Performance of Branch-Current Based Distribution System State Estimation

Sulaiman Almutairi
Department of Electrical Engineering
University of South Florida,
Tampa, FL 33620, USA
Prince Sattam Bin Abdulaziz University, KSA
Email: sza@mail.usf.edu

Zhixin Miao
Department of Electrical Engineering
University of South Florida,
Tampa, FL 33620, USA
Email: zmiao@usf.edu

Lingling Fan
Department of Electrical Engineering
University of South Florida,
Tampa, FL 33620, USA
Email: linglingfan@usf.edu

***Abstract*—The introduction of distributed generators (DG) and other emerging technologies such as electric vehicle charging (EV) to distribution networks have influenced the philosophy of operating the distribution networks. With the presence of these technologies, the nature of the distribution networks are changing form being passive networks to active networks and in order to accommodate these changes, the way of controlling and operating distribution systems must be reconfigured and distribution system state estimation based real-time model is needed for a secure control and protection in distribution systems. The objective of this paper is to present a comparison between the branch-current based distribution system state estimation in polar and rectangular coordinates. Moreover, the inclusion of the synchronized measurements, obtained from Micro-PMU is discussed. The methods are conducted on the IEEE-13 bus distribution test feeder and results are discussed.**

***Index Terms*—State Estimation, Distribution System, Micro-PMU Measurement, Branch-Current based, weighted least square.**

I. Introduction

Power systems consist of generation, transmission, and distribution systems. Operating, controlling and monitoring the power systems under economical and efficient way, along with maintaining the system in a secure state play an essential role in the power system industry. These objectives can be accomplished by accurately monitoring the system state conditions. A well-known tool for monitoring the power systems is state estimation (SE). The state estimator computes the best estimate of the system states based on collected measurements across the network and provides real-time data for the energy control centers for the porpuse of controling and monitoring the network. It is a critical tool for system reliability and for many operational functions in the power system [1] [2] [3].

Distributed generators and other emerging technologies such as electric vehicle charging connected to distribution systems have increased remarkably in the recent years. With the existance of these new technologies, the power flow changed form being uni-directional to become bidirectional flow which makes it difficult to determine the direction of the flow.Therefore, the nature of the distribution networks are changing form being passive networks to active networks. In order to accommodate these changes, the philosophy of operating and controling distribution systems must be reconfigured and distribution sytem state estimation based real-time model is needed for a secure control and protection in distribution systems [4] [5].

In transmission system, a balanced system is assumed in most cases. Therefore, a single-phase state estimator is utilized in transmission system state estimation (TSSE) . However, distribution networks have unbalanced nature and these features have significant impacts on the positive-sequence state estimator. Thus, the single-phase estimator is not applicable in the distribution level. Modifications on the algorithms used in TSSE and switching to a full three-phase representation of the network model are crucial to have an accurate and practical distribution system state estimation (DSSE) [4] [6].

Unlike TSSE where bus voltages are state variables, Branch-current based DSSE is the most common approach tested on the literature [12]. The method has better performance in terms of computation time compared to the node-voltage based method, especially in radial networks. This is mainly because of the complexity of the Jacobian matrix on the conventional method which leads to high execution time and more memory requirement [8].

