Benders Decomposition for Stochastic Programming-Based PV/Battery/HVAC Planning

Mohemmed Alhaider
Department of Electrical Engineering
University of South Florida
Tampa, FL 33620
Email:malhaide@mail.usf.edu

Lingling Fan
Department of Electrical Engineering
University of South Florida
Tampa, FL 33620
Email: linglingfan@usf.edu

Zhixin Miao
Department of Electrical Engineering
University of South Florida
Tampa, FL 33620
Email: zmiao@usf.edu

***Abstract*—In this article, a general Benders Decomposition is applied to solve a stochastic mixed integer programming formulation (SMIP) to obtain the optimal sizing of a photovoltaic system (PV) and battery energy storage system (BESS) to power a Residential Heating Ventilation and Air- Conditioning System (HVAC). The uncertainty of PV-output is stochastically modeled using different scenarios with the probability of their occurring. The total cost of the HVAC energy consumption and installing PVs and BESS is to be minimized cosidering the system is grid-connected and electricity price is varying. A simplified model of a space cooling is used while considering the thermal energy constraints. The optimization problem extends to find the optimal HVAC on/off states, and BESS charging- discharging states for a multi-horizon period.**

***Index Terms*—Bender Decomposition, HVACs, SMIP**

I. Introduction

A. Engineering problem

Demand side management (DSM) that can provide fast peak shaving and valley filling is receiving more attention than ever. A residential heating, ventilation and Air-conditioning (HVAC) system is important to fulfill the role of DSM. In most places, peak loads occur during the day with high temperature. HVAC loads account for a large portion of the peak loads that could reach to 40% - 60% of the loads in commercial and residential buildings [1]. Also during the day time, there is much of unexploited solar energy that could be utilized to enhance DSM functions. Therefore, it is essential to use Photovoltaic cells (PV) to harvest the sun energy and convert it to electricity that can power the HVAC units. The utilities may have some agreements with the consumers to shift part of their HVAC loads during peak loads hours. These agreements provide the customer with economic incentives and prevent the need for additional conventional generation during these peak hours. For example, NVEnergys cool share program is associated with approximately 145 MW remote controlled air conditioning loads [2]. These loads can be dispatched by NVEnergy through "raise/lower" thermostat commands as needed. A smart strategy is required to implement the load shifting and reduce the mismatch between the renewable generation and HVAC. However, the stochastic nature of intermittent PV output complicates the integration of operation and planning purposes. A hybrid power system composed of energy storage and renewable generation can alleviate the issues associated with the renewable power supply fluctuation. Storage of the renewable power generation in excess of the load makes it accessible for later release when the renewable generation is insufficient to supply the load. This paper deals with planning and operation of hybrid solar/storage/HVAC systems.

B. Stochastic programming

Addressing the issues of uncertainty of renewable energy has been studied in the literature, e.g., [3]–[6]. In the aforementioned papers, optimal design and operation of wind/solar hybrid systems is discussed. In [7], a hybrid system that combines renewable energy generation and energy storage system to meet a controllable HVAC load is studied. To address uncertainty, stochastic programming problems are usually formulated where numerous scenarios with probability are created. This results in large-scale optimization problems. To complicate the matter, when optimal sizing and HVAC on/off are considered, integer variables are introduced. This makes problem solving a challenging issue. Prior research related to mixed integer stochastic programming problems can be found in [8]–[12]. In [8], two-stage stochastic programming is used to deal with scheduling problems of chemical batch processes. In [9], two-stage stochastic programming is used to develop offering strategies for wind power production while considering the uncertainty in wind power and market prices. In [13], two-stage stochastic mixed-integer linear programming is used to design time-of-use (TOU) rates to deal with demand response options. Battery scheduling problems can also be found in the literature, namely in [10], where a deterministic MILP program is developed to schedule battery charging by a set of PV arrays on a space station. In [11], the collective discharge scheduling problem is addressed using a decision-making algorithm, while in [12] charge and discharge strategies are used to study the sensitivity of electric vehicle battery economics.

