Bender's Decomposition Algorithm for Model Predictive Control of a Modular Multi-level Converter

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***Abstract*— In this paper, we consider an optimization problem solving for model predictive control (MPC) of a modular multilevel converter (MMC). An MMC consists of a large number of submodules. The objective of the MPC is to determine the best switching sequences for the submodules in the MMC to track the phase current references for T time horizons. The MPC is formulated as a nonlinear mixed-integer programming (MIP) problem with the on/off status of submodules as binary decision variables and MMC dynamic states such as phase currents, circulating currents and submodule capacitor voltages as continuous decision variables. With a large number of submodules and a large number of time horizons, the dimension of the nonlinear MIP problem becomes difficult to handle. Our contribution is to formulate this problem and solve this problem using Bender's decomposition. An example 4-level single-phase MMC is demonstrated for the proposed algorithm.**

***Index Terms*— MMC; Nonlinear mixed-integer programming problem; Bender's decomposition**

I. INTRODUCTION

Compared with traditional two-level voltage source converter (VSC), MMCs have much lower harmonics in the output voltage, which significantly reduces the size of grid side filter [?]. MMCs have modular topology and the extensibility for several hundreds of output voltage levels. Therefore, MMC is ideal for high-voltage high-power applications, such as HVDC transmission [?], high-voltage motor drives [?], and electric railways [?]. Fig. 1 is the topology of a three phase MMC. For an $N+1$ level MMC, there are N submodules on each arm of the converter. Each sub-module is a half bridge dc-dc converter. Since the current flows through different sub-modules at different times, the voltages of submodules capacitors vary.

A. State-of-the-art MMC Switching Schemes

MMC control differs from two-level VSC control in two aspects: (i) switching sequence generation and (ii) the inclusion of circulating current mitigation control. In switching sequence generation, in two-level VSCs, the output from pulse width modulation (PWM) is the switching sequence directly fed to the gates. In MMCs, the output of pulse-width modulation or other types of switching schemes is the number of submodules to be turned on at each arm. Which submodules to be turned on then depends on additional submodule voltage balance consideration. The PWM switching schemes are also very different from that of two-level VSCs. Phase-disposition (PD)-PWM and Phase shifted-PWM are often adopted [?] (see Fig. 2 for an example of PD-PWM).

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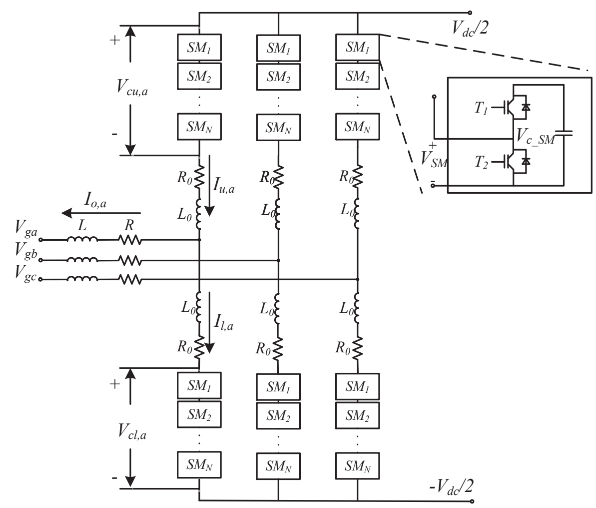

Fig. 1. Three phase MMC topology.

In MMC's PWM, there are usually many carrier signals for the reference sinusoidal signal to be compared to; while for two-level VSC's PMW, there is usually one triangular carrier signal.

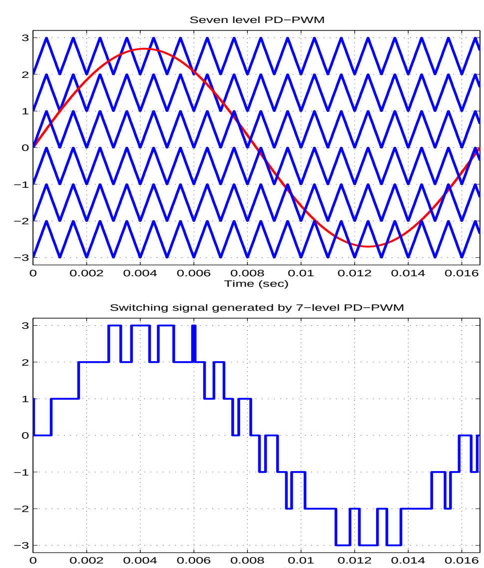

Fig. 2. (a) Three-level VSC PD-PMW scheme and switching status of a phase. (b) Seven-level VSC PD-PWM scheme and switching status of a phase.

