Loss Allocation in AC OPF-Based Financial Transmission Rights Auction Models

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***Abstract*—This paper presents two models of AC optimal power flow based Financial Transmission Rights (FTRs) allocation auction. Both models were compared with the conventional DC optimal power flow (DC-OPF) based model. Considering an FTR between two nodes as a real power injection at the source node and a power withdrawal at the sink node; the first model assumes that the net power due to FTRs equals zero. FTRs do not consider loss and the slack node is compensating for the total system losses. In the second model, it is suggested that each FTR's source node is taking care of a portion of the system loss by introducing a power injection. The portion of loss compensation is proportional to each FTR allocated amount, i.e., if one FTR is to use the grid more, then it should compensate more for the loss. To show the validity of all models and the advantages of the new suggested model, numerical examples of a 3-bus system and the IEEE 14-bus system are presented. All results are compared and explained.**

***Index Terms*—Financial Transmission Rights, FTR Allocation auction considering loss, DCOPF, ACOPF, MATLAB Optimization Toolbox, fmincon.**

I. Introduction

Wholesale electricity markets based on locational marginal pricing (LMP) technique periodically conduct a market clearing problem to determine the required amount of power generation and clearing prices across the system. This technique could produce various prices at different nodes of the system [1]. A receiving node could be 30% more expansive than the sending node due to congestion on that transmission line. To hedge against transmission congestion charges, power market participants could obtain transmission rights through auction markets [2]. Based on the policies of a power market, transmission rights can be defined in various ways depending on how they are allocated and how their holders are being compensated. The independent system operator (ISO) or the regional transmission organization (RTO) is the responsible party of defending those transmission rights, adopting a mechanism for allocating them and how its holders are compensated. In this paper, we assume that the system is monitored and controlled by an independent system operator (ISO) that allocates financial transmission rights (FTRs) periodically and run a day-ahead energy market to find LMPs daily.

FTRs was introduced by William Hogan and it has been adopted in several markets including PJM, New England and New York in the US and in New Zealand [3]. FTRs grant its holder the right to collect congestion rents equal to the LMP at the sink node minus the LMP at the source node $(LMP_j - LMP_i) \times FTR_{ij}$. FTRs is defined from a source node to a destination node. Also, they are specified in MW consistent with the transfer capability between these two nodes [4].

The allocation of point-to-point obligation FTRs is done by an auction performed periodically by the ISOs. The market participants send their bids which include bidding price ($/MWh), amount of MWs and the desired location between two nodes. After obtaining bids within a time window, the ISO run a modified power flow optimization problem which maximize the amount of FTR allocation subject to the system constrains. The problem's optimal solution yields the allocated amount of FTRs and clearance prices for all bidders.

The most common FTRs allocation technique applied by ISOs is found based on DC OPF-based models which neglect both reactive power effects on transmission lines and losses across the system. In [2], the authors used a DC OPF model to allocate FTRs and compute the clearance prices omitting the conductance of transmission lines for simplicity.

The objective of this paper is to propose an AC OPF-based FTR auction model with loss allocation. In contrast to DC OPF-based linear programming models, AC OPF-based models are nonlinear programming models. The optimization problem is solved using "fmincon," a built-in function in MATLAB's optimization toolbox which uses sequential quadratic programming or interior point method algorithm. To the best of the authors' knowledge, AC OPF-based FTR auction models with loss allocation have not been discussed in a literature.

The rest of the paper is organized as follows. Section II presents the existing DC OPF-based FTR auction model. Section III then presents our research: AC OPF-based FTR auction models. Two types of models are proposed depending how losses are allocated. Section IV gives two case studies. Section V concludes the paper.

II. DC OPF-based FTR Allocation Model

This model is mainly used because of its linearity, durability and high speed of convergence. It is a linearized form of the non-convex AC OPF-based model. The mathematical representation of the model is formulated as the following linear program [2] [5]:

$$\textbf{maximize} \quad \mathbf{c^T\ R} \tag{1a}$$

$$\textbf{subject to}$$

$$(W-U)\times\ \mathbf{R} - B\,\delta = 0 \qquad :\lambda \tag{1b}$$

$$S^{min} \le F\,\delta \le S^{max} \qquad :\mu^{\mathbf{min}},\mu^{\mathbf{max}} \tag{1c}$$

$$0 \ \le \ \mathbf{R} \ \le \ \mathbf{R^{max}} \qquad : \xi^{\mathbf{max}} \tag{1d}$$

where $\mathbf{c}$ is a vector of bidding prices while $\mathbf{R}$ represent a vector of the allocated FTRs amounts in MWs which will be computed by the optimization solver. B is a matrix of the transmission lines' susceptance. δ is a vector which includes voltage angles for each node except the slack node n. U and W are two binary matrices that represent the source node and the sink node of all FTRs respectively, as defined in (2) [2]. The equation $(W - U)$ generates a matrix that specifies the direction of each FTR on each node, whether it is an injection or withdrawal.