Pioneer studies on DSSE were done in 1990's [6]- [10]. Different types of estimator, such as taking node-voltage or branch-current in both coordinates, polar or rectangular, as system states, were tested. DSSE methods and formulations differ from each other based on the measurements types and the way they are handled [12]. In [9], the authors developed a three-phase DSSE formulation by adopting the node-voltages in polar coordinates as state variables. Since the relationship between the measurements functions and the state variables are non-linear, except for the voltage magnitude measurement, an iterative solution procedure must be applied. In [10], an approximation method was presented by applying phase transformation and removing the phase angle difference between phases. The approximation made the Jaocobian matrix constant and easier to be handled. Node-voltages in rectangular coordinates as state variables was adopted by [6]. The authors proposed a three-phase DSSE formulation which transforms all types of measurements into their equivalent currents. If all measurements are transformed, and the state variables (node-voltages) are represented in rectangular form then the Jocobian matrix are linear and equal to the branch admittance. A three-phase branch-current based formulation was developed in [8]. State variables are chosen to be branch-currents in rectangular form. Measurements (power and current) are converted into their equivalent current measurements which results in having a constant Jocobian matrix. After estimating the currents, a forward sweeping procedure is applied to find the bus voltages. The results were promising and computationally efficient compared to node-voltages. Branch-currents in polar form(magnitude and phase angle based method) instead of rectangular form were also proposed in [11]. The three-phases are decoupled, and the results show that the computational speed were improved compared to the coupled three-phases.

Synchronized phasor measurements (PMUs) are widely used in transmission systems. They have the ability of measuring voltage and current phasors by the use of Global Positioning System (GPS) [14]. Power Standards Laboratory have developed Micro-synchrophasors (Micro-PMUs) for distribution systems [15]. They measure voltage and current phasors with high-precision. Based on the literature, the possible applications of Micro-PMUs in distribution systems are state estimation, fault location, protective relaying, topology detection and others [15]. [13] presented an efficient branch-current based DSSE that can handle traditional, non-synchronized, measurements and synchronized measurements, acquired by Micro-PMUs. The proposed method shows that the inclusion of synchronized phasor measurements improve the the accuracy of the estimator.

In this paper, we compare the performance of the branch-current based DSSE in both formulations, polar and rectangular coordinates. Moreover, the influence of different types of measurements, traditional and synchrophasor, on the methods are discussed. Comparisons between the two formulations and the measurements types in terms of accuracy and computational time are analyzed and presented.

II. Branch-Current Based Distribution System State Estimation

The branch-current based three-phase DSSE adopts the branch current as the system state variables instead of using the node-voltage. Figure 1 shows a three-phase lines segment. As it can be seen, the shunt capacitance affects the branch current flowing into the line. There are three different currents. Hence the branch currents phasor which are at the sending end I_l^{ph} are selected as the state variables in this formulation. The branch currents are chosen as the state variables because if they are determined, all the node voltages can be found. To explain that based on the direction of I_l^{ph} in Fig.1, the following function can be obtained:

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}_j = \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}_i - \begin{bmatrix} Z^{aa} & Z^{ab} & Z^{ac} \\ Z^{ba} & Z^{bb} & Z^{bc} \\ Z^{ca} & Z^{cb} & Z^{cc} \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}_l \tag{1}$$

in a compact form:

$$V_j^{ph} = V_i^{ph} - Z_l^{ph} I_l^{ph} \tag{2}$$

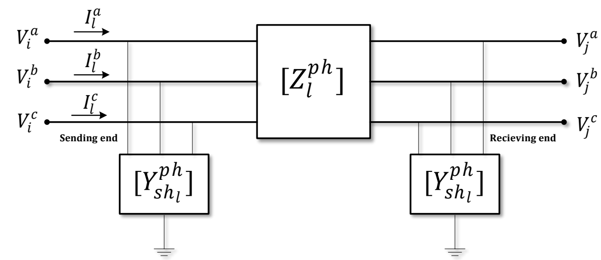

Fig. 1: Three-phase lines segment

where, V_i^{ph} and V_j^{ph} are the three-phase node voltages phasors at the sending and receiving end, respectively, Z_l^{ph} is the three-phase series impedance of the line, containing the self and mutual impedances, and ph is the phase index (a, b, c). Once the branch currents are known, we can apply (2) to calculate all the node voltages.