MMC PWM gives only the number of sub-modules to be switched on. To determine which modules to be switched on, another phase is required: capacitor voltage balancing. A capacitor voltage balancing block selects the proper sub-modules to be switched.

B. Our Contributions

As an advanced control method, MPC is very successful on its application for the control of power converters [**?**]. Its basic principle is to generate a system dynamic model based minimizing optimization problem, and provide the solutions to the controller for driving the system to reach the control targets (Generally they will be formulated as the objective function in the MPC optimization problem). A major challenge for implementing MPC on MMC is that the system dynamic model of MMC is nonlinear with binary terms, in other word, the MPC problem of MMC is a nonlinear MIP problem which generally is difficult to be solved.

In this paper, we adopt Bender's decomposition to solve the MPC problem. Our major focus is the derivation for the MPC problem formulation and its Bender's decomposition forms. In the case study section, the potential and practicability of this application are verified.

The rest of the paper is organized as follows. Section II gives the dynamic model of a single-phase MMC. Section III gives the details about the formulation of the MPC as a nonlinear MIP problem. Section IV gives the Bender's decomposition algorithm. Section V presents the case study and results. Finally, the paper is concluded in Section VI.

II. Dynamic Model of MMC

Fig. 1 shows the overall structure of a three-phase MMC consisting of six arms. Subscripts u and l denote upper and lower arms, respectively. There are N sub-modules and one inductor L_0 on each arm. A resistor R_0 is inserted to represent the switching loss of the IGBTs on each arm. The output voltage of each sub-module has two values, v_c (connected) and 0 (disconnected). When the number of sub-modules or the switching frequency is high enough, the voltage across whole sub-modules in each arm can be considered as continuous. Since the dc side capacitors are usually big enough, the voltage across the arm can be considered as a constant dc voltage. Thus, we can express a single-phase equivalent circuit of a MMC as Fig. 3.

In Fig. 3, i_u and i_l are the arm currents for upper and lower arms; i_o and v_o are the converter output current and voltage respectively. The circulating current flowing within the converter is denoted as i_{diff}. Since the upper and lower arm are symmetric, ideally both lower and upper arm currents contain half of the converter output current. Therefore, with Kirchhoff Current Law (KCL), we can get following equations.

$$\begin{cases} i_u = i_{\text{diff}} + \frac{i_o}{2} \\ i_l = i_{\text{diff}} - \frac{i_o}{2} \end{cases} \Rightarrow \begin{cases} i_{\text{diff}} = \frac{i_u + i_l}{2} \\ i_o = i_u - i_l. \end{cases} \tag{1}$$

The voltage across the arm resistance and inductance can be expressed by the arm current. Therefore, with Kirchhoff Voltage Law (KVL), we can have the voltage relationship as follows.

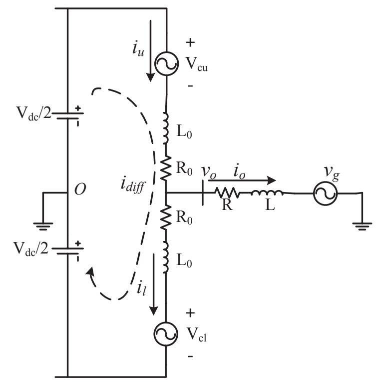

Fig. 3. Single phase equivalent circuit of MMC.

$$\begin{cases} v_u + i_u R_0 + L_0 \frac{\mathrm{d}i_u}{\mathrm{d}t} = \frac{V_{\text{dc}}}{2} - v_o \\ v_l + i_l R_0 + L_0 \frac{\mathrm{d}i_l}{\mathrm{d}t} = \frac{V_{\text{dc}}}{2} + v_o \end{cases} \tag{2}$$