$$U_{i,k} = \begin{cases} 1 & \text{if node i is a source node for } \mathbf{R_k} \textbf{ and } \mathbf{i \neq n} \\ 0 & \text{otherwise.} \end{cases} \tag{2a}$$

$$W_{i,k} = \begin{cases} 1 & \text{if node i is a sink node for } \mathbf{R_k} \textbf{ and } \mathbf{i \neq n} \\ 0 & \text{otherwise.} \end{cases} \tag{2b}$$

S^{max} and S^{min} are the power flow limits of the transmission lines in MVA. The power flow on line l from node i to node j is given by $S_l^{min} \le -B_{ij}\,(\delta_i - \delta_j) \le S_l^{max}$, where B_{ij} represent the line's susceptance. To generalize this line's power flow equation for all lines it can be written as:

$$S^{min} \le \mathbf{F}\delta \le S^{max} \tag{3}$$

where $\mathbf{F}$ is a matrix with a row for each transmission line and a column for each node except the slack node. For each transmission line l, the source node takes a negative sign susceptance of that line; the sink node takes a positive sign while the nodes where the line does not reach take zero [2].

$$\begin{cases} F_{li} = -B_{ij} \\ F_{lj} = B_{ij} \\ F_{lk} = 0, \quad k \neq i \text{ or } j \end{cases}$$

The dual variable of each constraint will be used for pricing. μ_{max}, μ_{min} alongside with λ are used to compute the compensation price for FTRs holders if the ISO is defining the rights as rights to revenue from the FTRs auction not from a day-ahead energy marker LMPs [2].

III. AC OPF-Based FTR Auction Models

Transmission lines' losses caused by its conductance G along with reactive power flow occupy some capacity which effect lines' maximum transfer capabilities. DC OPF model approximation neglects this fact which could give a miscalculation of about 5% in the LMP [6]. Those effects are taken into account in AC OPF-based models.

In the first model, we let the slack node compensate for the total power loss. The mathematical representation of the model is formulated as follows [5] [7]:

$$\textbf{maximize} \quad \mathbf{c^T \ R} \tag{4a}$$

subject to

$$(W - U) \times \ \mathbf{R} \ + \ P \ = 0 \qquad : \lambda \tag{4b}$$

$$S^{min} \ \le \ \sqrt{S_P^2 + S_Q^2} \ \le \ S^{max} \qquad : \mu_{\mathbf{min}}, \mu_{\mathbf{max}} \tag{4c}$$

$$|V_{min}| \ \le \ |V| \ \le \ |V_{max}| \qquad : \gamma_{\mathbf{min}}, \gamma_{\mathbf{max}} \tag{4d}$$

$$\delta_{min} \le \ \delta \ \le \ \delta_{max} \qquad : \psi_{\mathbf{min}}, \psi_{\mathbf{max}} \tag{4e}$$

$$0 \ \le \ \mathbf{R} \le R^{\,max} \qquad : \xi \tag{4f}$$

where P is a vector of total real power injection at each node such as $P_i = P_{G_i} - P_{D_i}$ and $i = 1, \cdots, n-1$ as the slack node is excluded. It can defined as [8]:

$$P_i = V_i \sum_{j=1}^{n} \ V_j \ [\ G_{ij}\cos(\delta_i - \delta_j) \ + \ B_{ij}\sin(\delta_i - \delta_j)] \tag{5}$$

S_P and S_Q are two vectors containing the average of real and reactive powers respectively flowing across each transmission line. It can be computed as [7]:

$$\begin{aligned} S_{P_k} &= \frac{P_{ij_k} - P_{ji_k}}{2} = \frac{G_k\,(V_i^2 - V_j^2)}{2} + B_k\ V_i\ V_j\ \sin\,(\delta_i - \delta_j) \\ S_{Q_k} &= \frac{Q_{ij_k} - Q_{ji_k}}{2} = \frac{B_k\,(V_i^2 - V_j^2)}{2} - G_k V_i V_j \sin(\delta_i - \delta_j) - \frac{y_c\,(V_i^2 - V_j^2)}{2} \end{aligned} \tag{6}$$

where y_c is the transmission line shunt capacitance. $|V_{min}|$, $|V_{max}|$ are two vectors representing minimum and maximum allowable voltage's magnitude at each node respectively.

