Mixed Integer Linear Programming Formulation for Chance Constrained Mathematical Programs with Equilibrium Constraints

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***Abstract*—This paper gives a mixed integer linear programming (MILP) formulation for a bi-level mathematical program with equilibrium constraints (MPEC) considering chance constraints. The particular MPEC problem relates to a power producer's bidding strategy: maximize its total benefit through determining bidding price and bidding power output while considering an electricity pool's operation and guessing the rival producer's bidding price. The entire decision-making process can be described by a bi-level optimization problem. The contribution of our paper is the MILP formulation of this problem. First, the lower-level pool operation problem is replaced by Karush-Kuhn-Tucker (KKT) optimality condition, which is further converted to an MILP formulation except a bilinear item in the objective function. Secondly, duality theory is implemented to replace the bilinear item by linear items. Finally, two types of chance constraints are examined and modeled in MILP formulation. With the MILP formulation, the entire MPEC problem considering randomness in price guessing can be solved using off-shelf MIP solvers, e.g., Gurobi. An example is given to illustrate the formulation and show the case study results.**

***Index Terms*—Chance Constraints, MILP, Pool Strategy, Electricity Market, MPEC.**

I. Introduction

Many problems arising from engineering and economics are mathematical problems with equilibrium constraints (MPEC) [1]. In this paper, a particular MPEC problem relates to a strategic power producer trying to maximize its total benefit through determining bidding price and bidding power output while considering an electricity pool's operation and guessing the rival producer's bidding price. The entire decision-making process can be described by a bi-level optimization problem. The upper-level tries to maximize the power producer's profit and minimize its cost while the lower-level emulates the decision making of a pool: minimize the total operation cost considering the bidding price from the power producer and the other producers while guaranteeing generation and load balance as well as network limits observed.

Optimal bidding strategy problem formulations have been seen in the literature, e.g., [2]–[6]. There are fundamentally two approaches for solving an MPEC problem [1]: (i) nonlinear optimization program solving techniques and (ii) MILP solving techniques.

In the first approach, the optimal bidding problem described by a bi-level program is converted to a nonlinear programming problem after the lower-level optimization problem is replaced by the KKT optimality condition. Naturally, nonlinear program solving techniques, e.g., interior point method, can be applied to solve the problem. Such approach can be found in the literature [7], [8]

In the second approach, the nonlinear programming problem should be converted to an MILP first. For example, in [6], MILP formulation is adopted to express the complementary slackness condition in the KKT optimality by introducing binary variables and adopting the Big-M technique. Further, duality theory is used to linearize the objective function with a bilinear term by an MILP formulation.

Uncertainty is also considered for the optimal bidding problems. In [6], a stochastic programming problem is formulated and solved. [4] considers a similar problem with wind generation included for both day-ahead and real-time market. [4] also incorporates risk management and the uncertainty of other strategic power producers through stochastic programming approach. Most recently, chance constrained mathematical program (CCMP) has found applications in power market to capture the randomness. For example, [9] investigated chance constrained unit commitment problems.

The objective of this paper is to formulate a mathematical programming problem related to optimal bidding strategy while considering uncertainty in guessing of the marginal cost of the rival generator.

The contribution of our paper is the MILP formulation of a chance constrained optimal bidding strategy problem. First, the lower-level pool operation problem is replaced by Karush-Kuhn-Tucker (KKT) optimality condition, which is further converted to an MILP formulation except a bilinear item in the objective function. Secondly, duality theory is implemented to replace the bilinear item by linear items. Finally, two types of chance constraints are examined and modeled in MILP formulation sing the method in [10]. With the MILP formulation, the entire MPEC problem considering randomness in price guessing can be solved using off-shelf MIP solvers, e.g., Gurobi. An example is given to illustrate the formulation and show the case study results.

The rest of the paper is organized as follows. Section II presents the bi-level optimization problem and the corresponding MPEC problem when the lower-level problem is replaced by the KKT conditions. Section III presents three steps of MILP formulation to deal with the complementary slackness, bilinear objective function, and finally chance constraints considering uncertainty in price guessing. Section IV gives the case study results.

II. The Optimal Bidding Strategy Problem

In this section, a simple two-generator system is used to explain the optimal bidding strategy problem. The system is shown in Fig. 1.