The formulation of the problem is based on the most common algorithm weighted least square (WLS). The WLS estimation mathematically represents the relationship between system states and measurements [10]. The general model of SE is expressed as:

$$z = h(x) + e \tag{3}$$

where, z is the measurements vector, $h(x)$ is the non-linear equation vector relating measurements to the system states, x is the system state variables and e is the measurements error vector. The WLS state estimator solution is based on solving the following optimization problem:

$$\begin{aligned} \min_m J(x) &= \sum_1^m w_i (z_i - h_i(x))^2 \\ &= [z - h(x)]^T W [z - h(x)] \end{aligned} \tag{4}$$

where W is a diagonal matrix representing the measurements weights. Equation (4) is solved iteratively using Newton method. The state variables x are updated at each iteration $x^{k+1} = x^k + \Delta x$:

$$G(x)\Delta x = H^T W [z - h(x)] \tag{5}$$

where, $G(x) = (H^T W H)$ is called the gain matrix and H is the Jacobian matrix of $h(x)$.

The branch-current based DSSE can be expressed in polar or rectangular coordinates. In the following, both terms will be discussed in their three-phase representation.

A. Branch-Current DSSE in Rectangular Form

This method adopts the branch current in rectangular coordinates as the state variables of the system. x vector can be expressed as:

$$x = \left[I_1^{ph,r} \dots I_l^{ph,r} \dots I_N^{ph,r}, I_1^{ph,m} \dots I_l^{ph,m} \dots I_N^{ph,m} \right]^T$$

where,

$$I_l^{ph,r} = \begin{bmatrix} I_l^{a,r} \\ I_l^{b,r} \\ I_l^{c,r} \end{bmatrix}, I_l^{ph,m} = \begin{bmatrix} I_l^{a,m} \\ I_l^{b,m} \\ I_l^{c,m} \end{bmatrix}$$

$I_l^{ph,r}$ and $I_l^{ph,m}$ represent the real and imaginary parts of the branch current at branch l, respectively and N represents the number of branches .

The measurements that are commonly utilized in distribution networks are adopted in this method. They are traditional and synchronized measurements. The traditional measurements considered here are power, current magnitude and voltage magnitude measurements. They are derived as follows:

Power flow Measurements: All power measurements are converted into their equivalent currents. The real power (P_l) and and reactive power flow (Q_l) of branch l can be converted into equivalent current using the following equation:

$$I_l^{ph} = \left(\frac{P_l^{ph} + jQ_l^{ph}}{V_i^{ph,k}} \right)^* = I_l^{ph,r} + jI_l^{ph,m} \tag{6}$$

where $V_i^{ph,k}$ is the estimated node voltage at the kth iteration. The reason of converting all power measurements is to have linear relationships between the equivalent currents and the state variables as it can be observed from (6).

Current Magnitude Measurement: current magnitude function can be written as:

$$|I_l^{ph}| = \sqrt{(I_l^{ph,r})^2 + (I_l^{ph,m})^2} \tag{7}$$

Voltage Measurement::
Voltage measurements V_m in terms of the state variables I_l can be written as:

$$|V_j^{ph}| = \left| V_s^{ph} - \sum_{l=1}^{N} Z_l^{ph} I_l^{ph} \right| \tag{8}$$

where V_s^{ph} is the substation voltage.
The power flow measurments are linear with respect to the state variables resulting in constant terms in the Jacobian matrix. The current and voltage magnitude functions are nonlinear as presented in (7) and (8). Therefore, they introduce non-constant terms to the Jacobian matrix. The Jacobian matrix are constructed by differentiating the measurements functions with respect to the state variables. The derivation of the Jacobian matrix entries can be found in [8] and [13].

In case of synchronized measurements, the measurements, voltage and branch current phasors, have linear relationship with the system states.

The rectangular formulation is computed with the forward sweeping method to find the state estimation solution. The procedure of the algorithm has the following steps: [8]

1) Start by sitting the iteration index $k = 0$.
2) Initialize the the node voltages V_i^{k-1}.
3) Convert all power measurements to $I_{l,eq}$ using (4).
4) Find the gain matrix $G(x)$.
5) Solve $\Delta x = (G(x))^{-1} H^T W [z - h(x)]$.
6) After obtaining the branch current estimate, update the node voltages using the forward sweeping approach.
7) Check for convergence $\Delta x \leq \epsilon$.
8) If no, update $k = k + 1$, back to step 3.
9) Else, stop.