δ_{min} , δ_{max} are two vectors representing minimum and maximum allowable voltage's angles at each node respectively.

A. AC OPF-Based Model Considering Loss Allocation

The previous model uses the slack node to compensate for all losses. We now consider that each FTR shares its responsibility of loss compensation by injecting compensated power. A kth FTR from node i to node j is considered to inject a certain amount of power $P_{comp,k}$ in addition to R_k into node i, and extract the same amount R_k from bus j.

Therefore, the total amount that is to be injected at node i should equal $R_k + P_{comp,k}$, and thus the amount to be extracted from node j would be R_k. In addition, the compensated power should be proportional to its FTR amount. An equality constraints to enforce this requirement is added to the optimization problem as shown in (7c).

Here is the mathematical representation of the model considering loss allocation.

$$\textbf{maximize} \quad \mathbf{c^T \ R} \ - \ \rho \sum_{k=1}^{K} P_{comp} \tag{7a}$$

subject to

$$(W - U) \times \ \mathbf{R} \ + \ P \ - \ U \times P_{comp} \ = 0 \quad : \lambda \tag{7b}$$

$$\frac{P_{comp,1}}{R_1} = \frac{P_{comp,2}}{R_2} = \ ..... \ = \frac{P_{comp,K}}{R_K} \tag{7c}$$

$$S^{min} \ \le \ \sqrt{S_P^2 + S_Q^2} \ \le \ S^{max} \qquad : \mu_{\mathbf{min}}, \mu_{\mathbf{max}} \tag{7d}$$

$$|V_{min}| \ \le \ |V_i| \ \le \ |V_{max}| \qquad : \gamma_{\mathbf{min}}, \gamma_{\mathbf{max}} \tag{7e}$$

$$\delta_{min} \leq \delta_i \leq \delta_{max} \quad : \psi_{\mathbf{min}}, \psi_{\mathbf{max}} \tag{7f}$$

$$0 \leq \mathbf{R} \leq R^{max} \quad : \xi \tag{7g}$$

where ρ is a penalty factor and K is the number of bids. Since the model now accounts for power loss allocation, there is no need to consider a slack node and all nodes' power injection equations can be listed in the optimization model.

B. Computing Clearance Prices

After the optimization problem is solved and the decision variables and the dual variables corresponding to the equality constraint (λ^*) are found, the clearance price of an FTR can be calculated using either one of these equations [5].

- To find a vector of all clearance prices, we refer each λ^* to a node:

$$CP_{FTR} = [\lambda^*]^T \, [W - U] \tag{8}$$

- To calculate the clearance price of a particular FTR that goes from node i to j:

$$CP_{FTR_k} = (\ \lambda_j^* - \lambda_i^*\ ) \tag{9}$$

Both equations give the same clearance price for FTRs.

IV. Numerical Examples

To demonstrates the implementation technique of the three previous models on a practical FTRs allocation auction, a 3-bus system as shown in figure 1 is used. Next, to validate all models on a large scale systems, IEEE 14-bus system which is shown in figure 2 is used in the same way and results are compared.

A. Three-Bus System

In this example, three bids from power market participates to purchase FTRs in different locations are received by the ISO as listed in table I. For the DC and AC without loss allocation models, bus 3 is considered as a slack bus (i.e. $V_3 = 1.0$ and $\delta_3 = 0$). The system power base equal 100 MVA.

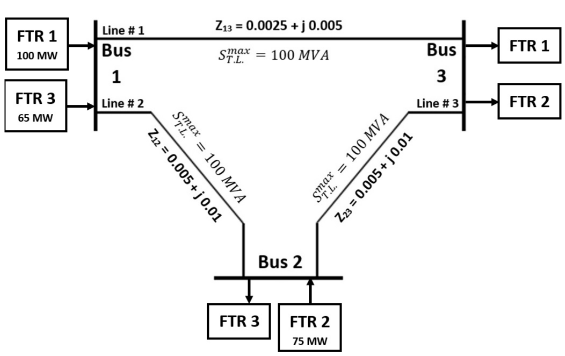

Figure 1. 3-bus system's diagram with the wanted FTRs.

