Space Vector-based Synchronous Machine Modeling

Zhengyu Wang, Lingling Fan
Department of Electrical Engineering
University of South Florida
Tampa, Florida 33620
Email: linglingfan@usf.edu

***Abstract*—In this article, a salient synchronous machine starting up process is emulated, and the simulation is carried out in MATLAB/Simulink. The dynamics of the simulation model include the three-order electromagnetic dynamics and two-order swing dynamics without the normal operation assumption. The model is derived based on the space vector concept and two-axis theory. Torque generation is also explained, and the harmonic analysis of torque matches the simulation results. Simulation can successfully show that the synchronous machine can achieve rated speed with three-phase stator excitation or single-phase stator excitation and increased rotor resistance.**

***Index Terms*—Synchronous Machine, Space Vector, Dynamics Simulation**

I. Introduction

Modeling a synchronous generator for power grid operation using Park's transformation is well covered in classic textbooks, e.g., [1]. Modeling a synchronous generator for starting up is usually not addressed in a power system text books. In power system dynamic study, the swing dynamics is usually simplified to consider nominal or near nominal operation. In fact, during that process, the synchronous generator works as an induction motor. Therefore, such simplification does not apply. Further, in classic induction machine textbooks, round-rotor machines are usually considered. Simulation of free acceleration for a salient machine cannot be found in a classic machine textbook, e.g., [2].

Therefore, the topic of modeling a synchronous generator's startup process covers two areas of knowledge: power systems and electric machines. Our contributions include the employment of space vector technique to model a salient machine and a thorough analysis of machine behavior and harmonics.

The introduction of space vector concept has made modeling of a rotating magnetic field straightforward and facilitate the understanding of physics [3]. In this paper, we will examine space vector-based synchronous machine modeling and use the dynamic model to simulate the starting process of a salient synchronous machine. The preliminary work has been documented in a previous report [4]. In this paper, additional analysis on torque is presented. We use the relative positions between two rotating magnetic fields to examine harmonics in the electromagnetic torque during the starting process. Further, we show that single-phase stator current can also initiate the starting process.

Suppose that a two-pole salient machine is standing still at the beginning and the stator windings are connected to a three-phase voltage source. The field winding is short circuited. The three-phase currents of the stator windings will form a rotating magnetic field in the airgap with the rotating speed the same as the electric frequency. This rotating magnetic field will induce a 60 Hz current in the rotor winding or the excitation winding FF'. The rotor current forms two rotating magnetic fields: one forward and one backward. Interactions between the rotor fields and the stator fields yield torque. The machine starts to rotate should the average torque be greater than 0. During the starting process, a synchronous machine acts as an induction motor. To have adequate start-up torque, rotor resistance may be increased.

The difference between a salient machine and an induction machine is that air gap in synchronous machine is not uniform. The minimum air gap distance g_d is at the rotor axis or the d-axis different, while the maximum air gap distance g_q is at the q-axis as shown in Fig. 1. The stator magnetomotive force (MMF), represented as a rotating space vector, can be decomposed into dq components. For the d-component, then the flux line path includes the air gap with a distance of g_d. For the q-component, the path of the flux lines passes the air gap with a distance of g_q.

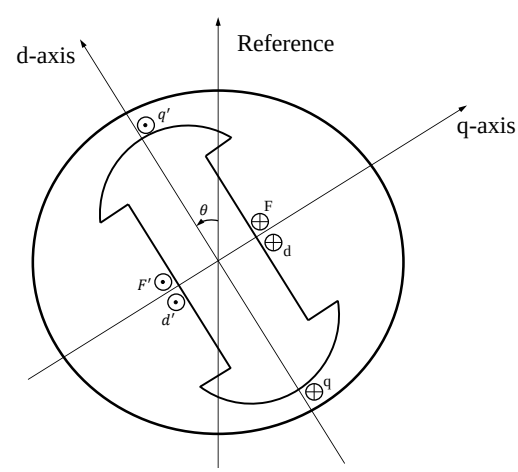

Fig. 1. Synchronous machine schematic with dq fictitious windings and excitation winding FF'.