B. Branch-Current DSSE in Polar Form

The formulation of the problem adopts the branch current magnitude I_l and phase angle α_l as the system state variables. x vector can be expressed as:

$$x = \left[\alpha_1^{ph} \ldots \alpha_l^{ph} \ldots \alpha_N^{ph}, I_1^{ph} \ldots I_l^{ph} \ldots I_N^{ph} \right]^T$$

$$I_l^{ph} = \begin{bmatrix} I_l^a \\ I_l^b \\ I_l^c \end{bmatrix}, \alpha_l^{ph} = \begin{bmatrix} \alpha_l^a \\ \alpha_l^a \\ \alpha_l^c \end{bmatrix}$$

The measurements considered here are the same as the ones in branch-current DSSE in rectangular form. Since the state variables are the branch current magnitudes and phase angles, the measurement functions should be expressed in terms of these variables. In case of traditional measurements, they can be expressed as follow:

Power flow Measurements:

$$P_l^{ph} + jQ_l^{ph} = V_i^{ph} I_l^{ph} [\cos(\delta_i^{ph} - \alpha_l^{ph}) + j \sin(\delta_i^{ph} - \alpha_l^{ph})] \tag{9}$$

where, V_i^{ph} is the voltage magnitude at bus i phase ph and δ_i^{ph} it its phase angle.

Current Magnitude Measurements:

$$|I_l^{ph}|(\text{measured}) = |\mathrm{I_l^{ph}}| \tag{10}$$

The voltage magnitude measurements are expressed as in (8). From the above equations, it can be observed that the relationship between the measurements and the state variables are non-linear, except for current magnitude measurements. Therefore, the entries of the Jacobian matrix H have non-constant terms and an iterative solution procedure must be applied. The derivation of the Jacobian matrix terms is taken as in [11].

In case of synchronized measurements, the measurements, voltage and branch current phasors, are expressed as follow:

Branch Current Phasor Measurements:

$$I_l^{ph}(\text{measured}) = \mathrm{I_l^{ph}} \tag{11}$$

where $I_l^{ph} = I_l^{ph} \angle \alpha_l^{ph}$.

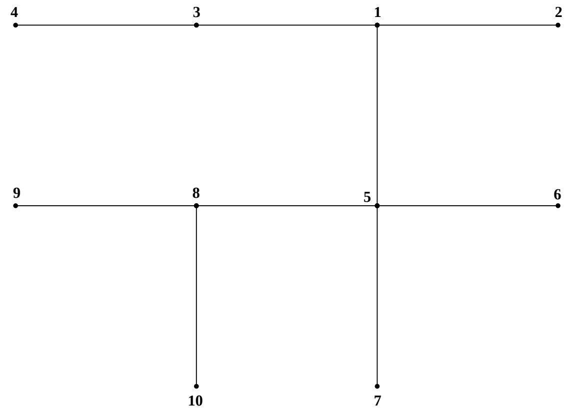

Fig. 2: [Modified] IEEE 13-bus Test Feeder. [18]

Voltage Phasor Measurements:

$$V_j^{ph} = V_i^{ph} - \sum_{l=1}^{N} Z_l^{ph} I_l^{ph}. \tag{12}$$

From (11), it is obvious that the state variables are directly measured which results in linear relationship with x. The voltage phasor measurements add non-constant terms to the Jacobian matrix as indicated in (12). The terms of the Jacobian can be found in [13].