Table I
A list of Bidding Prices, Amounts of MWs and Locations

Bids	From bus	To bus	Desired MWs	Bidding Price
1	1	3	100	7,000
2	2	3	75	8,500
3	1	2	65	7,500

1) DC OPF-Based Model: To solve this FTR auction optimization problem using DCOPF based model, we will ignore the transmission lines' conductances, neglect power losses and reactive power. The problem formulation in per unit will be [2]:

$$\textbf{maximize} \quad 7000 \times R_1 + 8500 \times R_2 + 7500 \times R_3$$

$$\textbf{subject to}$$

$$\begin{bmatrix} -1 & 0 & -1 \\ 0 & -1 & 1 \end{bmatrix} \times \begin{bmatrix} R_1 \\ R_2 \\ R_3 \end{bmatrix} - \begin{bmatrix} -300 & 100 \\ 100 & -200 \end{bmatrix} \times \begin{bmatrix} \delta_1 \\ \delta_2 \end{bmatrix} = [0]$$

$$\begin{bmatrix} -200 & 0 \\ -100 & 100 \\ 0 & -100 \end{bmatrix} \times \begin{bmatrix} \delta_1 \\ \delta_2 \end{bmatrix} \leq \begin{bmatrix} 1.0 \\ 1.0 \\ 1.0 \end{bmatrix}$$

$$-\begin{bmatrix} -200 & 0 \\ -100 & 100 \\ 0 & -100 \end{bmatrix} \times \begin{bmatrix} \delta_1 \\ \delta_2 \end{bmatrix} \leq -\begin{bmatrix} -1.0 \\ -1.0 \\ -1.0 \end{bmatrix}$$

$$\begin{bmatrix} R_1 \\ R_2 \\ R_3 \end{bmatrix} \leq \begin{bmatrix} 1.0 \\ 0.75 \\ 0.65 \end{bmatrix}$$

$$-\begin{bmatrix} R_1 \\ R_2 \\ R_3 \end{bmatrix} \leq \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

This linear program problem was solved using CVX, a package for specifying and solving convex programs [9]. The optimal allocation of FTRs and prices are listed and compared with the results from the ACOPF based models in the last part. The resulted voltage angles are $\delta_1 = -0.005$, $\delta_2 = -0.003$ and $\delta_3 = 0$. The optimal values of dual variable corresponding to the equality constraint λ^* are: $\lambda_1^* = -7,000$, $\lambda_2^* = -3,500$ and $\lambda_3^* = 0$. Note that because node 3 is considered slack, the program does not compute the value of λ_3 so we assume it to be zero. However, this assumption is inconsequential and does not effect the pricing of FTRs.

2) AC OPF-Based Model 1: Using "fmincon" which is built-in function in MATLAB's optimization toolbox, we can solve this nonlinear optimization problem without any assumptions or constraint relaxation. However, to maximize the objective function instead of minimizing it and to be able to solve this particular model, "fmincon" should be constricted as follow:

$$\underset{x}{\text{minimize}} \quad -f(x) \tag{10a}$$

subject to

$$c(x) \leq 0 \tag{10b}$$

$$ceq\,(x) = \; 0 \tag{10c}$$
$$LB \leq x \leq UB \tag{10d}$$

where x is a vector of decision variables, c(x) and ceq(x) are vectors contain nonlinear inequality constraints and nonlinear equality constraints respectively and LB and UB are the lower and upper limits of the decision variables. The solver has to start from an initial value for each decision variable which can be specified in x_0. The function should be called as shown in (11).

$$\begin{aligned}[x,&fval, flag, output, lambda] = \\ &fmincon(fun, x0, [], [], [], [], lb, ub, nonlcon)\end{aligned} \tag{11}$$

where fun is the objective function, $nonlcon$ is a function contains nonlinear constraints and $lambda$ is a structure of fields consist of the Lagrange multipliers at the optimal solution [10]. Thus, after converting all values into per unit system, the parameters of fmincon should be filled as shown in (12). The decision variables are listed in (12a), the lower and upper limits are shown in (12b)(12c) , the nonlinear inequality and equality constraints are shown in (12d)(12e).