C. Large-scale problem solving techniques and Benders decomposition

Commercial solvers such as CPELX or Gurobi are usually adopted. However, very long computing time is expected. For example, in [6], the model was run using 4 parallel CPU threads on a 256 GB RAM server running GAMS 23.0.2 and CPLEX 11.2.1 and the maximum execution time is 10 h. The second option is heuristic methods. Heuristic methods, e.g., genetic algorithm [7], has the similar scalability issues. The third option is to use commercial solvers through customized algorithm. Benders decomposition is a known efficient algorithm to handle large-scale mixed-integer problems. Benders decomposition has been applied in many power system applications, e.g., unit commitment problems considering wind [14] or transmission constraints [15], transmission planning considering wind [16]. The essential technique is to decompose decision variables into multiple sets [17], e.g., a set of mixed integer variables and a set of continuous variables. For each set of variables, optimization solving will be conducted. In this paper, Benders decomposition technique will be adopted for planning and operation of PV/battery/HVAC hybrid systems. A program has been developed and implemented in MATLAB and solved using the CVX solver package and CPELX. The numerical simulation has been performed on a 3.4-GHz based processor with 8GB of RAM and the maximum execution time is 35 minutes. On the other hand, CVX keeps running after 24 hours for the original problem without decomposition.

D. Contribution of this paper

This work is extension to our previous work [18] where MIP optimization problems are formulated to investigate the potential benefits of using HVACs in demand response applications. Here, this work proposes an HVAC system powered by PV-battery system. A stochastic linear programming formulation is developed to find the optimal size of the PV-battery system to meet HVAC loads. This model finds the optimal operating schedule of HVAC units (on/off states) and the optimal charging-discharging schedule of the PV-battery system. This model is investigated under the uncertainty of the PV-output considering the grid-connected mode while the electricity price is varying. The main contribution of this paper is modeling the uncertainty of solar energy and using Benders decomposition to investigate the potential benefits of PV-BESS with HVAC loads. This formulation has shown a great ability to deal with big number of scenarios which cannot be handled by commercial solvers without applying decomposition methods.

II. Thermal Dynamics Models of an HVAC Unit

The equivalent thermal parameters model of a residential HVAC is shown below in Fig. 1 [19], where Q denotes heat rate for HVAC unit (Btu/hr or W);
UA denotes standby heat loss coefficient (Btu/^{o}F.hr or $W/^o$C);
R_1 denotes 1/UA;
R_2 denotes 1/UA_{mass};
T_o denotes ambient temperature (^{o}F or^{o}C);
T_i denotes air temperature inside the house (^{o}F or ^{o}C);
T_m denotes mass temperature inside the house (^{o}F or ^{o}C);
C_a denotes air heat capacity (Btu/^{o}F or $J/^o$C);
C_m denotes mass heat capacity (Btu/^{o}F or $J/^o$C);

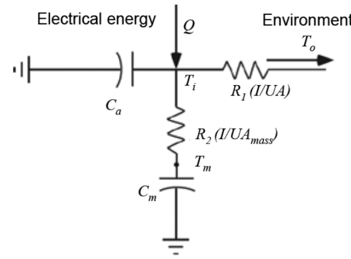


Fig. 1: The HVAC model.

The state space description of the ETP model is described as follows.

