location	Obj	I_F^a	I_F^b	I_F^b	I_{flow}^a	I_{flow}^b	I_{flow}^b
725.abc	265	1757.7	1904.1	1807.7	1956.6	2059.5	1966.6
744.abc	152.7	2142.3	2282.5	2152.9	2399.9	2472.2	2336.3
732.abc	153.2	1864.8	2015.8	1899.3	2092.1	2182.2	2058.3
737.abc	204	1631.1	1787.8	1683.1	1836.2	1938.5	1820.9

VI. CONCLUSION

This paper has presented a computing method for fault location and fault current identification in radial networks. in addition, it examines microPMU placement for tree networks. In order to test the effectiveness the proposed mixed integer programming-based technique on three-phase unbalanced distribution network, a heavily loaded and very unbalanced feeder (IEEE-37 test feeder) was chosen for demonstration. Different type of faults and transformer configurations were tested. The proposed method has shown high accuracy rate in detecting the faulted phases and buses and in identifying the fault current phasor even for complicated scenarios such as Delta connection.

In real life, knowing the faulted bus eliminates the time needed by utility crew members for inspection, which yields to fast system restoration, and then reliability indices improvement. The proposed method has the potential to serve that purpose.

Future work in this topic will be testing the algorithm in larger distribution networks with different line segments (not only three-phase line segments) and address more complicated scenarios.

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