Achieving Economic Operation and Secondary Frequency Regulation Simultaneously Through Feedback Control

Zhixin Miao, *Senior Member, IEEE,* Lingling Fan, *Senior Member, IEEE*

***Abstract*—This article presents an important finding: the parameters of secondary frequency control based on integral or proportional integral control can be tuned to achieve economic operation and frequency regulation simultaneously. We show that if the power imbalance is represented by frequency deviation, an iterative dual ascent based economic dispatch solving is equivalent to integral control. An iterative method of multipliers based economic dispatch is equivalent to proportional integral control. Similarly, if the controller parameters of the secondary frequency controls are chosen based on generator cost functions, these secondary frequency controllers achieve both economic operation and frequency regulation simultaneously.**

***Index Terms*—Economic dispatch; dual decomposition; method of multipliers; secondary frequency control**

I. Introduction

Suppose the system has a load increase, due to primary frequency control, the system will show frequency deviation. Moreover, at steady state, power imbalance and the frequency deviation are linearly related. Conventional centralized secondary frequency control will respond and change the turbine governor set points to bring frequency back to 60 Hz. Each generator has its share based on participation factors [1]. In this paper assume that each generator is equipped with a secondary frequency controller. Then the gains of the integral control used in the secondary frequency control decide which generator increases more and which increases less. Intuitively, we may let the gain of a generator with cheap cost greater. Our paper shows that this point can be proved theoretically using iterative optimization solving procedure, e.g., dual ascent and method of multipliers.

We will start with a two-generator system to explain our finding. In Section II, dual decomposition based iterative economic dispatch problem is presented. The corresponding continuous dynamic model is then derived. In Section III, method of multipliers based iterative approach is presented. The continuous dynamic model is again derived. We show that if the power imbalance can be represented by frequency deviation, the former approach is similar to integral control and the latter approach is similar to proportional integral control.

This finding has a *important* meaning for the power industry. The parameters of the feedback control for each generator (input: frequency deviation, output: turbine-governor's power reference) can be selected according to the quadratic coefficients of the cost functions. This selection will lead to frequency regulation and economic operation *simultaneously*.

Z. Miao and L. Fan are with Department of Electrical Engineering at University of South Florida, Tampa, FL 33620 (Emails: zmiao@usf.edu, linglingfan@usf.edu).

II. Dual Decomposition Based Iterative Economic Dispatch

For a two-area system, each area with a generator (dispatch level P_i) and a load (D_i), the economic dispatch problem is expressed as follows.

$$\min_{P_1,P_2} \; C_1(P_1)+C_2(P_2) \quad \text{s.t.: } P_1+P_2=D_1+D_2. \tag{1}$$

where $C_i(P_i)=a_iP_i^2+b_iP_i+C_i$ is the cost function.

The dual problem is as follows.

$$\max_{\lambda}\left(\min_{P_1} C_1(P_1)+\lambda(D_1-P_1)+\min_{P_2} C_2(P_2)+\lambda(D_2-P_2)\right)$$

For a given λ, Area 1 and Area 2 can carry out minimization problems separately. The price λ is then updated to maximize the objective function of the dual problem. In addition, the power imbalance will be reflected by the frequency deviation assuming that there exists primary frequency control and the system reaches steady-state. This second assumption is reasonable should the secondary frequency control is designed with a much lower bandwidth than that of the rest system dynamics. The iterative procedure of λ update is as follows.

$$\lambda^{k+1}=\lambda^k+\alpha(D_1+D_2-P_1^k-P_2^k) \tag{2}$$

where $\alpha>0$ is the step size. When primary frequency control is considered, the power imbalance is linearly related with frequency deviation $\Delta P=-\sum_i \frac{1}{R_i}\Delta f$, where R_i is the droop control parameter. Therefore, $\lambda^{k+1}=\lambda^k-K\Delta f^k$, where $K=-\alpha\sum_i\frac{1}{R_i}$. K is related to the step size α and system characteristics. For dual ascent, the step size has a limit to have converged solution. Therefore K has a limit. In addition, when power system dynamics are considered, additional constraints may be posed on K.

Ignoring the generator limits, the marginal costs of the generators should equal to the price at each step.

$$\lambda^k=2a_1P_1^k+b_1=2a_2P_2^k+b_2. \tag{3}$$

Therefore, the iteration for the power commands that will be sent to the turbine governors are:

$$P_i^{k+1}=P_i^k-\frac{K}{2a_i}\Delta f^k, i=1,2 \tag{4}$$

The continuous dynamic model of the above procedure can be obtained using Forward Euler approximation for derivatives.

$$\tau \dot{P}_i = -\frac{K}{2a_i}\Delta f, i = 1, 2 \tag{5}$$

where τ is the step size of the discrete iteration.