$$x = [\, V_1 \,,\, V_2 \,,\, \delta_1 \,,\, \delta_2 \,,\, R_1 \,,\, R_2 \,,\, R_3 \,] \tag{12a}$$
$$lb = [0.9 \,,\, 0.9 \,,\, -2\pi \,,\, -2\pi \,,\, 0 \,,\, 0,\, 0\,] \tag{12b}$$
$$ub = [\, 1.1 \,,\, 1.1 \,,\, 2\pi \,,\, 2\pi \,,\, 1 \,,\, 0.75 \,,\, 0.65\,] \tag{12c}$$

$$c\,(x) = \begin{bmatrix} \sqrt{S_P^2 + S_Q^2} \leq \quad S^{max} \\ -\sqrt{S_P^2 + S_Q^2} \leq -S^{min} \end{bmatrix} \tag{12d}$$

$$= \begin{bmatrix} \begin{bmatrix} S_{P_{13}}(x) \\ S_{P_{12}}(x) \\ S_{P_{23}}(x) \end{bmatrix}^2 + \begin{bmatrix} S_{Q_{13}}(x) \\ S_{Q_{12}}(x) \\ S_{Q_{23}}(x) \end{bmatrix}^2 - \begin{bmatrix} 1.0 \\ 1.0 \\ 1.0 \end{bmatrix}^2 \\ \begin{bmatrix} -1.0 \\ -1.0 \\ -1.0 \end{bmatrix}^2 - \begin{bmatrix} S_{P_{13}}(x) \\ S_{P_{12}}(x) \\ S_{P_{23}}(x) \end{bmatrix}^2 - \begin{bmatrix} S_{Q_{13}}(x) \\ S_{Q_{12}}(x) \\ S_{Q_{23}}(x) \end{bmatrix}^2 \end{bmatrix}$$

$$ceq\,(x) = \left[\, (W - U) \times \, \mathbf{R} \, + \, P \right] \tag{12e}$$

$$= \begin{bmatrix} \begin{bmatrix} -1 & 0 & -1 \\ 0 & -1 & 1 \end{bmatrix} \times \begin{bmatrix} R_1 \\ R_2 \\ R_3 \end{bmatrix} + \begin{bmatrix} P_1(x) \\ P_2(x) \end{bmatrix} \end{bmatrix}$$

P_1 and P_2 are calculated using (5). The average real power flow across transmission lines $S_{P_{13}}$, $S_{P_{12}}$ and $S_{P_{23}}$ are calculated using (6) and the average reactive power flows $S_{Q_{13}}$, $S_{Q_{12}}$ and $S_{Q_{23}}$ are calculated using (6). For instance:

$$\begin{aligned} P_1(x) = V_1 \times ([&V_1 \cdot G_{1,1} \cdot \cos(\delta_1 - \delta_1) + B_{1,1} \cdot \sin(\delta_1 - \delta_1)] \\ &+ [V_2 \cdot G_{1,2} \cdot \cos(\delta_1 - \delta_2) + B_{1,2} \cdot \sin(\delta_1 - \delta_2)] \\ &+ [G_{1,3} \cdot \cos(\delta_1) + B_{1,3} \cdot \sin(\delta_1)]) \end{aligned}$$

$$S_{P_{13}}(x) = \frac{G_{1,3} \times (V_1^2 - 1)}{2} + B_{1,3} \times V_1 \times \sin(\delta_1)$$

$$S_{Q_{13}}(x) = \frac{B_{1,3} \times (V_1^2 - 1)}{2} - G_{1,3} \times V_1 \times \sin(\delta_1)$$

The optimal allocation of FTRs and prices are listed in the following part. The resulted voltage magnitudes and angles are $V_1 = 0.9978$, $V_2 = 0.9889$, $V_3 = 1.0$, $\delta_1 = 0.0051$, $\delta_2 = -0.0018$ and $\delta_3 = 0$. The optimal values of dual variable corresponding to the equality constraint λ^* are: $\lambda_1^* = -7,000$, $\lambda_2^* = -3750.46$ and $\lambda_3^* = 0$.