Considering that the output voltage v_o can be written as $v_g + i_o R + L \frac{\mathrm{d}i_o}{\mathrm{d}t}$ and (1), by substracting the two equations from (2) we have:

$$\frac{v_u - v_l}{2} + \left(R + \frac{1}{2}R_0\right) i_o + \left(L + \frac{1}{2}L_0\right) \frac{\mathrm{d}i_o}{\mathrm{d}t} + v_g = 0 \tag{3}$$

It is obvious that the term $\frac{v_u - v_l}{2}$ in (2) drives the output current of the converter, therefore we name this term as e, which is the inner EMF of the converter. We can have an equivalent circuit of MMC as Fig. 4.

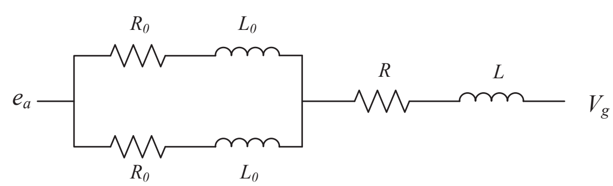

Fig. 4. An equivalent circuit of one phase of MMC.

Adding the two equations in (2) leads to the plant model of the circulating current control:

$$i_{\text{diff}} R_0 + L_0 \frac{\mathrm{d}i_{\text{diff}}}{\mathrm{d}t} = \frac{V_{\text{dc}}}{2} - \frac{v_u + v_l}{2}. \tag{4}$$

Considering Fig. 1, since v_u and v_l are the sum of all submodule voltages on the correspond arm, we can express v_u and v_l through the following equations.

$$\begin{cases} v_u = \sum_{i=1}^{N} V_{SMi}, & \text{upper arm} \\ v_l = \sum_{i=N+1}^{2N} V_{SMi}, & \text{lower arm} \end{cases} \tag{5}$$

For convenience, we name the voltage of the capacitor on each submodule as $v_c(i)$, and the state of the correspond IGBT as $u(i)$. Apparently, $V_{SMi} = v_c(i)u(i)$, combine it with the equation (5), then we have:

$$\begin{cases} v_u = \sum_{i=1}^{N} v_c(i)u(i), & \text{upper arm} \\ v_l = \sum_{i=N+1}^{2N} v_c(i)u(i), & \text{lower arm} \end{cases} \tag{6}$$

III. Nonlinear MIP Optimization Problem Formulation

We propose to discretize the continuous dynamic model of MMC which described by (1)–(6). It means we consider:

$$\frac{di_o}{dt} = \frac{i_o(k+1) - i_o(k)}{h}, \tag{7}$$

$$\frac{di_{\text{diff}}}{dt} = \frac{i_{\text{diff}}(k+1) - i_{\text{diff}}(k)}{h}, \tag{8}$$

where h is the step size of the discretized signal, $k \in \{1, 2, \cdots, T\}$ is the index of the time step. Combining (7) and (8) respectively with (3) and (4), we obtain the following equations:

$$i_o(k+1) = i_o(k) + \frac{h}{L+\frac{L_0}{2}} \left[-\left(R + \frac{R_0}{2}\right) i_o(k) - v_g(k) - \frac{v_u(k)-v_l(k)}{2} \right] \tag{9}$$

$$i_{\text{diff}}(k+1) = i_{\text{diff}}(k) + \frac{h}{L} \left[-R_0 i_{\text{z}}(k) + \frac{V_{dc}}{2} - \frac{v_u(k)+v_l(k)}{2} \right] \tag{10}$$

where

$$\begin{cases} v_u(k) = \sum_{i=1}^{N} v_c(i,k)u(i,k) \\ v_l(k) = \sum_{i=N+1}^{2N} v_c(i,k)u(i,k) \end{cases} \tag{11}$$