The implementation of the WLS algorithm is adopted from [11] and has the following procedure:

1) Start by sitting the iteration index $k = 0$.
2) Initialization:
   Branch current magnitude and phase angle initialization plays an important role on the convergence speed of the method. The backward-forward method is used to obtain the initial value. The backward approach is used to obtain the initial value branch current. Then, the forward approach is used to obtain the node-voltage initial value based on the branch current calculated in the previous approach.
3) Find the gain matrix $G(x) = H^T W H$.
4) Solve $\Delta x = (G(x))^{-1} H^T W [z - h(x)]$.
5) Check for convergence, $\Delta x^k \leq \epsilon$?
6) If no, update the state variables, $x^{k+1} = x^k + \Delta x^k$, back to step 3.
7) Else, stop.

III. Tests and Results

Case studies have been conducted to perform the comparison between the two formulations. The modified IEEE-13 bus radial distribution test feeder used in [18] is chosen for the study . Figure 4 shows the one line diagram of the modified feeder. The feeder is unbalanced which is composed of three-phase, two-phase and single-phase lines and loads. Due to the unbalanced features of the test feeder, the three-phase model of the lines is considered. The distribution feeder branches are modeled by 3×3 impedance matrix containing the self and mutual impedance of the lines as described in [16].

In order to obtain set of measurements, a load flow calculation has been carried out using the Matlab code developed by A. Garces in [17]. The results of the load flow computing are taken as the true values of the system states. The results of voltage phasor, current phasor, voltage magnitude, current magnitude and real and reactive power flow are used as the bases of the traditional and synchronized measurements. All the loads are taken as pseudo-measurements in order to make the system observable.

All measurements are generated by adding a random percent of error to the true values using (13), where Z_{true} is the true values obtained from the load flow and e is the error vector. As for the traditional measurements, the following random percent of error are considered: 2% for voltage magnitude measurements with a standard deviation σ of 0.0067 and 3% for current magnitude and power flow measurements with a standard deviation σ of 0.01 is assumed; whereas for synchronized measurements, 0.5% for the magnitude and 0.01 rad for the angle accuracy are considered as specified in [15].

The pseudo-measurements are assumed to have 30% error since they are based on historical load data.

$$Z_m = Z_{\text{true}} + e \tag{13}$$

The WLS algorithm is implemented and coded in MATLAB R2017a environment on a system with Intel i7 and 8 GB RAM. The data for the IEEE-13 feeder are given in [18].

To evaluate the two algorithms, the results in terms of accuracy and the convergence speed for the methods are compared and analyzed.

Two cases are considered for the study:

1) Traditional Measurements: real and reactive power flow, voltage magnitude and current magnitude.
2) Micro-PMU Measurements: voltage phasors and branch current phasors.

Pseudo-measurements (loads) are used for both cases to overcome the observability issue. The same measurements

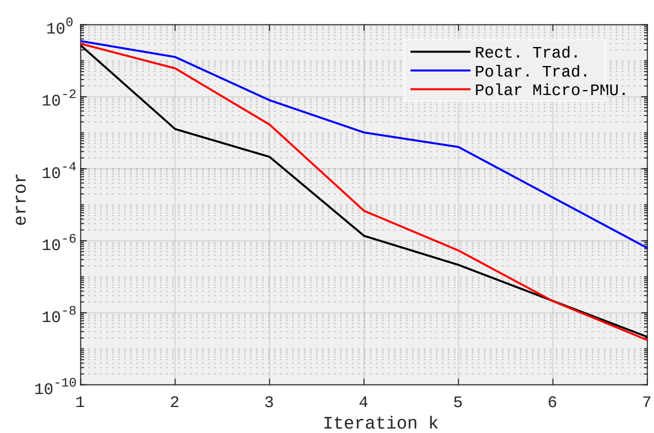

Fig. 3: Convergence of branch-current DSSE (Avg.)