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du \quad (1)$$

where $x = \begin{bmatrix} T_i & T_m \end{bmatrix}^T$,

$$A = \begin{bmatrix} -(\frac{1}{R_2C_a} + \frac{1}{R_1C_a}) & \frac{1}{R_2C_a} \\ \frac{1}{R_2C_m} & -(\frac{1}{R_2C_m}) \end{bmatrix},$$
$$B = \begin{bmatrix} \frac{T_0}{R_1C_a} + \frac{Q}{C_a} \\ 0 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad D = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Based on the above state-space model, when the cooler is turned OFF, the room temperature at time is described by

$$T^{t+1}_{\text{room}} = T^{t+1}_o - (T^{t+1}_o - T^t_{\text{room}})e^{-\Delta t/RC}. \quad (2)$$

When the cooler is turned ON, the room temperature at time is described by

$$T^{t+1}_{\text{room}} = T^{t+1}_o + QR - (T^{t+1}_o + QR - T^t_{\text{room}})e^{-\Delta t/RC} \quad (3)$$

where:
T_{room} denotes room temperature (^{o}F or ^{o}C)
C denotes equivalent heat capacity (Btu/^{o}F)
R denotes equivalent thermal resistance (^{o}C/W)
Q denotes equivalent heat rate (W)
t denotes air temperature inside the house (minute)
Δt refers time step (1 minute)
The above two equations will be combined to one thermal constraint through the introduction of a binary variable W^t (1: the cooler is on. 0: the cooler is off).

$$T^{t+1}_{\text{room}} = T^{t+1}_o + W^tQR - (T^{t+1}_o + W^tQR - T^t_{\text{room}})e^{-\frac{\Delta t}{RC}} \quad (4)$$

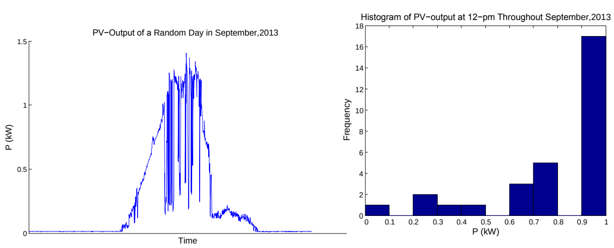
III. Optimization Problem Formulation

A. Treatment of Uncertainty in PV-Output

Scenarios of PV-output are developed based on real data collected from PV panels that are installed on campus of University of South Florida at Saint Petersburg. PV-output of a random day is shown in Fig. 2a. After a quick observation, one can notice that there are extreme power inconsistencies which are due to the weather uncertainty. Therefore, the solar power collected at each hour span is averaged to have more consistent power during each hour. Then, statistical analysis is followed by discretizing the solar power into different levels where the obtained average of PV-output would fall in. Now, throughout the whole month, for each hour the frequency of each level would be counted. Fig.2b shows an example of the histogram for hour 12-pm. From this histogram, we can create our scenarios based on the desired number of scenarios we would consider. Here, we would consider creating three scenarios. Then, 60% of PV-output occurred between 0.9 and 1kW, 25% occurred between 0.6 and 0.8kW, and 15% occurred between 0.2 and 0.5kW. This method is applied to the rest hours and three different PV-output at each hour with its probability of occurring are developed and Table.I follow the same pattern in creating three different output at each hour.

TABLE I: Different expected PV-output of a random day

Hour	10 am	11 am	12 pm	1 pm	2 pm	3 pm	4pm	5pm
1st event	0.90	0.80	0.95	1.20	1.30	0.90	0.80	0.70
Probability	0.75	0.73	0.64	0.71	0.70	0.60	0.75	0.72
2nd event	0.70	0.50	0.80	0.90	1.00	0.60	0.60	0.40
Probability	0.15	0.18	0.24	0.16	0.19	0.35	0.15	0.21
3rd event	0.30	0.40	0.20	0.70	0.80	0.40	0.30	0.20
Probability	0.10	0.09	0.12	0.13	0.11	0.05	0.10	0.90



(a) PV-output of a random day. (b) Histogram of PV-output at 12 pm.

Fig. 2: PV power characteristics.

The total number of scenarios that can be created from Table.I are 3^8 which is 6561 scenario. However, the probability that most of these scenarios occur is very small. To reduce the computational burden, the tree scenario is created and the probability of each scenario occur is calculated. Then, the most likely 600 scenarios to occur are considered in our analysis. The total probability of those 600 scenarios to occur is more than 88% and the rest 12% is divided proportionally between them.