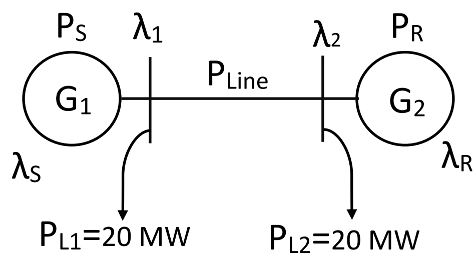

Fig. 1. A simple power system with two producers. λ_s and λ_R are marginal cost of the two generators and are given. P_S, P_R, λ_1 and λ_2 are to be determined by the strategy.

Gen S will carry out the optimal bidding strategy decision making process to determine its bidding price α_s and bidding power P_S. Gen S will treat the marginal cost of the rival generator Gen R (λ_R) as known.

A. Upper Level Problem

The upper level problem is to determine the bidding price α_S and the power P_S. The objective is maximizing the total profit of the strategic producer, with the bidding price α_S not appearing. α_S will appear in the objective function of the lower-level problem. α_S will affect the bidding power P_S.

$$\min_{\alpha_S, P_S} \quad \lambda_S P_S - \lambda_1 P_S \tag{1}$$

where λ_S is the marginal cost of the strategic power producer, and P_S is power produced by the strategic power producer. The first term sums the total cost. In the second term λ_1 is the locational marginal price at bus 1. The second term is the total revenue generated. λ_1 will be determined by the pool or the lower-level problem. Therefore, the objective function maximizes the total profit by minimizing the cost and maximizing the revenue. The generation P_S belongs to the feasible region defined by the lower level problem.

B. Lower Level Problem

The lower level problem is formulated to represent the market clearing process, and is shown as follows along with their duel variables:

$$\min_{P_S, P_R} \quad \alpha_S P_S + \lambda_R P_R \tag{2}$$

$$\text{s.b.} \quad P_S - P_{L1} = P_{Line} : \lambda_1 \tag{3}$$

$$P_R - P_{L2} = -P_{Line} : \lambda_2 \tag{4}$$

$$P_{Line} \le P_{Line}^{max} : \mu_{Line} \tag{5}$$

$$0 \le P_S \le P_S^{max} : \mu_S^{min}, \mu_S^{max} \tag{6}$$

$$0 \le P_R \le P_R^{max} : \mu_R^{min}, \mu_R^{max} \tag{7}$$

where λ_R is the marginal cost of the rival power producer, P_R is the power produced by the rival power producer, P_{Line} is the power flow in the Transmission line from bus 1 to bus 2, and P_{L1} and P_{L2} are constant loads at buses 1 and 2, respectively. P_{Line}^{max} is the transmission capacity of line 1-2, P_S^{max} is the upper limit of the strategic power producer and P_R^{max} is the upper limit of the rival power producer. Constraints (3) and (4) enforce power balance in the network, and constraints (5)-(7) are power bounds for their respective variables.

C. Model Conversion to MPEC

We first replace the lower-level problem by the KKT conditions and convert the bi-level problem to an MPEC problem.

The Lagrangian function of the lower-level problem is formulated as follow.

$$\mathcal{L}\ (P_S, P_R, \lambda_1, \lambda_2, \mu_S^{min}, \mu_S^{max}, \mu_R^{min}, \mu_R^{max}, \mu_{Line}) = \alpha_S P_S + \lambda_R P_R + \lambda_1 (P_{Line} + P_{L1} - P_S) + \lambda_2 (P_{L2} - P_{Line} - P_R) + \mu_S^{max}(P_S - P_S^{max}) - \mu_S^{min} P_S + \mu_{Line}(P_{Line} - P_{Line}^{max}) + \mu_R^{max}(P_R - P_R^{max}) - \mu_R^{min} P_R$$

The KKT conditions are listed as follows.