TABLE I: State variables estimates (Polar formulation)

Branches	True Value		Traditional		Micro-PMU		True Value		Traditional		Micro-PMU		True Value		Traditional		Micro-PMU	
	Phase a						Phase b						Phase c					
	Mag.(pu)	Ang.(rad)	Mag.(pu)	Ang.(rad)	Mag.(pu)	Ang.(rad)	Mag.(pu)	Ang.(rad)	Mag.(pu)	Ang.(rad)	Mag.(pu)	Ang.(rad)	Mag.(pu)	Ang.(rad)	Mag.(pu)	Ang.(rad)	Mag.(pu)	Ang.(rad)
1-2	0.034	-0.597	0.043	-0.675	0.043	-0.675	0.026	-2.756	0.034	-2.951	0.034	-2.951	0.026	1.446	0.028	1.442	0.028	1.442
1-3	0	0	0	0	0	0	0.111	-2.551	0.121	-2.543	0.114	-2.549	0.036	0.96	0.028	0.819	0.031	1.019
3-4	0	0	0	0	0	0	0.046	-2.607	0.04	-2.51	0.048	-2.609	0.046	-2.709	0.035	-2.941	0.039	-2.842
1-5	0.189	-0.346	0.194	-0.397	0.191	-0.351	0.090	-2.324	0.099	-2.262	0.096	-2.352	0.215	1.684	0.236	1.59	0.229	1.674
5-6	0.086	-0.0434	0.076	-0.051	0.09	-0.047	0.027	-0.978	0.02	-0.991	0.024	-0.974	0.052	2.014	0.046	2.031	0.053	2.001
5-7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5-8	0.027	-0.652	0.035	-0.547	0.031	-0.624	0	0	0	0	0	0	0.024	2.217	0.034	2.112	0.029	2.203
8-9	0	0	0	0	0	0	0	0	0	0	0	0	0.023	2.204	0.036	2.005	0.027	2.101
9-10	0.027	-0.632	0.033	-0.653	0.03	-0.636	0	0	0	0	0	0	0	0	0	0	0	0

TABLE II: State variables estimates (Rectangular formulation)

Branches	True Value		Traditional		Micro-PMU		True Value		Traditional		Micro-PMU		True Value		Traditional		Micro-PMU	
	Phase a						Phase b						Phase c					
	Mag.(pu)	Ang.(rad)	Mag.(pu)	Ang.(rad)	Mag.(pu)	Ang.(rad)	Mag.(pu)	Ang.(rad)	Mag.(pu)	Ang.(rad)	Mag.(pu)	Ang.(rad)	Mag.(pu)	Ang.(rad)	Mag.(pu)	Ang.(rad)	Mag.(pu)	Ang.(rad)
1-2	0.034	-0.597	0.043	-0.675	0.043	-0.675	0.026	-2.756	0.034	-2.951	0.034	-2.951	0.026	1.446	0.028	1.442	0.028	1.442
1-3	0	0	0	0	0	0	0.111	-2.551	0.120	-2.545	0.115	-2.547	0.036	0.961	0.028	0.819	0.031	1.019
3-4	0	0	0	0	0	0	0.046	-2.607	0.04	-2.51	0.048	-2.609	0.046	-2.709	0.035	-2.941	0.039	-2.842
1-5	0.189	-0.346	0.195	-0.395	0.190	-0.353	0.090	-2.324	0.099	-2.262	0.096	-2.352	0.215	1.684	0.236	1.59	0.229	1.674
5-6	0.086	-0.0434	0.076	-0.051	0.09	-0.047	0.027	-0.978	0.021	-0.990	0.024	-0.975	0.052	2.014	0.046	2.031	0.053	2.001
5-7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5-8	0.027	-0.652	0.035	-0.547	0.031	-0.624	0	0	0	0	0	0	0.024	2.217	0.034	2.113	0.028	2.201
8-9	0	0	0	0	0	0	0	0	0	0	0	0	0.023	2.202	0.034	2.006	0.027	2.102
9-10	0.027	-0.632	0.034	-0.653	0.030	-0.636	0	0	0	0	0	0	0	0	0	0	0	0

configurations are considered for both cases. Every case is tested for number of times and average root mean square error (RMSE) and computational time are calculated.