B. Optimization Model

The optimization model here is developed based on the assumption that there is a battery energy storage system (BESS) which is installed and can be charged and store the energy from the grid, and can be discharged and supply the HVAC with power.

C. Decision Variables

$$X = [C_b \;\; E_b \;\; N_{pv} \;\; P_{in}^{s,j} \;\; P_b^{s,j} \;\; W_k^j] \tag{5}$$

where:

j denotes jth period, $j \in J = \{1, 2, \cdots, 96\}$. 24-hour is considered with each period 15 minutes.

k denotes kth No. of HVAC unit. $k \in K = 1, 2, 3;$

s denotes sth scenario. $s \in S$, where S contains 600 scenarios in the case study.

C_b denotes the power rating of the battery energy system.

E_b denotes the energy rating of the battery energy system.

N_{pv} denotes the power rating of the battery energy system.

$P_{in}^{s,j}$ denotes purchased power at the jth period of the sth scenario.

W_k^j denotes a binary variable that is equal to 1 if the HVAC kth is on at the jth period of the sth scenario and 0 otherwise.

$P_b^{s,j}$ denotes the battery power at jth period of the sth scenario.

D. Objective Function

$$\min \left\{ \beta_1 C_b + \beta_2 E_b + \alpha N_{pv} + \sum_{s=1}^{S} \rho_s \left(\lambda^j P_{in}^{s,j} \right) \right\} \tag{6}$$

where

β_1 denotes the cost of 1 kW rating of the BESS.

β_2 denotes the cost of 1 kWh rating of the BESS.

α denotes the cost of installation of PV panel.

ρ_s denotes the probability of s-th scenario

λ^j denotes the energy price at the jth period

E. Constraints

Constraint set (7) describes the room temperature dynamics and ensures that thermostat setting is enabled where the temperature of the room must be greater than the minimum temperature and less than the maximum temperature. This set does not contain probability scenarios since the HVAC operation is assumed to be deterministic for the possible scenarios.

$$T_k^{j+1} = T_o^{j+1} + W_k^j QR - (T_o^{j+1} + W_k^j QR - T_k^j) e^{\frac{-\Delta t}{RC}}$$
$$T_{min} \leq T_k^{j+1} \leq T_{max} \qquad \forall j \in J \tag{7}$$

Constraint set (9) guarantees that there is enough available power when the HVAC unit is turned on. It also ensures that the purchased power is within the power limits.

$$P_{in}^{s,j} + P_b^{s,j} + N_{pv} P_{PV}^{s,j} \geq \sum_{i=1}^{k} W_k^j P_{ac,k} \tag{8}$$

$$P_{min} \leq P_{in}^{s,j} \leq P_{max} \qquad \forall j \in J, \forall k \in K, \forall s \in S \tag{9}$$

Constraint set (11) makes sure that at any hour and any scenario, the battery charging or discharging does not exceeding the battery power rating and the energy at any hour will not exceed the energy limits.

$$-C_b \leq P_b^{s,j} \leq C_b \;\; \forall j \in J, \forall s \in S \tag{10}$$

$$\underline{E_b} \leq E_0 + \sum_{i=1}^{j} P_b^{s,i} \leq \overline{E_b} \;\; \forall j \in J, \forall s \in S \tag{11}$$

where:

C_b denotes the power rate of the battery energy system.

E_b denotes the energy rate of the battery energy system.

$\underline{E_b}$ and $\overline{E_b}$ refer to the minimum and the maximum energy limit of the battery unit respectively.