$$\alpha_S - \lambda_1 - \mu_S^{min} + \mu_S^{max} = 0 \tag{8}$$

$$\lambda_R - \lambda_2 - \mu_R^{min} + \mu_R^{max} = 0 \tag{9}$$

$$P_S - P_{L1} = P_{Line} \tag{10}$$

$$P_R - P_{L2} = -P_{Line} \tag{11}$$

$$0 \le P_{Line}^{max} - P_{Line} \perp \mu_{Line} \ge 0 \tag{12}$$

$$0 \le P_S \perp \mu_S^{min} \ge 0 \tag{13}$$

$$0 \le P_R \perp \mu_R^{min} \ge 0 \tag{14}$$

$$0 \le P_S^{max} - P_S \perp \mu_S^{max} \ge 0 \tag{15}$$

$$0 \le P_R^{max} - P_R \perp \mu_R^{max} \ge 0 \tag{16}$$

III. MILP Formulation

The MPEC problem will be formulated into an MILP problem. We carry out linearization in the objective function as well as in the constraints. The MPEC model obtained after applying the KKT conditions, includes the following nonlinearities:

1) The term $\lambda_1 P_S$ in the objective function.
2) The complementarity conditions in the constraints.

A. Bilinear term in the objective function

To find a linear expression for $\lambda_1 P_S$, we can use the strong duality condition and some of the KKT equalities. The strong duality theorem says that if a problem is convex, the objective functions of the primal and dual problems have the same value at the optimum. The complementarity conditions in the constraints can be dealt with using the slack variables and the "big-M" method. This way achieves a MILP formulation [6].

According to the strong duality theorem, at optimal point $Z_{primal} = Z_{dual}$
where $Z_{primal} = \alpha_S P_S + \lambda_R P_R$

$Z_{dual} = -\mu_{Line} P_{Line}^{max} - \mu_S^{max} P_S^{max} - \mu_R^{max} P_R^{max}$,
Considering equation (8) and multiplying it with P_S,

$$Z_{primal} = Z_{dual} \tag{17}$$

$$\alpha_S P_S - \lambda_1 P_S - \mu_S^{min} P_S + \mu_S^{max} P_S = 0 \tag{18}$$

From (13),

$$\mu_S^{min} P_S = 0 \tag{19}$$

From (15),

$$\mu_S^{max} P_S = \mu_S^{max} P_S^{max} \tag{20}$$

Substituting (18) - (20) in (17),

$$\lambda_1 P_S = -\lambda_R P_R - \mu_{Line} P_{Line}^{max} - \mu_R^{max} P_R^{max}$$

The objective function becomes,

$$\min \ \lambda_S P_S + \lambda_R P_R + \mu_{Line} P_{Line}^{max} + \mu_R^{max} P_R^{max} \tag{21}$$

The linear constraints are (8)-(11).

B. Complementary slackness

The complementary slackness conditions (12)-(16) can be converted to MILP formulations by introducing a binary variable for each complementary slackness condition.

$$0 \le P_{Line}^{max} - P_{Line} \le \omega_{max}^{Line} M \tag{22}$$

$$0 \le \mu_{Line} \le (1 - \omega_{max}^{Line}) M \tag{23}$$

$$0 \le P_S \le \omega_{min}^S M \tag{24}$$

$$0 \le \mu_S^{min} \le (1 - \omega_{min}^S) M \tag{25}$$

$$0 \le P_R \le \omega_{min}^R M \tag{26}$$

$$0 \le \mu_R^{min} \le (1 - \omega_{min}^R) M \tag{27}$$

$$\omega_{max}^{Line}, \omega_{min}^S, \omega_{min}^R \in \{0, 1\} \tag{28}$$

$$0 \le P_S^{max} - P_S \le \omega_{max}^S M \tag{29}$$

$$0 \le \mu_S^{max} \le (1 - \omega_{max}^S) M \tag{30}$$

$$0 \le P_R^{max} - P_R \le \omega_{max}^R M \tag{31}$$

$$0 \le \mu_R^{max} \le (1 - \omega_{max}^R) M \tag{32}$$

$$\omega_{max}^S, \omega_{max}^R \in \{0, 1\} \tag{33}$$

For example ω_{max}^S is introduced to indicate if the generator's upper bound is hit ($\omega_{max}^S = 0$) or not ($\omega_{max}^S = 1$). M is a big number. When the limit is hit, $0 \le P_S^{max} - P_S \le \omega_{max}^S M$ is equivalent to $P_S = P_S^{max}$. When the limit is not hit, $0 \le \mu_S^{max} \le (1 - \omega_{max}^S) M$ is equivalent to $\mu_S^{max} = 0$.

The model is now a mixed integer linear program (MILP), with (21) as objective function, and (8)-(11), (22)-(33) as constraints.