As for the estimators results, Table II and **??** present the two algorithms estimated values of the system state variables and compare them with the true values obtained from the load flow calculation. Table II shows the results for branch current magnitudes and phase angles for the branch-current DSSE in polar coordinates with traditional and Micro-PMU measurements. Table **??** presents the results for the branch-current DSSE in rectangular coordinates. The results are expressed in polar coordinates to clarify the comparison. The branch current magnitudes and phase angles results are shown.

As it can be observed from the estimated results on Table II and **??**, both formulations are able to give similar results. Therefore, the accuracy of the results are the same regardless of the chosen state variables, polar or rectangular coordinates, in the estimator. Basically, the results do not rely on the used state variables in the estimators but on the available measurements and their types. Moreover, it is obvious to observe that Micro-PMU Measurements effects on the result of the estimates. They have effects on both magnitudes and phase angles.
In addition, Table III present the estimated results accuracy in terms of RMSE. It can be observed that the accuracy are the same for both formulations. The error is reduced with synchrophasors measurements. It can be concluded that the estimated results are similar in terms of accuracy in the two algorithms and the use of synchrophasors measurements improve the accuracy of both the magnitude and the phase angle.

Fig. 4 shows the rate of the convergence speed for both

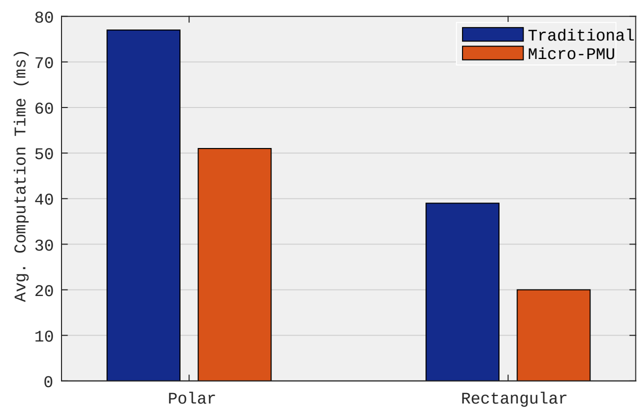

Fig. 4: Average computational time

TABLE III: Branch current average RMSE

Formulation	Measurement	Mag. (%)	Ang. (crad)
Polar	Traditional	6.6	4.1
	Micro-PMU	5.9	3.2
Rectangular	Traditional	6.5	4.1
	Micro-PMU	5.9	3.19

formulations with the all cases. The convergence tolerance is 10^{-3}. In the figure, the blue line indicates the results for the formulation in polar form with Case 1, the red line represents the same formulation with Case 2, while the black line represents the formulation in rectangular form with Case 1. The results show that the rectangular branch-current DSSE have the best performance in terms of convergence speed. It is the fastest with the use of Micro-PMU measurements because of the linearity between the measurements and the state variables which result in a constant Jacobian matrix. Moreover, the rectangular formulation with traditional measurements has less iteration numbers and faster convergence compared to the polar formulation with both traditional and Micro-PMU measurements.

Moreover, Fig. 5 shows the average computational time. It can be noticed that the rectangular formulation has the fastest performance. Both formulations have better execution time in the presence of Micro-PMU measurements.

IV. Conclusion

In this paper, a branch-current based distribution system state estimation is presented. Both rectangular and polar formulations are conducted and the results are compared in terms of accuracy and computational time. Furthermore, the use of synchrophasors measurements (Micro-PMUs) is discussed and compared with the traditional measurements . The two algorithms are coded and implemented in MATLAB environment. The results indicates that both formulations have similar accuracy regardless of the chosen the state variables. The impacts on accuracy depend on the available measurements and their types. Rectangular formulation have the best performance with less number of iterations. It is computationally more efficient with the presence of traditional or synchrophasors due to the linear relationship of power, current phasors and voltage phasors measurements. In terms of measurements types, the results show that Micro-PMU measurements impact on both magnitudes and phase angles of the estimates and they have the shortest computational time.

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