In (11), N is the numbers of switches on one arm, $i \in \{1, 2, \cdots, 2N\}$ is the index of switches. We expect to control the MMC output current to track the current reference which is a sinusoidal waveform. This can be expressed by solving the following optimization problem:

$$\min \quad \sum_{k=1}^{T} \left[i_o^{\text{ref}}(k) - i_o(k) \right]^2 \tag{12}$$

$$\begin{aligned}
\text{subject to} \quad & (9), (10), \\
& v_u(k) = \sum_{i=1}^{N} v_c(i,k)u(i,k) \\
& v_l(k) = \sum_{i=N+1}^{2N} v_c(i,k)u(i,k)
\end{aligned}$$

$$v_c(i,k+1) = \begin{cases} v_c(i,k) + \frac{h}{c} \cdot u(i,k) \left[\frac{i_o(k)}{2} + i_{\text{diff}}(k) \right], \\ \qquad i = 1, 2, \cdots, N. \\ v_c(i,k) + \frac{h}{c} \cdot u(i,k) \left[-\frac{i_o(k)}{2} + i_{\text{diff}}(k) \right], \\ \qquad i = N+1, \cdots, 2N \end{cases}$$

$$\sum_{i=1}^{2N} u(i,k) = N$$

$$u(i,k) \in \{0, 1\}$$

where i_o^{ref} is our reference current, $u(i,k)$ is the state of the ith switch at the kth time step, $v_c(i,k)$ is the voltage on the ith switch capacitor at the kth time step.

If we replace the binary constraint $u(i,k) \in \{0, 1\}$ by an equality constraint $u(i,k)(1 - u(i,k)) = 0$ with $u(i,k)$ as continuous variable, then this problem can be solved by nonlinear programming solver $fmincon$ using sequential quadratic programming (SQP) algorithm.

IV. Bender's Decomposition Formulation

To implement Bender's decomposition for solving the problem (12), we separate the problem into a master problem and a subproblem. In the subproblem, the binary variable u is considered as fixed value. Therefore the subproblem could be solved as a linear programming problem. Iteratively, the dual variables which are solved from the subproblem that will be used to generate Bender's cuts and add them to the master problem. And then, the solution of u from the master problem is returned to the subproblem. This iteration process is repeated until the stop criteria is met.

A. Subproblem

In our case, decision variables in the subproblem are: i_o, i_{diff}, v_u, v_l, and v_c, and define any given u as $\hat{u}$, its primal problem could be expressed as follow:

$$v_{\text{ub}} = \min \quad \sum_{k=1}^{T} \left[i_o^{\text{ref}}(k) - i_o(k) \right]^2 \tag{13}$$

$$\begin{aligned}
\text{subject to} \quad & (9), (10) \\
& v_u(k) = \sum_{i=1}^{N} v_c(i,k)\hat{u}(i,k) \\
& v_l(k) = \sum_{i=N+1}^{2N} v_c(i,k)\hat{u}(i,k)
\end{aligned}$$

$$v_c(i,k+1) = \begin{cases} v_c(i,k) + \frac{h}{c} \cdot \hat{u}(i,k) \left[\frac{i_o(k)}{2} + i_{\text{diff}}(k) \right] \\ \qquad i = 1, 2, \cdots, N. \\ v_c(i,k) + \frac{h}{c} \cdot \hat{u}(i,k) \left[-\frac{i_o(k)}{2} + i_{\text{diff}}(k) \right] \\ \qquad i = N+1, \cdots, 2N. \end{cases}$$

where v_{ub} is the value of the objective function. If $\hat{u}$ is a feasible solution that can make $\sum_i \hat{u}(i,k) = N$, then v_{ub} is an upper bound (UB) of the original nonlinear MIP problem.

To generate Bender's cuts, we need to find dual variables which correspond with the constraints that include the binary variable u. To achieve this, first we define a Lagrangian function to aggregate the objective function and the constraints that are related to u.

$$\begin{aligned}
L = & \sum_{k=1}^{T} \left[i_o^{\text{ref}}(k) - i_o(k) \right]^2 \\
& + \sum_{i=1}^{N} \sum_{k} \lambda_1(i,k) \left\{ v_c(i,k+1) - v_c(i,k) - \frac{h}{c} u(i,k) \left[\frac{i_o(k)}{2} + i_{\text{diff}}(k) \right] \right\} \\
& + \sum_{i=N+1}^{2N} \sum_{k} \lambda_2(i,k) \left\{ v_c(i,k+1) - v_c(i,k) - \frac{h}{c} u(i,k) \left[-\frac{i_o(k)}{2} + i_{\text{diff}}(k) \right] \right\} \\
& + \sum_{k} \lambda_3(k) \left[v_u(k) - \sum_{i=1}^{N} v_c(i,k)u(i,k) \right] \\
& + \sum_{k} \lambda_4(k) \left[v_l(k) - \sum_{i=1+N}^{2N} v_c(i,k)u(i,k) \right].
\end{aligned}$$