C. Chance Constraints

In this subsection, we consider the uncertainty in λ_R, the guessed marginal cost of a rival generator. Two types of chance constraints will be examined. The first chance constraint is as follows. Given the randomness of the marginal cost of Gen R, we would like to obtain the bidding price and the bidding power that can represent more than $1-\epsilon$ chance. This condition can be represented by chance constraints. The representation is not straightforward, though. The second chance constraint is more straightforward. Given the randomness of the marginal cost of Gen R, we would like to obtain the bidding power that will be less than a certain value, say 22 MW with a chance greater than or equal to $1 - \epsilon$.

Both types of chance constraints can be modeled in MILP formulation by enumerating scenarios and introducing a binary variable φ_υ [10] . $\varphi_\upsilon = 0$ means that Scenario υ is included in the representing scenarios. $\varphi_\upsilon = 1$ means that this scenario is not a representative scenario that can make the chance constraint be satisfied.

Case I:
In Case 1, we use the chance constraint to represent the probability of the solution of the bidding strategy is more than $1 - \epsilon$. Since this probability is difficult to be written in a mathematical format, we proceed to explain the MILP formulation.

We define auxiliary variables $P_{S\upsilon}^C$ for $P_{S\upsilon}$, $P_{R\upsilon}^C$ for $P_{R\upsilon}$, $\mu_{Line,\upsilon}^C$ for $\mu_{Line,\upsilon}$ and $\mu_{R\upsilon}^{maxC}$ for $\mu_{R\upsilon}^{max}$ in chance constrained mathematical program (CCMP). The objective function (21) is now replaced by the following objective function.

$$\begin{aligned} \min \quad & \sum_{\upsilon} \pi_\upsilon (\lambda_S P_{S\upsilon}^C + \lambda_R P_{R\upsilon}^C + \mu_{Line\upsilon}^C P_{Line\upsilon}^{max} \\ & + \mu_{R\upsilon}^{maxC} P_{R\upsilon}^{max}) \end{aligned} \tag{34}$$

where π_υ is the probability of Scenario υ is happening.

Note that this objective function already considers enumeration of all scenarios due to the uncertainty of λ_R. The uncertainty of λ_R is represented by many scenarios, each with a probability.

The auxiliary variables will be 0 if this particular scenario is not a representing scenario. The auxiliary variables will be the same as the variables if the particular scenario is a representing scenario. This condition can then be expressed by MILP formulations with a binary variable φ_υ introduced.

By using a binary controller variable and "Big-M" coefficient, the chance constrained problem can be formulated with a set of linear constraints as shown in the following where M is a sufficiently large number.

$$\sum_{\upsilon \in \Upsilon} \pi_\upsilon \varphi_\upsilon \le \epsilon \tag{35}$$

$$P_{S\upsilon}^C \ge P_{S\upsilon} - \varphi_\upsilon M \tag{36}$$

$$P_{R\upsilon}^C \ge P_{R\upsilon} - \varphi_\upsilon M \tag{37}$$

$$\mu_{Line\upsilon}^C \ge \mu_{Line\upsilon} - \varphi_\upsilon M \tag{38}$$

$$\mu_{R\upsilon}^{maxC} \ge \mu_{R\upsilon}^{max} - \varphi_\upsilon M \tag{39}$$

$$P_{S\upsilon}^C, P_{R\upsilon}^C, \mu_{Line\upsilon}^C, \mu_{R\upsilon}^{maxC} \ge 0 \tag{40}$$

$$\varphi_\upsilon \in \{0, 1\} \tag{41}$$

(35) represents the total probability of the non-representing scenarios (when $\varphi_\upsilon = 1$) should be less than ϵ. (36) -(39) represent the relationships between the auxiliary variables and the variables. For example, when $\varphi_\upsilon = 1$, or Scenario υ is not a representing one, then $P_{S\upsilon}^C$ should be greater than a large negative number. Since $P_{S\upsilon} \ge 0$, this constraint (36)

is basically a relaxed one without imposing any constraint. Due to the minimization problem's objective function which includes a term $\lambda_S P_{S\upsilon}^C$, $P_{S\upsilon}^C$ will be 0.