IV. Large-scale Problem Solving Using Benders Decomposition

The main philosophy is to separate the decision variables into two sets: those related to HVAC, and those not related to HVAC. The first set includes integer variables W which denotes HVAC on/off, number of PV and room temperatures. The second set includes the rest of variables including battery power dispatch level, battery power and energy ratings. The second set variables are all continuous.

a) Step.1: In this step, we assume that the power and energy ratings of a battery, power demand are all zero. Only the HVAC operation is considered. Form the master problem and we find its optimal solution as follows.

$$\min_{N_{pv},W_k^j} \quad Z_{lower} \tag{12}$$

$$\text{subject to } Z_{lower} \geq \alpha N_{pv} \tag{13}$$

$$T_k^{j+1} = T_o^{j+1} + W_k^j QR - (T_o^{j+1} W_k^j QR - T_k^j) e^{\frac{-\Delta t}{RC}} \tag{14}$$

$$T_{min} \leq T_k^{j+1} \leq T_{max} \ \ \forall j \in J, k \in K \tag{15}$$

b) Step.2: Form the subproblem and use the obtained optimal solution for the integer variables from the master problem ($\hat{N}_{pv}$,$\hat{W}_j^k$).

$$\min_{\substack{C_b,E_b \\ P_{in}^{s,j} P_b^{s,j}}} \left\{ \beta_1 C_b + \beta_2 E_b + \sum_{s=1}^{S} \rho_s \left(\lambda^j P_{in}^{s,j} \right) \right\} \tag{16}$$

$$\text{s.t.} \quad P_{in}^{s,j} + P_b^{s,j} + \hat{N}_{pv} P_{PV}^{s,j} \geq \sum_k \hat{W}_k^j P_{ac,k} \tag{17}$$

$$P_{min} \leq P_{in}^{s,j} \leq P_{max} \tag{18}$$

$$\underline{E_b} \leq E_0 + \sum_{i=1}^{j} P_b^{s,i} \leq \overline{E_b} \tag{19}$$

$$-C_b \leq P_b^{s,j} \leq \ C_b \quad \forall s \in S, j \in J, k \in K \tag{20}$$

In this step, the subproblem could be infeasible when the available power cannot meet the total demand. In this case, a feasibility cut is generated and added to the master problem as follow:

$$\min_{Y,P_{in}^{s,j},P_b^{s,j}} \quad \mathbf{1}^T Y \tag{21}$$

$$\text{s.t. } P_{in}^{s,j} + P_b^{s,j} + Y^{s,j} \geq \sum_k \hat{W}_k^j P_{ac,k} - \hat{N}_{pv} P_{PV}^{s,j}$$

After generating a feasibility cut, $u_{r,j}^s$, the dual associated with this problem is taken. The general form of this cut is:

$$\sum_{s=1}^{S} \sum_j u_{r,j}^s \sum_k W_k^j P_{ac,k} - \sum_s \sum_j u_{r,j}^s P_{PV}^{s,j} N_{pv} \leq 0$$

In the case where the problem is feasible, then Z_{upper} is calculated and the convergence behavior is tested. If the convergence is not approached, then an optimality cut is generated and added to the master problem.

$$Z_{upper} = \left\{ \beta_1 \hat{C}_b + \beta_2 \hat{E}_b + \alpha \hat{N}_{pv} + \sum_{s=1}^{S} \rho_s \left(\lambda^j \hat{P}_{in}^{s,j} \right) \right\}$$

We assume that $u_{p,j}^s$ are the dual variables associated with constraints (17). The optimality cut is formed as the following

$$Z_{lower} \geq \alpha N_{pv} + \sum_{s=1}^{S} \sum_{j=1}^{J} u_{p,j}^s P_{PV}^{s,j} N_{pv} + \sum_{s=1}^{S} \sum_{j=1}^{J} u_{p,j}^s \sum_k W_k^j P_{ac,k}$$

c) Step.3: The master problem with the added constraints from step.2 will be solved and Form the subproblem and use the obtained optimal solution for the integer variables from the master problem to solve and find the optimal solution for the other variables.

V. Case Studies and Numerical Examples

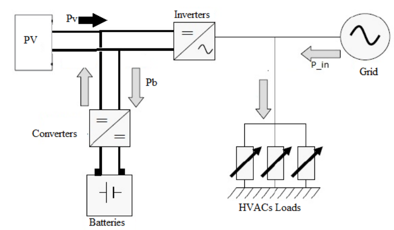


Fig. 3: The study system.