The effect of stator currents is thus the same as the effect of two rotor currents from the fictitious windings, notated as dd' and qq' in Fig. 1. The electromagnetic dynamics will be developed based on the three flux linkages linked to FF', dd', and qq' due to the airgap flux.

The rest of the paper is organized as follows. Section II presents three-phase and single-phase stator current space vectors. Section III also presents analysis of electromagnetic torque. Section IV presents the dynamic model. Section V presents the case study results. The conclusion of the paper is given in Section VI.

II. Space Vector Concept

Three-phase balanced stator current will form a rotating magnetic field in the airgap. This rotating field can be represented by a space vector with the magnitude and the angular position (where the maximum of the waveform is located) specified. In this section, we show that three-phase balanced current forms a forward rotating magnetic field using space vector concept while the single-phase current forms both a forward and backward rotating magnetic fields. Further, analysis of the electromagnetic torque is carried out.

A. Three-phase current derivation

Assume a balanced three-phase voltage source is connected to stator for excitation and its frequency is ω_e. The excitation current is expressed by space vector as $\vec{i}$. Let the magnitude of the induced three-phase current be I_m, then phase currents can be written as:

$$\begin{cases} i_a = I_m \cos(\theta_a) \\ i_b = I_m \cos(\theta_a - \frac{2\pi}{3}) \\ i_c = I_m \cos(\theta_a + \frac{2\pi}{3}) \end{cases}$$

where θ_a is the a phase position. By introducing θ_{a0} to express the initial position of a phase, θ_a can be expressed as $\omega_e t + \theta_{a0}$. Since the three-phase current is balanced, $\vec{i}$ can be found by adding $\vec{i}_a$, $\vec{i}_b$, and $\vec{i}_c$ together:

$$\begin{aligned} \vec{i} &= \frac{2}{3}\left(i_a e^{j0} + i_b e^{j\frac{2\pi}{3}} + i_c e^{-j\frac{2\pi}{3}}\right) \\ &= I_m e^{j\theta_a} = I_m e^{j\theta_{a0}} e^{j\omega_e t} \end{aligned} \tag{1}$$

$\vec{i}$ can be decomposed by projecting to both d-axis and q-axis as shown in Fig. 2. The dq components are:

$$\begin{cases} \vec{i}_d = I_m \cos(\theta - \theta_a) e^{j\theta} \\ \vec{i}_q = I_m \sin(\theta - \theta_a) e^{j\left(\theta - \frac{\pi}{2}\right)} \end{cases}$$

where θ is the rotor position. θ_0 is the initial position of rotor referred to reference axis, therefore, $\theta = \omega_e t + \theta_0$.

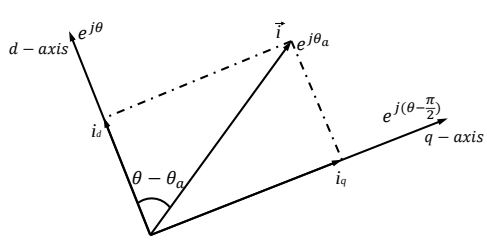

Fig. 2. Three-phase current space vector decomposition on dq frame

B. Single-phase current derivation

Assume only a phase stator winding is connected to the voltage source. Then $\vec{i}$ is:

$$\begin{aligned} \vec{i} &= \frac{2}{3}(i_a e^{j0} + 0e^{j\frac{2\pi}{3}} + 0e^{-j\frac{2\pi}{3}}) \\ &= \frac{2}{3} I_m \cos\theta_a e^{j0} = \frac{1}{3} I_m \left(e^{j\theta_a} + e^{-j\theta_a}\right) \end{aligned} \tag{2}$$

Because angle of the current space vector is 0, the dq frame with $\vec{i}$ can be formed together as shown in Fig. 3.