Case II:

For the second type of chance constraint, it is easy to write a mathematical expression as:

$$\text{Probability}\left\{\ P_S(\upsilon) \le P_S^0\ \right\} \ge 1-\epsilon$$

where P_S^0 is a given value.

This constraint can be expressed in MILP formulation by enumeration of scenarios and introducing the binary variable φ_υ.

$$\sum_{\upsilon\in\Upsilon} \pi_\upsilon \varphi_\upsilon \le \epsilon \tag{42}$$

$$P_{S\upsilon} + \varphi_\upsilon M \le P_S^0 \tag{43}$$

$$\varphi_\upsilon \in \{0,1\} \tag{44}$$

For this type of chance constraint, there is no need to introduce auxiliary variables.

Hence, our model for the mixed integer linear programing model with chance constrained mathematical program has its objective function (34) and constraints (8)-(11),(22)-(33) for every scenario υ, and the additional constraints (35)-(41) or (42-44) related to chance constraints.

IV. Illustrative Examples

The MILP problems are solved using GUROBI 6.5 [11] interfaced with Python 2.7 on an Intel(R) Core(TM) i7-6700 with processors at 3.4 GHz and 16 GB RAM. The results have also been verified using SCIP version 3.2.1 optimizer [12].

A. Two-generator system: Case I

Using the parameters given in Table I, we will solve our model example described in Fig.1, first with CCMP and then without CCMP. The probabilities and the corresponding values for different scenarios are generated using the command uniform in python. Fig. 2 presents the seven scenarios with different probabilities. The model is solved for a time horizon of 24 hours, considering different risk tolerance levels, and the results are tabulated in Table II and Table III.

TABLE I
Parameters for two bus network shown in Fig.1.

λ_s ($/MWh$)	P_S^{max} (MW)	P_R^{max} (MW)
12	22.8	22.8

TABLE II
Result table for the solution of illustrative example case I

ϵ(%)	0%	8%	20%	30%	50%
profit ($)	7094	7318	7676	7997	8675

Table II shows that if $\epsilon = 0$, that is, all seven scenarios are included as the representative scenarios, the total profit is $7094. With ϵ increasing, that is, the chance of $1-\epsilon$ decreasing, the worst scenarios will be excluded, which leads to the increase of the profit.

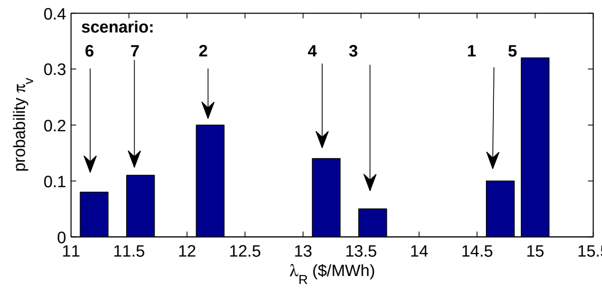

Fig. 2. Generated λ_R ($/MWh) values for different scenarios.

TABLE III
Scenario selection. 1.0 stands for not selected

ϵ	υ_1	υ_2	υ_3	υ_4	υ_5	υ_6	υ_7
0.00	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.05	0.0	0.0	**1.0**	0.0	0.0	0.0	0.0
0.08	0.0	0.0	0.0	0.0	0.0	**1.0**	0.0
0.20	0.0	**1.0**	0.0	0.0	0.0	0.0	0.0
0.30	**1.0**	**1.0**	0.0	0.0	0.0	0.0	0.0
0.50	**1.0**	0.0	0.0	0.0	**1.0**	**1.0**	0.0

The details on scenario selection is presented in Table III. For example, where $\epsilon = 0$, all scenarios are selected. This problem is same as a stochastic programming problem. When $\epsilon = 0.08$, scenario 6 with $\lambda_R = 11.2$ ($/MWh) is not selected. This exclusion leads to an increase of the rival's marginal cost on average. Therefore, the strategic power producer's benefit will be improved.

B. Two-Generator System: Case II

Using the parameters given in Table IV, we will solve our model example described in Fig.1. The probabilities and the corresponding values for different scenarios are used from Fig. 2. The model is solved for a time horizon of 24 hours, considering different risk tolerance levels, and the results are tabulated in Table V. Increasing ϵ means relaxing the constraint of $P_S \le P_S^0$. Therefore, the profit increases with ϵ increasing.