The partial dual of the original problem can be formulated as follows.

$$\max_{\lambda} \quad \min_{i_o, i_{\text{diff}}, v_u, v_l, v_c} L$$

$$\text{subject to} \quad (9),(10)$$

B. Cuts introduced by the subproblem

Given a $\hat{u}$, we may formulate a cut using the following inequality constraint:

$$\mu \geq v_{\text{ub}}^{l}\left[\hat{u}^{l}(i,k)\right] + \sum_{k}\sum_{i} g^{l}(i,k)\left[u(i,k) - \hat{u}^{l}(i,k)\right] \tag{14}$$

where superscript $l \in \{1, 2, \cdots, K\}$ is the index of the cuts (capital K denotes the last Bender's iteration step), matrix g is defined as:

$$g = \begin{bmatrix} \frac{\partial L(\hat{\lambda}_1,\hat{\lambda}_2,\hat{\lambda}_3,\hat{\lambda}_4)}{\partial u(1,1)} & \cdots & \frac{\partial L(\hat{\lambda}_1,\hat{\lambda}_2,\hat{\lambda}_3,\hat{\lambda}_4)}{\partial u(1,T)} \\ \vdots & \ddots & \vdots \\ \frac{\partial L(\hat{\lambda}_1,\hat{\lambda}_2,\hat{\lambda}_3,\hat{\lambda}_4)}{\partial u(2N,1)} & \cdots & \frac{\partial L(\hat{\lambda}_1,\hat{\lambda}_2,\hat{\lambda}_3,\hat{\lambda}_4)}{\partial u(2N,T)} \end{bmatrix}$$

thus

$$g(i,k) = \begin{cases} -\hat{\lambda}_1(i,k)\frac{h}{c}\left[\frac{i_o(k)}{2} + i_{\text{diff}}(k)\right] - \hat{\lambda}_3(k)v_c(i,k), \\ \quad i = 1, 2, \cdots, N. \\ -\hat{\lambda}_2(i,k)\frac{h}{c}\left[-\frac{i_o(k)}{2} + i_{\text{diff}}(k)\right] - \hat{\lambda}_4(k)v_c(i,k), \\ \quad i = N+1, \cdots, 2N. \end{cases}$$

C. Master problem

Associating Bender's cuts, the master problem can be written as follows:

$$\begin{aligned} v_{\text{lb}} = \min \quad & \mu \\ \text{subject to} \quad & \sum_{i=1}^{2N} u(i,k) = N \\ & u(i,k) \in \{0,1\} \\ & \mu \geq v_{\text{ub}}^{l}\left[\hat{u}^{l}(i,k)\right] \\ & + \sum_{k}\sum_{i} g^{l}(i,k)\left[u(i,k) - \hat{u}^{l}(i,k)\right] \end{aligned}$$

where v_{lb} is a lower bound (LB) of the original nonlinear MIP problem.

Apparently, the above problem is a mixed integer linear programming problem which can be solved by gurobi or mosek. Therefore, through the iteration, if the objective functions' values of the master problem and subproblem are close enough to reach a stop criteria, we can consider the optimal solutions are found. Generally, the criteria is defined as:

$$\epsilon \geq |v_{\text{ub}} - v_{\text{lb}}|$$

where ϵ is a fixed small value.