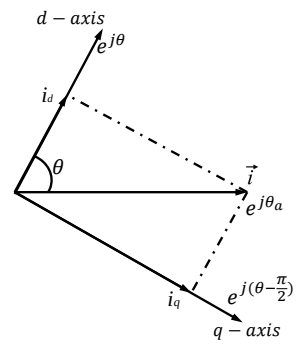

Fig. 3. Single-phase current space vector decomposition on dq frame

Based on the figure, $\vec{i}$ can be decomposed to dq-axis as:

$$\begin{cases} \vec{i}_d = \frac{2}{3} I_m \cos(\theta_a)\cos(\theta) e^{j\theta} \\ \vec{i}_q = \frac{2}{3} I_m \cos(\theta_a)\sin(\theta) e^{j(\theta - \frac{\pi}{2})} \end{cases}$$

C. Electromagnetic torque

From physics, the interactions of the stator MMF and rotor MMF result in torque.

Fig. 4 shows the space vector formed stator MMF, $\vec{F}_s$.

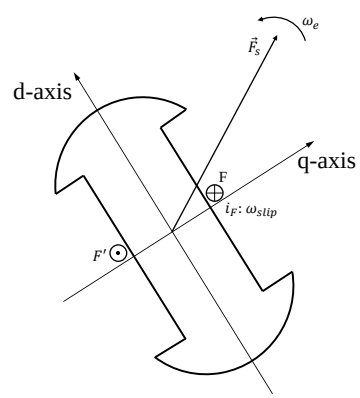

Fig. 4. Stator MMF space vector on rotor referenced dq frame.

For rotor, during the acceleration process, machine speed (ω_m) is increasing from 0 to full speed (ω_e). Before reaching ω_e, the difference between the actual speed and desired speed is called slip speed, which is expressed by ω_{slip}. For rotor, due to excitation from stator circuit, a current of slip frequency in the rotor coil is produced and it is expressed by i_F. According to revolving theory: i_F forms two rotating magnetic field as

shown in (2): one forward and one backward. $\vec{F}_{r1}$ and $\vec{F}_{r2}$ are introduced to represent the forward and backward MMFs. The rotating speed is the slip speed $s\omega_e$ if the rotor is standing still. Since the rotor is also rotating at ω_m, the rotating speeds for the two MMFs are: ω_e and $(1-2s)\omega_e$, as shown Fig. 5.

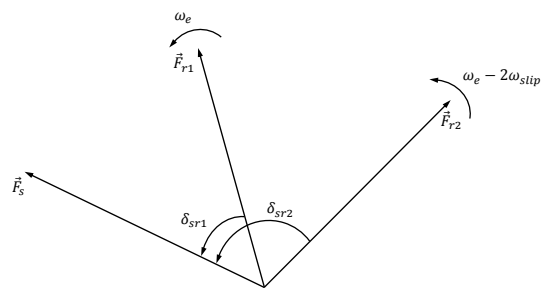

Fig. 5. All MMF diagram, angle between $\vec{F}_s$, excitation current is three-phase.

As shown in Fig 5, $\vec{F}_{r1}$ is relatively static to $\vec{F}_s$ and the relative speed of $\vec{F}_{r2}$ to $\vec{F}_s$ is $-2\omega_{slip}$.

It is known that the relationship between torque and two forces is proportional. In this case, the relationship among the electromagnetic torque and MMFs can be expressed as:

$$T_{ei} \propto F_s F_{ri} \sin(\delta_{sri}) \tag{3}$$

With a three-phase excitation current, electromagnetic torque should have two components, one is a dc torque and the other one has a frequency of $2\omega_{slip}$.

TABLE I
ELECTROMAGNETIC TORQUE FREQUENCY

MMF	F_s
F_{r_1}	0
F_{r_2}	$2\omega_{slip}$

III. GENERATOR DYNAMICS MODELING

The generator dynamic model consists of the electromagnetic dynamics and the swing dynamics.

A. Three-order electromagnetic dynamics

According to Faraday's Law, the relationship among stator's voltage, current, and flux linkages viewed from a static reference can be concluded as:

$$\vec{v}_s(t) = -r_s \vec{i}_s(t) - \frac{d\vec{\lambda}(t)}{dt} \tag{4}$$

where the r_s is the resistance in stator circuit.