Remarks: The continuous dynamic model not only indicates that dual decomposition-based economic dispatch is equivalent to an integrator in secondary frequency control, but also indicates that if the gains of the integral controllers for generators are inversely proportional to the quadratic coefficients of the generators' cost functions, the local feedback control can realize economic dispatch and frequency regulation simultaneously.

III. Method of Multipliers Based Iterative Economic Dispatch

In method of multipliers, an additional term related to an equality constraint is added to the objective function. The advantage of method of multipliers is to achieve faster convergence compared to dual ascent method [2]. The modified economic dispatch problem now becomes:

$$\begin{aligned} &\min \;\; C_1(P_1) + C_2(P_2) + \frac{\rho}{2}(P_1 + P_2 - D_1 - D_2)^2 \\ &\text{subject to: } P_1 + P_2 = D_1 + D_2. \end{aligned} \tag{6}$$

where $\rho > 0$ and ρ is the penalty parameter.

Again, the power imbalance can be reflected by frequency deviation. Applying dual ascent, the λ update procedure now becomes

$$\lambda^{k+1} = \lambda^k + \rho(D_1 + D_2 - P_1^k - P_2^k) = \lambda^k - K\Delta f^k. \tag{7}$$

Note that in the update procedure, a specific step size ρ is used. Use of ρ as the step size guarantees that the iterate is dual feasible for the original dispatch problem. This feature makes the method of multiplier having improved convergence properties.

For a given λ^k, P_1^k and P_2^k should minimize the following objective function:

$$\begin{aligned} L_\rho(P_1, P_2) = C_1(P_1) + C_2(P_2) + \frac{\rho}{2}(P_1 + P_2 - D_1 - D_2)^2 \\ + \lambda^k(D_1 + D_2 - P_1 - P_2). \end{aligned} \tag{8}$$

The arguments that minimize $L_\rho(P_1, P_2)$ can be found by setting the gradients related to P_1 and P_2 to zeros.

$$\frac{\partial L_\rho}{\partial P_i} = 0 \tag{9}$$

The above condition implicates the following relationship of P_1^k, P_2^k versus λ^k.

$$\lambda^k = 2a_i P_i^k + b_i - \rho(D_1 + D_2 - P_1^k - P_2^k) \tag{10}$$

$$\lambda^k = 2a_i P_i^k + b_i + K\Delta f^k \tag{11}$$

The iteration for the power commands can be found by replacing λ^{k+1} and λ^k in (7) by (11).

$$2a_i(P_i^{k+1} - P_1^k) + K(\Delta f^{k+1} - \Delta f^k) = -K\Delta f^k$$

Using the Forward Euler approximation for derivatives, the continuous dynamic model is the found to be:

$$2a_i \tau \dot{P}_i + K\tau \dot{\Delta f} = -K\Delta f$$

The transfer functions for the above model are as follows.

$$\frac{P_i}{\Delta f} = \frac{-K}{2a_i}\left(1 + \frac{1}{\tau s}\right) \tag{12}$$

Remarks: The continuous dynamic model not only indicates that method of multipliers-based economic dispatch is equivalent to a proportional integral (PI) controller, but also indicates that if the gains of the PI controllers are inversely proportional to the quadratic coefficients, the local feedback control can realize economic dispatch and frequency regulation simultaneously.

The two dynamic models also indicate that method of multiplier-based iteration can achieve faster convergence compared to dual decomposition-based iteration. The former is viewed as a PI control while the latter is viewed as an integral controller. The PI controller should lead to faster response than the integral controller.

In practical applications, once the set points of turbines are fixed at the initial hour according to economic dispatch, for the rest time, the generators can automatically take care of load increase and decrease with their secondary frequency control. This is all done by local feedback control. Using the proposed method, economic dispatch can be realized along with frequency regulation within hundreds of seconds.

Our paper shows the connection between dual ascent, the method of multipliers and feedback control. The work has significant meaning due to the following aspects: (i) from the methodology point of view, it bridges the gap between optimization and feedback control; (ii) from the application point of view, it shows that decentralized feedback control can realize an optimal goal. Utilities can set their secondary frequency control parameters and realize economic dispatch along with frequency regulation.

IV. Conclusion

In this article, we claimed findings of important practical value for the power industry. The finding is regarding secondary frequency control: the parameters of the controls can be set based on generators' cost functions to realize economic operation. We discovered the findings by examining the approximate continuous models for two iterative approaches: the method of multipliers and dual decomposition. The former is equivalent to PI control while the latter is equivalent to integral control.

References

[1] A. J. Wood and B. F. Wollenberg, *Power generation, operation, and control*. John Wiley & Sons, 2012.

[2] S. Boyd, N. Parikh, E. Chu, B. Peleato, and J. Eckstein, "Distributed optimization and statistical learning via the alternating direction method of multipliers," *Foundations and Trends® in Machine Learning*, vol. 3, no. 1, pp. 1–122, 2011.