Design A Robust Power System Stabilizer on SMIB Using Lyapunov Theory

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* Motivation of robust power system stabilizer (PSS)

* Robust design based on Lyapunov stability criterion
  * Our contribution

* Application in PSS design

* Case study
Motivation

* Power system stabilizer (PSS) is used to providing damping to electromechanical oscillation modes for a synchronous generator.
  * There is damping issue related to automatic voltage regulator (AVR) at high power transfer level [1].

* Conventional PSS design is based on the linearized model of a typical operating condition.
  * At another operating condition, PSS may not work well.

* Robust PSS can work for a wide range of operating conditions.
  * Lyapunov stability theory:

\[ \dot{x} = Ax. \] If there exists a \( P \geq 0 \), that makes \( A^T P + PA \leq 0 \) true. Then, the system is stable.

If \( A^T P + PA \leq -\beta P \), the system is exponentially stable where \( \beta \) is positive.

Robust design based on Lyapunov stability criterion

*For different conditions, there are different $A$ matrix and $B$ matrix.

\[ \dot{x} = A_1 x + B_1 u \]
\[ \dot{x} = A_2 x + B_2 u \]
\[ \ldots \]
\[ \dot{x} = A_n x + B_n u \]

$u = KCx$ that can stabilize all the closed-loop systems.

\[ \dot{x} = (A_i + B_i KC)x \Rightarrow (A_i + B_i KC)^T P + P(A_i + B_i KC) \leq -\beta \cdot P \]

Not LMIs!
* Using two variables $X$ and $Y$ to replace two unknown matrices, $K$ and $P$ [2].

\[
0 \geq (A_i + B_i KC)^T P + P (A_i + B_i KC) + \beta P
\]

If $X = P^{-1}$ and $Y = KCP^{-1}$

* Then, MATLAB CVX toolbox is used to find $X$ matrix and $Y$ matrix to satisfy this inequality.

\[
0 \geq X A_i^T + Y^T B_i^T + A_i X + B_i Y + X \beta
\]

* $K$ matrix is estimated easily based on $X$ and $Y$.

\[
K = Y X^{-1} C^{-1}
\]
Our contribution

* $H_\infty$ design is based on a nominal system and considers a bounded uncertainty [3].
  * Solve one or two LMIs.

* Our design is based on many operating conditions.
  * Solve 50 LMIs.
  * Not possible without the advancement in computing of convex programming tools.

* The power system can be presented by the state space matrix.

\[
\begin{aligned}
\dot{x} &= A_i x + B u \\
y &= C x
\end{aligned}
\]

* Several values are changed under different operating conditions.

Block diagram of EMT+AVR model of SMIB system
Application of PSS design

* Conventional PSS is applied to EMT+AVR model.

* Two zeros and one pole are added to change the system’s root locus.
Robust PSS is applied to EMT+AVR model.

If all of state variables are considered as outputs, $C$ matrix will be a unit one.

\[ K = YX^{-1}C^{-1} \]

\[ K = KC = YX^{-1} \]
Application of PSS design

* 50 conditions are considered totally.
  * 25 combinations of real power and reactive power generated by synchronous machine.
  * fault on one of transmission line

* 10 conditions are selected to verify if designed robust PSS works.