To find the expression of relationship on the dq-frame, all elements are replaced by space vectors and it will lead to:

$$\bar{v}_{sdq} e^{j\theta} = -r_s \bar{i}_{sdq} e^{j\theta} - \frac{d\bar{\lambda}_{dq}}{dt} e^{j\theta} - j\omega_m \bar{\lambda}_{dq} e^{j\theta}. \tag{5}$$

Separating into d-axis and q-axis leads to:

$$\begin{cases} v_{sd} = -r_s i_d - \frac{d\lambda_d}{dt} - \omega_m \lambda_q \\ v_{sq} = -r_s i_q - \frac{d\lambda_q}{dt} + \omega_m \lambda_d. \end{cases}$$

The rotor flux linkage, rotor voltage and rotor current is expressed as follows. The rotor voltage is set to be zero for processing self-starting.

$$v_F = -r_F i_F - \frac{d\lambda_F}{dt} = 0 \tag{6}$$

The current and flux relationship in synchronous machine is as shown below.

$$\begin{cases} \lambda_F = M_F \frac{3}{2} i_d + L_F i_F \\ \lambda_d = L_d i_d + M_F i_F \\ \lambda_q = L_q i_q \end{cases}$$

Now the dynamics equation can be formed for electromagnetic transient process as:

$$\begin{bmatrix} \frac{d\lambda_F}{dt} \\ \frac{d\lambda_d}{dt} \\ \frac{d\lambda_q}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{L_d r_F}{L_d L_F - k(M_F^2)} & -\frac{-M_F r_F}{L_d L_F - k(M_F^2)} & 0 \\ -\frac{-kM_F r_F}{L_d L_F - k(M_F^2)} & -\frac{L_d r_s}{L_d L_F - k(M_F^2)} & -\omega_m \\ 0 & \omega_m & -\frac{r_s}{L_q} \end{bmatrix} \begin{bmatrix} \lambda_F \\ \lambda_d \\ \lambda_q \end{bmatrix} - \begin{bmatrix} 0 \\ v_d \\ v_q \end{bmatrix} \tag{7}$$

where the $k = \sqrt{\frac{3}{2}}$.

The electromagnetic torque can be expressed by current and flux linkages as:

$$T_e = \frac{3}{2}(\lambda_d i_q - \lambda_q i_d) \tag{8}$$

So far, the three-order electromagnetic dynamics is fully derived. It requires the machine's angular speed for simulation.

B. Two-order swing dynamics

In the swing equation without simplification of normal operation assumption, the derived angular speed is related to inertia (J) and net torque. In this paper, the net torque is $-T_e$ since $T_m = 0$ to emulate free acceleration (self-start). The equation is shown as:

$$\frac{d\delta}{dt} = \omega_m - \omega_e \tag{9}$$

$$J\frac{d\omega_m}{dt} = T_m - T_e \tag{10}$$

where δ is the angle between the rotor axis and the stator voltage's space vector.

The output of swing equation is derived angular speed, so it needs an integrator to translate to ω_m. Till now, all the dynamics computations are done and implemented system will be shown and explained in detail in Section IV.

IV. SIMULATION

In this non-linear simulation of a two-pole synchronous machine, the inputs of simulation system are rotor voltage (v_F) and the dq-reference frame voltage, v_{sd} and v_{sq}. To perform the free acceleration operation, rotor voltage is set to 0, and the magnitude of excitation voltage(V_m) is set to 1 pu. The simulation in block diagram is shown in Fig. 6.

According to three-phase current dq decomposition, similarly, v_{sd} and v_{sq} can be computed based on following equations:

$$v_{sd} = V_m \cos(\theta - \theta_v) = V_m \cos(\omega_e t - \omega_m t) \tag{11}$$

$$v_{sq} = V_m \sin(\theta - \theta_v) = -V_m \sin(\omega_e t - \omega_m t) \tag{12}$$

where ω_e is stator current frequency and ω_m is machine angular speed.