A. The Study System

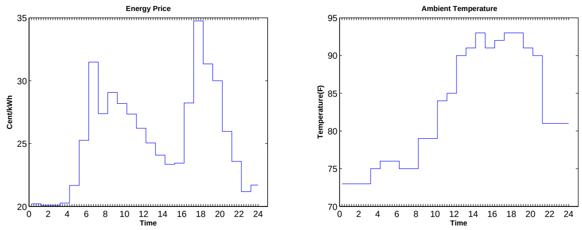
The study system shown in Fig. 3 consists of three HVAC units (rated at 15 kW) in their cooling modes. HVAC units consume electricity from the grid at a varying price, shown in Fig. 4a, during known periods. Rooms temperature should be maintained within a defined range by the consumer. Here, the consumer is to set thermostat point settings to 71 F as minimum limit and 75 F as maximum limit. The ambient temperature is shown in Fig.4b. The parameters C , R, and Q are shown in Table II.

TABLE II: Parameter Values For HVAC

	$Q(W)$	$R(F/W)$	$C(J/F)$
Values	400	0.1208	3599.3

B. Result and Analysis

Here, we consider operating HVACs while the system is connected to the grid and can purchase the power from the grid at varying price. Three cases would be considered to study the effect of the uncertainty. Table.III shows the results of those cases. The first case, $Case-1$, is done to study the behavior of the system without installing PV-BESS. In the second case, installing PV-BESS is considered in the deterministic mode



(a) Energy price. (b) Ambient temperature.

Fig. 4: Input data to the case study.

to study the effect of the availability of the solar energy. Here, the model is run twice. In the first run, $Case-2A$, it is considered to have high availability of solar energy. In the second run, $Case-2B$, it is considered to have poor availability of solar energy. The last case, $Case-3$, the proposed method in this paper using Benders decomposition method is applied to deal with the uncertainty. The 600 scenarios with the greatest probabilities are considered. It can be seen that the deterministic case with high availability of solar energy is the best result. However, in the last case, $Case-3$, the uncertainty is solar energy is considered with reasonable result and more reliable than the previous one. Fig. 5 shows that the optimal value converged to 22.32 when Benders decomposition method applied.

TABLE III: Simulation Restults

Case	$BESSenergy$ kWh	$BESSPower$ kW	N_{pv}	$Cost$ \$
Case-1	N/A	N/A	N/A	30.14
Case-2A	34.8	8.7	7	20.80
Case-2B	43.8	10.8	14	25.5
Case-3	37.6	9.40	7	22.32

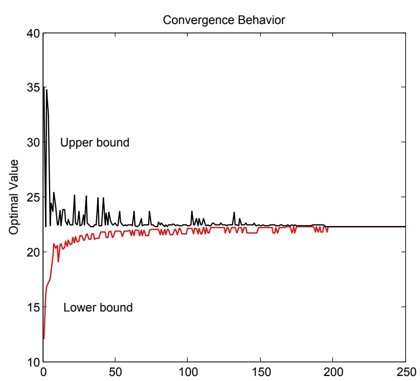


Fig. 5: Lower and upper bound convergence.

VI. Conclusion

In this paper, stochastic mixed integer programming optimization problems are formulated to determine the optimal sizing of PV-BESS to power HVAC loads. Benders Decomposition is used to solve the problem and deal with the uncertainty of PV-output. The optimization problem can find the optimal HVAC on/off states, and BESS charging- discharging states for a multi-horizon period. The main contribution of this paper is modeling the uncertainty of solar energy using and application of Benders decomposition to investigate the potential benefits of PV-BESS with HVAC loads. This formulation has shown a great ability to deal with a big number scenarios.