V. Case Study

In this section, a 5 level MMC is used as the platform to test the effectiveness of Bender's algorithm. The parameters of the MMC and MPC time step size are listed in Table I. Matlab CVX toolbox with Mosek 7.1 solver are applied to solve the master and subproblem. The stop criteria constant is $\epsilon = 1 \times 10^{-5}$. To examine the performance of Bender's

TABLE I
Parameters table

Items	Values
Submodule capacitor (C_{sm})	2500 μF
Insert inductor (L_0)	10 mH
Insert resistor (R_0)	0.1 Ω
Terminal inductor (L)	2 mH
Terminal resistor (R)	0.03 Ω
DC voltage (V_{dc})	40 kV
Rated frequency(f)	60 Hz
Grid voltage magnitude (v_s)	20 kV
Reference current magnitude (i_o^{ref})	5 kA
Prediction step size (h)	25 μs

algorithm, we have also developed a group of matlab codes to solve the problem (12) via $fmincon$ function based SQP method. The solutions of two methods about their objective values and solving time cost for different predict horizons are listed separately in Table II.

Note: As the calculate mechanisms are different. In our implementation, SQP and Bender's decomposition adopt different methods to calculate time cost. In Bender's algorithm, the time cost is the time consuming when the stop criteria is reached at first time (when $T = 10$, as it dose not reach the criteria, the time cost is the time consuming for 50 iterations). In SQP method, time cost t_{cost} is calculated through: $t_{cost} = t_{sum}/m$, where m is the numbers of different initial values that has been tested, t_{sum} is the total time cost for all of the testing.

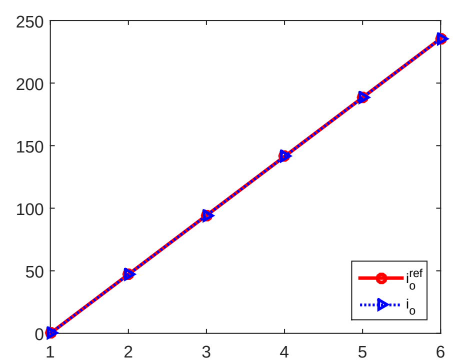

Fig. 5. Current tracking of Bender's algorithm when $T = 5$

Fig. 5 and 6 present, when using Bender's algorithm, the current tracking performance and the convergence of the objective value for subproblem and master problem when $T = 5$; Fig. 7 and 8 show the performance and convergence when the predict horizon is 7; Table III lists the solutions of the integer variables for the correspond horizon T. From those results, we can see the performance of Bender's algorithm is examined: the current tracking is nearly prefect, the solutions of the integer values are completely feasible, and the objective value of the master problem (LB) and subproblem (UB) are converged within a reasonable iteration steps. Moreover, according to the data in Table II, which

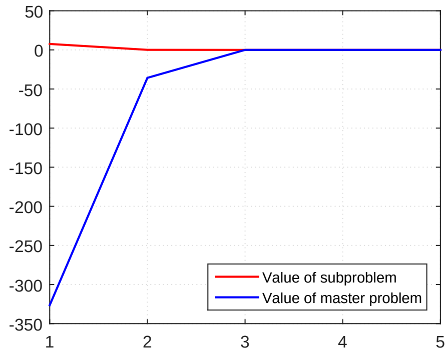

Fig. 6. Convergence of the objective value for the master problem and subproblem when $T = 5$.

is proposed to compare the solutions of the problem (12) by Bender's algorithm and SQP method, we can see the Bender's algorithm has some obvious advantages than SQP method.

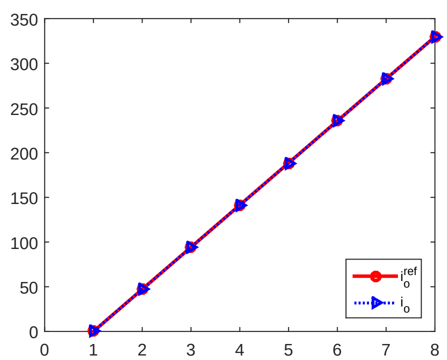

Fig. 7. Current tracking of Bender's algorithm when $T = 7$.