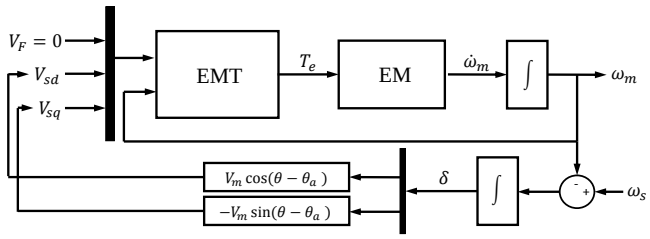

Fig. 6. Logical block diagram of synchronous machine with major dynamics.

Note the angular speed ω has to be fed back to the EMT block. v_{sd} and v_{sq} are computed using δ.

The detailed MATLAB/Simulink simulation block is shown as Fig. 7. Additionally, current and electromagnetic torque are also monitored.

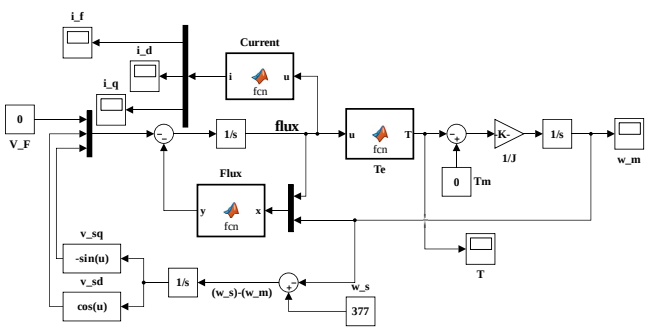

Fig. 7. MATLAB/Simulink implemented model block of synchronous machine.

Above is the simulation block when the excitation circuit on stator is three-phase, and it is different if excitation circuit is single-phase. Assume only phase a is connected. Based on the understanding of single-phase excitation current dq reference frame decomposition, v_{sd} and v_{sq} can be rewritten as:

$$v_{sd} = \frac{2}{3} V_m \cos(\omega_s t) \cos(\omega_m t) \tag{13}$$

$$v_{sq} = -\frac{2}{3} V_m \sin(\omega_s t) \cos(\omega_m t) \tag{14}$$

Then the blocks of v_{sd} and v_{sq} should be modified, and the updated parts are shown in Fig. 8 as below:

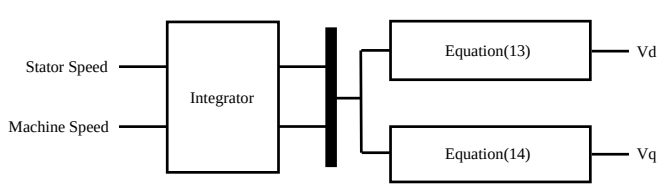

Fig. 8. Modified logic blocks when only phase a is connected.

For detailed computation involved in simulation, all the necessary parameters are selected and shown in Table. II.

TABLE II
PARAMETERS OF A TWO-POLE SYNCHRONOUS MACHINE

V_m	1 V	L_d	0.00398 H
ω_s	377 rad/sec	L_F	0.00265 H
r_s	0.002 Ω	M_F	0.00357 H
r_F	0.00133 Ω	J	0.0000029
L_d	0.00398 H	Poles	2
N(Turns)	1	k	1.225

V. CASE STUDIES

A. Case Study I: Synchronous machine with original r_F and three-phase excitation current

When the stator current is three-phase and the rotor resistance (r_F) is original, it has been found the synchronous machine cannot reach full speed, which is designed as 377 rad/sec (60 Hz). In steady-state region, the angular speed of machine saturated at 188.5 rad/sec. The angular speed (ω_m) and electromagnetic torque (T_e) curves are shown in Fig. 9.

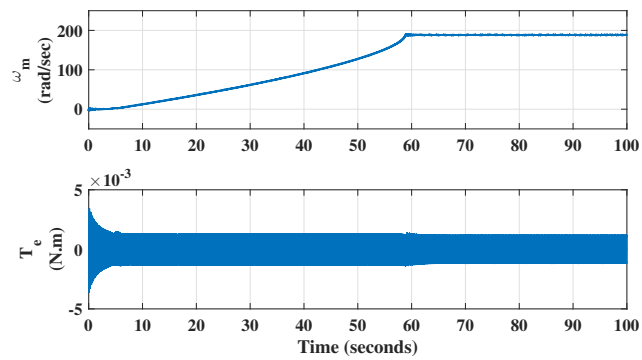

Fig. 9. ω_m and T_e plots, $r_F = 0.00133\ \Omega$, Stator current is three-phase.