References

[1] [Online]. Available: http://www1.eere.energy.gov/buildings/

[2] [Online]. Available: https://www.nvenergy.com/

[3] S. Dutta and R. Sharma, "Optimal storage sizing for integrating wind and load forecast uncertainties," in *Innovative Smart Grid Technologies (ISGT), 2012 IEEE PES*, Jan 2012, pp. 1–7.

[4] S. Bahramirad, W. Reder, and A. Khodaei, "Reliability-constrained optimal sizing of energy storage system in a microgrid," *Smart Grid, IEEE Transactions on*, vol. 3, no. 4, pp. 2056–2062, Dec 2012.

[5] A. S. Siddiqui and C. Marnay, "Distributed generation investment by a microgrid under uncertainty," *Energy*, vol. 33, no. 12, pp. 1729 – 1737, 2008. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S0360544208001989

[6] G. Cardoso, M. Stadler, M. Bozchalui, R. Sharma, C. Marnay, A. Barbosa-Pvoa, and P. Ferro, "Optimal investment and scheduling of distributed energy resources with uncertainty in electric vehicle driving schedules," *Energy*, vol. 64, no. 0, pp. 17 – 30, 2014. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S0360544213009596

[7] A. Arabali, M. Ghofrani, M. Etezadi-Amoli, M. S. Fadali, and Y. Baghzouz, "Genetic-algorithm-based optimization approach for energy management," *Power Delivery, IEEE Transactions on*, vol. 28, no. 1, pp. 162–170, 2013.

[8] G. E. Sand, "Modeling and solving real-time scheduling problems by stochastic integer programming," *Computers and Chemical Engineering*, vol. 28, pp. 1087 – 1103, 2004. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S0098135403002448

[9] H. Pousinho, V. Mendes, and J. Catalaoi, "A stochastic programming approach for the development of offering strategies for a wind power producer," *Electric Power Systems Research*, vol. 89, pp. 45 – 53, 2012. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S0098135403002448

[10] T. Sheskin, "Mixed integer program to shedule loads and charge batteries on space station," *Annual Conference on Computers and Industrial Engineering*, p. 2731, 2004.

[11] S. Sastry, O. Gimdogmus, T. Hartley, and R. Veillette, "Coordinated discharge of a collection of batteries," *Journal of Power Sources*, p. 284296, 2007.

[12] J. Neubauer, A. Brooker, and E. Wood, "Sensitivity of battery electric vehicle economics to drive patterns, vehicle range, and charge strategies," *Journal of Power Sources*, p. 269277, 2012.

[13] M. Nikzad, B. Mozafari, M. Bashirvand, and S. Solaymani, "Designing time-of-use program based on stochastic security constrained unit commitment considering reliability index," *Energy*, 2012.

[14] J. Wang, M. Shahidehpour, and Z. Li, "Security-constrained unit commitment with volatile wind power generation," *Power Systems, IEEE Transactions on*, vol. 23, no. 3, pp. 1319–1327, 2008.

[15] H. Ma, S. Shahidehpour, and M. Marwali, "Transmission constrained unit commitment based on benders decomposition," in *American Control Conference, 1997. Proceedings of the 1997*, vol. 4. IEEE, 1997, pp. 2263–2267.

[16] L. Kuznia, B. Zeng, G. Centeno, and Z. Miao, "Stochastic optimization for power system configuration with renewable energy in remote areas," *Annals of Operations Research*, vol. 210, no. 1, pp. 411–432, 2013.

[17] L. Fan. (2015) Tutorial on cutting plane methods for economic dispatch problems. [Online]. Available: http:power.eng.usf.edudocspapers2015NAPScutting_NAPS15.pdf

[18] M. Alhaider and L. Fan, "Mixed integer programming for hvacs operation," in *Power Energy Society General Meeting, 2015 IEEE*, July 2015, pp. 1–5.

[19] N. Lu, "An evaluation of the hvac load potential for providing load balancing service," *Smart Grid, IEEE Transactions on*, vol. 3, no. 3, pp. 1263–1270, 2012.