3) AC OPF-Based Model 2: In this model, we will expand the last model's code and continue to use the function "fmincon". However, we will introduce a new vector of decision variables P_{comp} to compute the required compensation power as shown in (13a). These new variables must be limited with lower and upper limits between 0 and one. Furthermore, the objective function will be modified to account for the penalizing of this compensation power as in (7a). finally, the nonlinear equality constraints vector (7c) is added to $ceq(x)$ as in (13b).

$$\begin{aligned} x = [\, &V_1 \,,\, V_2 \,,\, V_3 \,,\, \delta_1 \,,\, \delta_2 \,,\, \delta_3 \,,\, R_1 \,,\, R_2 \,,\, R_3 \,, \\ &P_{comp_1} \,,\, P_{comp_2} \,,\, P_{comp_3} \,] \end{aligned} \tag{13a}$$

$$ceq\,(x) = \begin{bmatrix} (W - U) \times \, \mathbf{R} \, + \, P \, - \, U \times P_{comp} \\ \frac{P_{comp\,1}}{R\,1} = \frac{P_{comp\,2}}{R\,2} = \frac{P_{comp\,3}}{R\,3} \end{bmatrix} \tag{13b}$$

$$= \begin{bmatrix} \begin{bmatrix} -1 & 0 & -1 \\ 0 & -1 & 1 \\ 1 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} R_1 \\ R_2 \\ R_3 \end{bmatrix} + \begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} P_{comp_1} \\ P_{comp_2} \\ P_{comp_3} \end{bmatrix} \\ \frac{P_{comp\,1}}{R\,1} - \frac{P_{comp\,2}}{R\,2} \\ \frac{P_{comp\,2}}{R\,2} - \frac{P_{comp\,3}}{R\,3} \end{bmatrix}$$

The optimal allocation of FTRs and prices for this model are listed in the following part. The resulted voltage magnitudes and angles are $V_1 = 0.9097$, $V_2 = 0.9$, $V_3 = 0.9122$, $\delta_1 = -0.0061$, $\delta_2 = 0.0022$ and $\delta_3 = 0$. The optimal values of dual variable corresponding to the equality constraint λ^* are: $\lambda_1^* = -1,250$, $\lambda_2^* = 2,000$ and $\lambda_3^* = 5,760$.

4) 3-bus System's Results Comparison: The objective function's optimal value for the DC OPF-based model equals \$15,100/hr while in the first AC OPF-based model it equals \$15,067/hr and in the second model it equals 15,098. The resulted clearance prices after computing them using equation (8) as shown in (14) are listed in Table II.

$$CP_{FTR} = [\lambda^*]^T \; [W - U]$$
$$CP_{FTR} = \begin{bmatrix} \lambda_1^* \\ \lambda_2^* \\ \lambda_3^* \end{bmatrix}^T \times \left(\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \right) \tag{14}$$

Table II
CLEARANCE PRICES (\$/100 MW)

Bids (\$/100 MW)	DC Model	AC Model 1	AC Model 2
1	7,000	7,000	7,009.8
2	3,500	3,750.5	3,759.7
3	3,500	3,249.5	3,250.1
Total	14,000	14,000	14,019.5

The allocated FTRs amounts are converted from per unit to actual values and listed in Table III. The results show that AC models account for the reserved capacity on transmission lines by power losses $I^2 \times R$ and reactive power and allocate less value compared to the DC-model if the line's capacity constraint is binding. The required compensation powers along with each FTR to compensate for power loss are P_{comp_1} = 0.43 MW, P_{comp_2} = 0.59 MW and P_{comp_3} = 0.51 MW,

Table III
ALLOCATED FTRS AMOUNTS IN MWS

Bids	DC Model	AC Model 1	AC Model 2
1	55.0	54.18	54.97
2	75.0	75.00	75.00
3	65.0	65.00	65.00
Total	195.0	194.18	194.97

The total cost of purchasing an FTR for each bidder for the DC and AC without loss allocation models is calculated using the formula ($CP_{FTR_k} \times FTR_k$). For the AC based model with loss allocation, the total cost is calculated using $CP_{FTR_k} \times (FTR_k + P_{comp_k})$. The purchaser of each FTR will pay for a small amount proportional to the amount of its FTR to guarantee getting the required amount at the sink node. The results are listed in table (IV).

Table IV
FTRS TOTAL COST OF PURCHASE ($)

Bidder	DC Model	AC Model 1	AC Model 2
1	385,000	379,272.8	388,338
2	262,500	281,284.6	284,177
3	227,500	211,219.9	212,901
Total	875,000	871,777.5	885,417

B. IEEE 14-Bus System

To test and verify the effectiveness of all three models on a larger case study, the IEEE 14-Bus system as shown in Fig. 2 is used. In this example, there are eight bids from power market participates listed in Table V. The system power base equal 100 MVA and all transmission lines are assumed to have a maximum power transfer capacity of 130 MVA.