To verify the frequency relationships among magnetomotive forces and torque, machine angular speed (ω_m), rotor current (i_F), and electromagnetic torque (T_e) curves are selected. In this case, the comparison results are shown in Fig. 10. Three periods of time are chosen to clearly show the frequency relationship. Row (a) is from 10 to 10.11 seconds, row (b) is from 40 to 40.11 seconds, and row (c) is from 80 to 80.11 seconds. Data of ripple frequency of ω_m, i_F, and T_e is collected and shown in Table. III.

TABLE III
RIPPLE FREQUENCIES FROM CASE STUDY I

	ω_m	i_F	ω_{slip}	T_e
(a)	2×60 Hz	60 Hz	60 Hz	2×60Hz
(b)	1.5×60Hz	0.75×60Hz	0.75×60 Hz	1.5×60 Hz
(c)	60 Hz	0.5×60 Hz	0.5×60 Hz	60 Hz

From comparing all the ripple frequencies of each components, it is obvious that results are capable to match previous analysis that ripple frequency of electromagnetic torque is double of the slip frequency, which is $2 \times \omega_{slip}$.

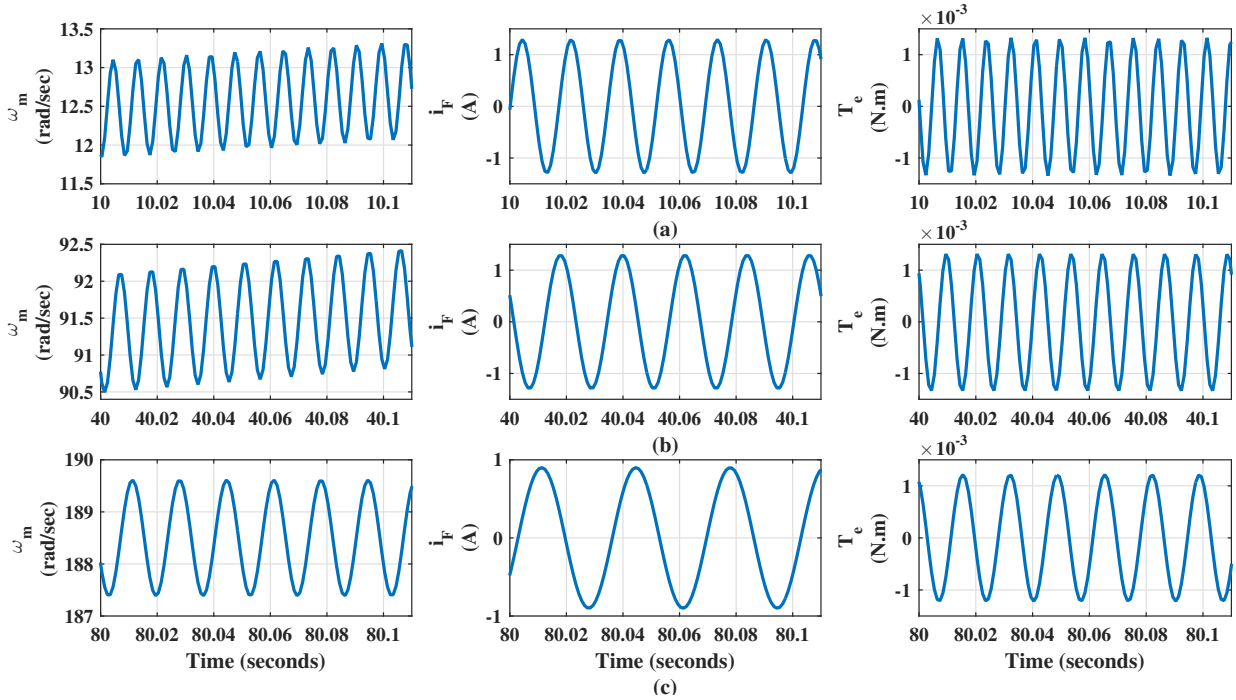

Fig. 10. $r_F = 0.00133\ \Omega$ for all time, stator current is three-phase.