TABLE II

RESULTS COMPARISON OF BENDER'S DECOMPOSITION AND SQP ALGORITHM

T	Nonlinear(SQP)		Benders		
	Obj	T (sec)	UB	LB	T (sec)
5	2.90	11.19	1.11e-16	1.11e-16	0.77
7	7.24	39.96	2.80e-6	2.80e-6	1.87
10	22.28	40.70	0.16	-0.72	56.61

However, from the data in Table II when $T = 10$, we can see, with more time cost than the SQP method, the difference between the master problem and subproblem still can not reach the stop criteria. In fact, when $T = 10$, even spend more than one hour to run the Bender's algorithm to 70 iteration steps, the convergence result can not be changed significantly and not reach the stop criteria. The reasons of this situation are the optimal value of the subproblem can not be guaranteed to decrease during the iteration [**?**], and the master problem became way too complicated after too

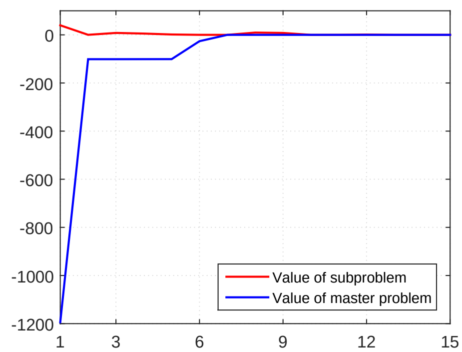

Fig. 8. Convergence of the objective value for the master problem and subproblem when $T = 7$.

TABLE III

BINARY SOLUTION

$T = 5$					$T = 7$						
0	1	0	0	1	0	1	1	0	0	1	1
0	0	1	1	0	0	0	0	0	1	1	0
1	0	0	1	0	1	0	0	1	1	0	1
0	1	1	0	0	1	0	1	0	1	1	1
1	0	0	1	0	1	1	1	1	1	0	1
1	0	1	0	1	1	1	0	0	0	0	0
1	1	1	1	1	0	1	1	1	0	1	0
0	1	0	0	1	0	0	0	1	0	0	0

many cuts added. Although the stop criteria has not been reached for 10 horizon, from Fig. 9 and 10, we can see the tendency of convergence is obvious for the master problem and subproblem. Thus, we can expect, if long enough time is provided for iteration, or those two problems which have been mentioned above are solved, Bender's algorithm will be practicable when $T \geq 10$.

As we only implement classic Bender's algorithm to solve the problem, there is no any step to deal with the problems about the convergence of the subproblem and the numbers of cuts. According to [**?**] [**?**] and [**?**], some modified Bender's algorithm could improve the computational effectiveness than the classic one. However, given the time limit, we can not finish all the studies about these improved Bender's algorithms. Thus, more research achievements about this topic are expected to be included in our further papers.

VI. CONCLUSION

In this paper, a dynamic model of MMC are derived and then formulated to a MPC problem. We separated the original MPC problem into a master problem and a subproblem to implement Bender's decomposition to solve the problem. A 5 level MMC is selected to test the proposed algorithm, and the solutions for different predict horizons are presented and compared with the solutions of SQP. According to the results, Bender's decomposition has greater performance on solving the low horizon MPC problem than SQP method. However for longer horizon ($T \geq 10$), even with the presented

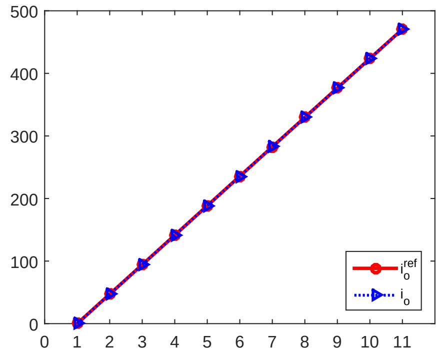

Fig. 9. Current tracking of Bender's algorithm when $T = 10$.

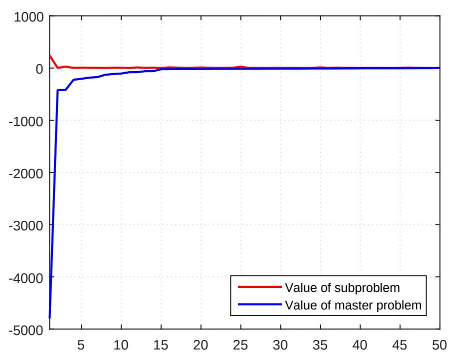

Fig. 10. Convergence of the objective value for the master problem and subproblem when $T = 10$.

potential to solve the problem, the Bender's algorithm still need further adjustment to reduce the computational load.

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