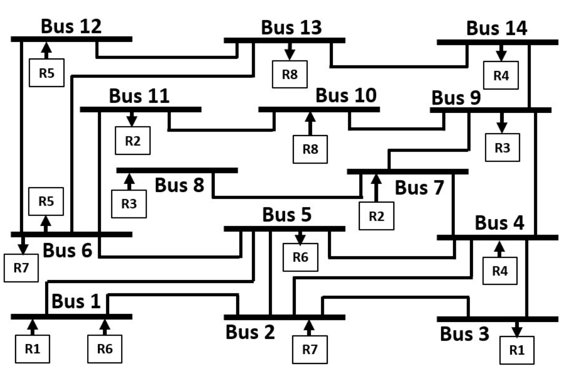

Figure 2. IEEE 14-bus system's diagram with the wanted FTRs.

Table V
A LIST OF BIDDING PRICES, AMOUNTS OF MWS AND LOCATIONS

Bids	From bus	To bus	Desired MWs	Bidding Price
1	1	3	150	6,500
2	7	11	125	8,000
3	8	9	45	5,000
4	4	14	95	9,500
5	12	6	85	7,700
6	1	5	90	9,000
7	2	6	100	5,900
8	10	13	45	8,200

The same coding approaches which were used to solve the 3-bus system for all three models were applied to the 14-bus system. The resulted objective function of the DC model equals 49,180.4 while in the AC model 1 it equals 51,202.83 and in the AC model 2 it equals 43,394.35 due to the addition of the penalty factor. The voltage's magnitude and angle of all nodes in pu are listed in Table VI. The optimal values of the dual variable corresponding to the equality constraint λ^* for the three models are listed in Table VII. FTRs clearance

Table VI
NODES' VOLTAGE MAGNITUDE AND ANGLE

$\mathbf{V}$	DC-Model	AC-Model 1	AC-Model 2
$\mathbf{V_1}$	- $\angle$ 0.5852	1.1 $\angle$ 0.3659	1.0339 $\angle$ 0.5194
$\mathbf{V_2}$	- $\angle$ 0.5001	1.077 $\angle$ 0.3015	1.0097 $\angle$ 0.4465
$\mathbf{V_3}$	- $\angle$ 0.3314	1.076 $\angle$ 0.1251	0.9682 $\angle$ 0.2891
$\mathbf{V_4}$	- $\angle$ 0.4159	0.9 $\angle$ 0.2644	0.9936 $\angle$ 0.36
$\mathbf{V_5}$	- $\angle$ 0.3789	0.9 $\angle$ 0.2213	0.9838 $\angle$ 0.3294
$\mathbf{V_6}$	- $\angle$ 0.0735	0.9983 $\angle$ -0.1244	1.0606 $\angle$ 0.035
$\mathbf{V_7}$	- $\angle$ 0.4415	1.091 $\angle$ 0.2581	1.0902 $\angle$ 0.3441
$\mathbf{V_8}$	- $\angle$ 0.4639	0.9789 $\angle$ 0.2623	0.9545 $\angle$ 0.3677
$\mathbf{V_9}$	- $\angle$ 0.2985	1.1 $\angle$ 0.1387	1.1 $\angle$ 0.2245
$\mathbf{V_{10}}$	- $\angle$ 0.2444	1.1 $\angle$ 0.07	1.1 $\angle$ 0.1846
$\mathbf{V_{11}}$	- $\angle$ 0.0150	1.0271 $\angle$ -0.1524	1.0478 $\angle$ 0.0187
$\mathbf{V_{12}}$	- $\angle$ 0.2187	1.0702 $\angle$ -0.0205	1.1 $\angle$ 0.1328
$\mathbf{V_{13}}$	- $\angle$ 0.0461	1.0242 $\angle$ -0.1166	1.0306 $\angle$ 0.0297
$\mathbf{V_{14}}$	- $\angle$ 0	1.0 $\angle$ 0	1.0 $\angle$ 0

Table VII
LAMBDA VALUES ($/MW)

λ	DC Model	AC Model 1	AC Model 2
λ_1^*	-11165	-11294	5827
λ_2^*	-3982	-3778	11620
λ_3^*	-4665	-4794	12060
λ_4^*	-5284	-5369	10990
λ_5^*	-6059	-6419	10742
λ_6^*	1918	2122	17623
λ_7^*	-5747	-6047	9766
λ_8^*	-5747	-6047	9766
λ_9^*	-747	-1047	14751
λ_{10}^*	-294	-1026	15256
λ_{11}^*	772	1953	17751
λ_{12}^*	1729	1668	15328
λ_{13}^*	1462	1659	18071
λ_{14}^*	0	1234	18436

prices after computing them using equation (8) are listed in Table VIII. The allocated FTRs amounts are converted from per unit to actual values and listed in Ttable IX.