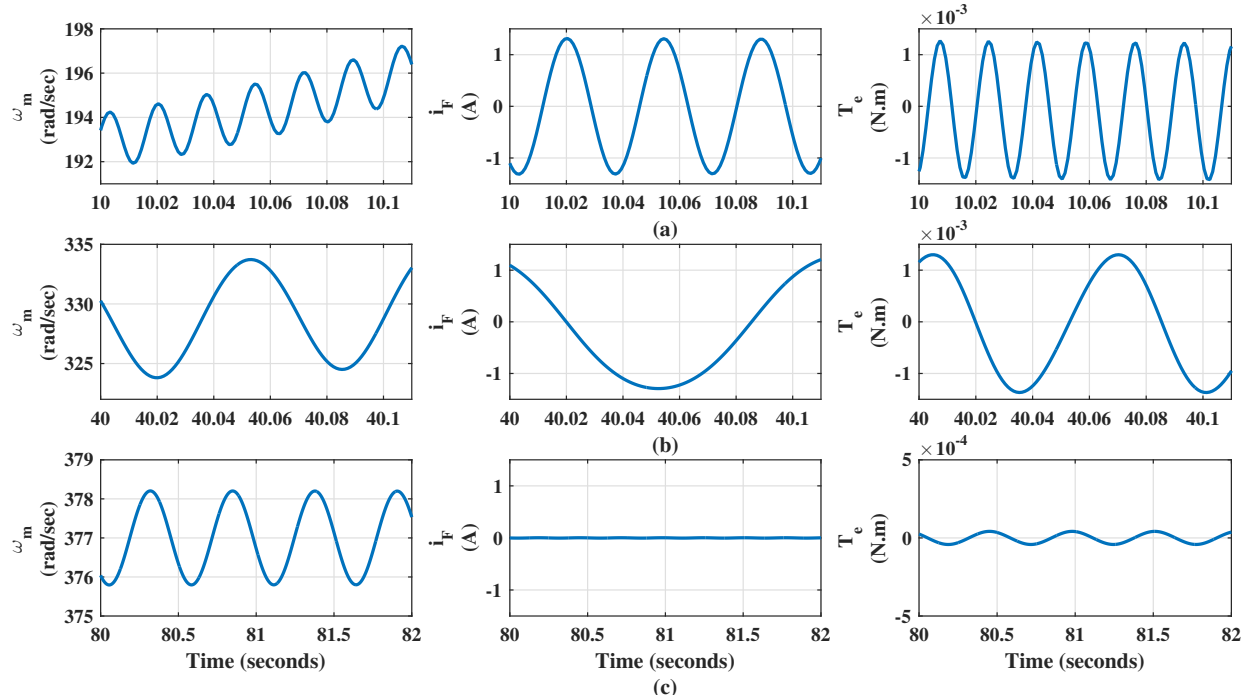

Fig. 11. r_F is initially set as 13 times of original resistance for first 10 seconds then reduced to origin, stator current is three-phase.

B. Case Study II: Synchronous machine with increased r_F for 10 seconds and three-phase excitation current

To finish free acceleration process, the r_F is increased according to previous analysis. After test synchronous machine simulation with increasing rotor resistance (r_F), it has been observed that the steady-state machine angular speed reaches 377 rad/sec when the r_F is increased to 13 times of original and excitation current is three-phase. In order to reduce power

loss in resistance, the r_F has been initially set to 13 times of original and reduced to original resistance after machine reaching 60% of full speed, which is 226.2 rad/sec [2]. For this case, the machine angular speed (ω_m) and electromagnetic torque (T_e) are shown in Fig. 12. From the plot, it is obvious that synchronous machine successfully operates the free acceleration process, but slightly slower than the system with larger r_F for all time.