TABLE IV
Parameters for two bus network shown in Fig.1.

λ_s ($/MWh$)	P_S^{max} (MW)	P_R^{max} (MW)	P_0^S (MW)
12	40	40	30

TABLE V
Result table for the solution of illustrative example case II

ϵ(%)	0%	8%	20%	30%	50%
profit ($)	7348	7571	7929	8277	8950

Table VI presents the computed $P_{S\upsilon}$ values and Table VII presents the selection of the representative scenarios. Table

TABLE VI
P_{Sv} IN EVERY SCENARIO FOR CASE II.

ϵ	v_1	v_2	v_3	v_4	v_5	v_6	v_7
0.00	30.0	30.0	30.0	30.0	30.0	25.0	25.0
0.08	30.0	30.0	30.0	30.0	30.0	25.0	25.0
0.20	**35.0**	30.0	30.0	30.0	30.0	30.0	30.0
0.30	**35.0**	**35.0**	30.0	30.0	30.0	30.0	30.0
0.50	**35.0**	**35.0**	**35.0**	**35.0**	30.0	30.0	30.0

TABLE VII
SCENARIO SELECTION. 1.0 STANDS FOR NOT SELECTED

ϵ	v_1	v_2	v_3	v_4	v_5	v_6	v_7
0.00	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.08	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.20	**1.0**	0.0	0.0	0.0	0.0	0.0	0.0
0.30	**1.0**	**1.0**	0.0	0.0	0.0	0.0	0.0
0.50	**1.0**	**1.0**	**1.0**	**1.0**	0.0	0.0	0.0

VII can be compared with Table VI. When $P_{Sv} > 30$, this scenario will not be selected.

C. Six-bus system 24-hour example

This subsection presents the implementation of the model to network based on the description given in [6]. The model including the CCMP type 1 chance constraints is applied to the network, considering the extensions such as, dc network model, unit ramp up and ramp down limits, different consumer bids in different hours of the day and susceptance of the transmission lines. We consider seven scenarios with different probabilities. The scenarios differ on rival producers offer λ_{Rv}, and on consumer bids λ_{Liv} for all i related to load buses. We can generate a set of scenarios by multiplying the above two terms given in [2] by the entries of vector [1.15, 1.1, 1.092, 1, 1.05, 0.89, 0.69]. The corresponding probabilities are given as [0.1, 0.2, 0.3, 0.1, 0.1, 0.1, 0.1].

The average LMP for different hours is plotted in Fig. 3.

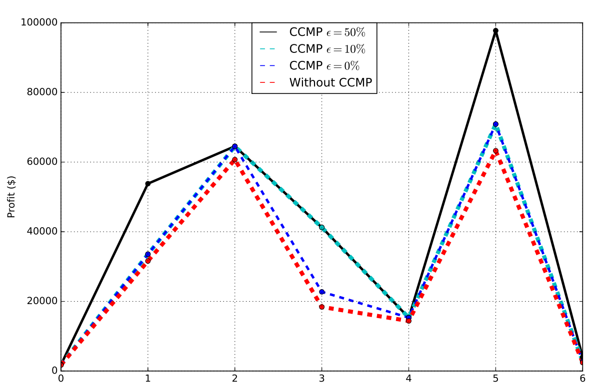

Fig. 3. A comparison of profit contribution versus scenario for different risk tolerance levels

V. CONCLUSION

This paper presents a mixed integer linear programming (MILP) formulation for a bi-level mathematical program with equilibrium constraints (MPEC) considering chance constraints. The particular MPEC problem relates to a power producer's bidding strategy: maximize its total benefit through determining bidding price and bidding power output while considering an electricity pool's operation and guessing the rival producer's bidding price. The entire decision-making process can be described by a bi-level optimization problem. The contribution of our paper is the MILP formulation of this problem. First, the lower-level pool operation problem is replaced by Karush-Kuhn-Tucker (KKT) optimality condition, which is further converted to an MILP formulation except a bilinear item in the objective function. Secondly, duality theory is implemented to replace the bilinear item by linear items. Finally, two types of chance constraints are examined and modeled in MILP formulation. With the MILP formulation, the entire MPEC problem considering randomness in price guessing can be solved using off-shelf MIP solvers, e.g., Gurobi. An example is given to illustrate the formulation and show the case study results.

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