Table VIII
CLEARANCE PRICES ($/MW)

Bids	DC Model	AC Model 1	AC Model 2
1	6,500	6,500	6,233.4
2	6,519	8,000	7,985
3	5,000	5,000	4,985
4	5,284.3	6,604	7,446.2
5	189.3	454	2,295.7
6	5,105.9	4,874.3	4,914.9
7	5,900	5,900	6,003.5
8	1,756.6	2,685.5	2,815.7
Total	36,255.02	40,017.8	42,679.46

Table IX
ALLOCATED FTRS AMOUNTS IN MWS

Bids	DC Model	AC Model 1	AC Model 2
1	124.4	124.06	115.42
2	125.0	124.42	101.09
3	12.75	2.57	13.15
4	95.0	95.0	95.0
5	85.0	85.0	85.0
6	90.0	90.0	90.0
7	52.49	96.56	62.09
8	45.0	45.0	45.0
Total	629.64	662.608	606.759

The required compensation power injections along with each FTR to compensate for power loss are listed in Table X. The system total power injection $\sum_i^n P_i = P_{loss}$ equals 38.7656 MW which is very close to the total required compensation power.

Table X
THE REQUIRED COMPENSATION POWER FOR EACH FTR

FTRs	P_{comp} (MWs)
FTR_1	7.37
FTR_2	6.46
FTR_3	0.84
FTR_4	6.07
FTR_5	5.43
FTR_6	5.75
FTR_7	3.97
FTR_8	2.88
Total	38.766

The total cost of purchasing an FTR for each bidder is listed in Table XI.

Table XI
FTRS TOTAL COST OF PURCHASE ($)

Bidder	DC Model	AC Model 1	AC Model 2
1	808,588	806,404	765,444
2	814,879	995,323	858,796
3	63,766	12,870	69,746
4	502,005	627,377	752,584
5	16,087	38,594	207,600
6	459,531	438,683	470,602
7	309,682	569,686	396,593
8	79,046	120,849	134,804
Total	3,053,584	3,609,786	3,656,169

V. CONCLUSION

In this paper, we have proposed a loss allocation method for AC OPF-based FTR auction model. FTRs are required to take responsibility of the system loss according to their usage of the system. Each FTR should inject a certain amount of compensation power at its source node and the ratios of compensated power to FTRs amounts are expected to be the same. A nonlinear programming AC OPF-based problem is formulated with the loss allocation requirement posed as additional constraints. Two test systems are examined for the proposed method and comparison with the conventional DC OPF-based auction results is conducted.

REFERENCES

[1] G. Zakeri, "Financial transmission rights markets: An overview," October 26, 2010.

[2] P. R. Gribik, D. Shirmohammadi, J. S. Graves, and J. G. Kritikson, "Transmission rights and transmission expansions," *IEEE Transactions on Power Systems*, vol. 20, no. 4, pp. 1728–1737, 2005.

[3] S. Oru, "Financial transmission rights: A coupled electricity-ftr model," 2013.

[4] K. Lyons, H. Fraser, and H. Parmesano, "An introduction to financial transmission rights," *The Electricity Journal*, vol. 13, no. 10, pp. 31–37, 2000.

[5] W. Hogan, "A concurrent auction model for transmission congestion contracts, january 1997," *John F. Kennedy School of Government, Harvard University, Cambridge, MA*.

[6] F. Ferris, Z. Friedman, D. Molzahn, C. DeMarco, and B. Lesieutre, "Acopf models: extending data, formulations, and solution methodology," 2013.

[7] W. W. Hogan, "Contract networks for electric power transmission," in *From Regulation to Competition: New frontiers in electricity markets*. Springer, 1994, pp. 175–199.

[8] A. Bergen and V. Vittal, *Power Systems Analysis*. Pearson/Prentice Hall, 2000.

[9] M. Grant and S. Boyd, "CVX: Matlab software for disciplined convex programming, version 2.1," http://cvxr.com/cvx, Mar. 2014.

[10] "Matlab documentation center, optimization toolbox, constrained optimization, fmincon." [Online]. Available: https://www.mathworks.com/help/optim/ug/fmincon.html