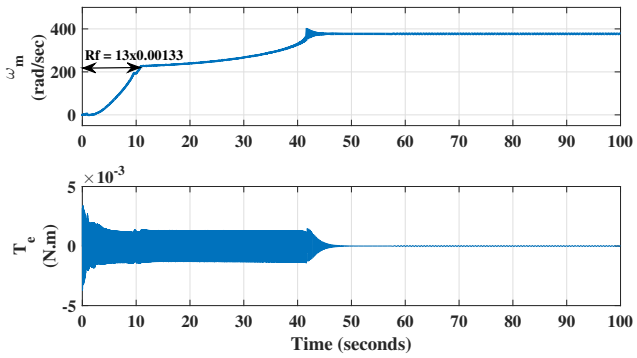

Fig. 12. ω_m and T_e plots, r_F is set to 13 times of original for about 10 seconds, r_F is set back to 0.00133 Ω when machine angular speed reaches 226.2 rad/sec, Stator current is three-phase.

To explore how frequency relationships will be in this case, same method is applied here. ω_m, i_F, ω_{slip}, and T_e are chosen, and the results are plotted and shown in Fig. 11. Note the last period of time for collecting data is from 80 to 82 seconds, which is different from previous case. The reason is rotor current tends to be dc current (0 Hz) when the ω_m is very close to nominal frequency (60 Hz). All the ripple frequencies are organized in Table. IV.

TABLE IV
RIPPLE FREQUENCIES FROM CASE STUDY II

	ω_m	i_F	ω_{slip}	T_e
(a)	60 Hz	0.5×60 Hz	0.5×60 Hz	60Hz
(b)	$0.\dot{3} \times 60$Hz	$0.1\dot{6} \times 60$Hz	$0.1\dot{6} \times 60$ Hz	$0.\dot{3} \times 60$ Hz
(c)	2 Hz	1 Hz	1 Hz	2 Hz

Although i_F plot in row (c) shows its frequency is nearly 0 Hz, the ripple frequency of T_e is still found as 2 Hz. So the analyzed theory is also proved in case study II.

C. Case Study III: Synchronous machine with increased r_F and single-phase excitation current

Assume a single-phase excitation circuit is now connected to stator. With default settings and original r_F, the machine cannot even accelerate to half of full speed because of the stator current is only $\frac{1}{3}$ of origin since only one phase is connected. In this case, the r_F is required to be enlarged more. Finally, it has been fund that steady-state synchronous machine angular speed can reach 377rad/sec when the r_F is set to 22 times of original resistance. The ω_m and T_e curves are shown in Fig. 13.

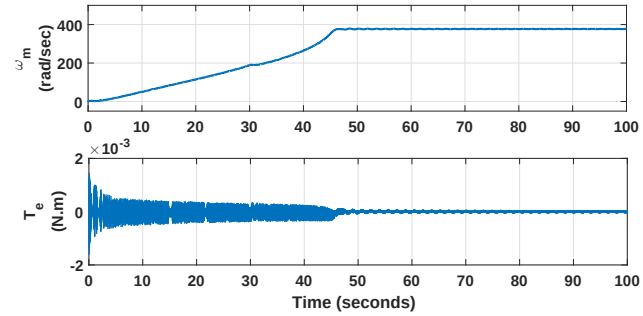

Fig. 13. ω_m and T_e plots, r_F is set to 22 times of origin for all time, Stator current is single-phase.

VI. CONCLUSION

Based on all the experimental results and comparisons, the nonlinear dynamic simulation model is fully capable to emulate the self-start process of a two-pole salient synchronous machine. From simulation, the theory has been proved that a synchronous machine can be accelerated to nominal speed with an external excitation current and an increased rotor resistance. Furthermore, harmonic phenomenon has also been proved accurately by detailed frequency analysis in selected time periods. During the implementation of this simulation model, directly applying space vector concept helps simplify the majority of computations, especially dynamics computations, which has also proved its reliability in this simulation.

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