<table>
<thead>
<tr>
<th>P(p.u.)</th>
<th>Q(p.u.)</th>
<th>$X_L = 0.4$</th>
<th>$X_L = 0.2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.28</td>
<td>-0.8</td>
<td>Cond1</td>
<td>Cond26</td>
</tr>
<tr>
<td>1.28</td>
<td>-0.5</td>
<td>Cond2</td>
<td>Cond27</td>
</tr>
<tr>
<td>1.28</td>
<td>0</td>
<td>Cond3</td>
<td>Cond28</td>
</tr>
<tr>
<td>1.28</td>
<td>0.5</td>
<td>Cond4</td>
<td>Cond29</td>
</tr>
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<td>1.28</td>
<td>0.8</td>
<td>Cond5</td>
<td>Cond30</td>
</tr>
<tr>
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<td>-0.8</td>
<td>Cond6</td>
<td>Cond31</td>
</tr>
<tr>
<td>1.26</td>
<td>-0.5</td>
<td>Cond7</td>
<td>Cond32</td>
</tr>
<tr>
<td>1.26</td>
<td>0</td>
<td>Cond8</td>
<td>Cond33</td>
</tr>
<tr>
<td>1.26</td>
<td>0.5</td>
<td>Cond9</td>
<td>Cond34</td>
</tr>
<tr>
<td>1.26</td>
<td>0.8</td>
<td>Cond10</td>
<td>Cond35</td>
</tr>
<tr>
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<td>-0.8</td>
<td>Cond11</td>
<td>Cond36</td>
</tr>
<tr>
<td>1.0</td>
<td>-0.5</td>
<td>Cond12</td>
<td>Cond37</td>
</tr>
<tr>
<td>1.0</td>
<td>0</td>
<td>Cond13</td>
<td>Cond38</td>
</tr>
<tr>
<td>1.0</td>
<td>0.5</td>
<td>Cond14</td>
<td>Cond39</td>
</tr>
<tr>
<td>1.0</td>
<td>0.8</td>
<td>Cond15</td>
<td>Cond40</td>
</tr>
<tr>
<td>0.8</td>
<td>-0.8</td>
<td>Cond16</td>
<td>Cond41</td>
</tr>
<tr>
<td>0.8</td>
<td>-0.5</td>
<td>Cond17</td>
<td>Cond42</td>
</tr>
<tr>
<td>0.8</td>
<td>0</td>
<td>Cond18</td>
<td>Cond43</td>
</tr>
<tr>
<td>0.8</td>
<td>0.5</td>
<td>Cond19</td>
<td>Cond44</td>
</tr>
<tr>
<td>0.8</td>
<td>0.8</td>
<td>Cond20</td>
<td>Cond45</td>
</tr>
<tr>
<td>0.6</td>
<td>-0.8</td>
<td>Cond21</td>
<td>Cond46</td>
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<tr>
<td>0.6</td>
<td>-0.5</td>
<td>Cond22</td>
<td>Cond47</td>
</tr>
<tr>
<td>0.6</td>
<td>0</td>
<td>Cond23</td>
<td>Cond48</td>
</tr>
<tr>
<td>0.6</td>
<td>0.5</td>
<td>Cond24</td>
<td>Cond49</td>
</tr>
<tr>
<td>0.6</td>
<td>0.8</td>
<td>Cond25</td>
<td>Cond50</td>
</tr>
</tbody>
</table>
* KC can be found to satisfy all of fifty LMIs by running the following CVX code in MATLAB.

```matlab
cvx_begin sdp
variable X(n,n) symmetric
variable Y(1,n)
X*A1’ + Y’*B’ + A1*X + B*Y + X*beta <= 0
X*A2’ + Y’*B’ + A2*X + B*Y + X*beta <= 0
.  
.  
.  
X >= eye(n)
cvx_end
KC = Y*inv(X)
```

KC =

```
1.0e+03 *
```
```
0.0080  7.6136  -0.1016  -0.0004
```
Comparison of eigenvalues
* Case 1: no PSS
* Case 2: conventional PSS
* Case 3: robust PSS

Nonlinear simulation results
* Case 2: conventional PSS
* Case 3: robust PSS
Comparison of Eigenvalues

* Closed-loop eigenvalues under selected 10 conditions.

<table>
<thead>
<tr>
<th>Case</th>
<th>Condition</th>
<th>Between 1, 2</th>
<th>Between 3, 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>EMT+A VR</td>
<td>-4.2411 &amp; -9.3056</td>
<td>-8.7674 ± 53.0810i</td>
</tr>
<tr>
<td>Case 2</td>
<td>EMT+A VR+Conventional PSS</td>
<td>-6.5566 ± 2.6118i</td>
<td>-8.9842 ± 53.0798i</td>
</tr>
<tr>
<td>Case 3</td>
<td>EMT+A VR+Robust PSS</td>
<td>-3.4965 ± 7.9962i</td>
<td>-12.0443 ± 53.1654i</td>
</tr>
<tr>
<td></td>
<td>Closed-loop eigenvalues under selected 10 conditions.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
* Calculating initial values based on Cond1 and Cond7
* Step change on input, 0.01\textit{p.u.}
Nonlinear Simulation Results

* The right plot showed that the system became stable after a transient while the left plot presented an unstable system.

* The oscillation frequency was around $5.10\,\text{rad/s}$ ($0.81\,\text{Hz}$).

Cond 1: $-2.1795 \pm 5.3098i$
$0.9911 \pm 4.7780i$

Cond 7: $-0.8991 \pm 5.7661i$
$-0.2979 \pm 5.0637i$
* The system was stable under both of conditions.

* Robust PSS has the faster response speed and smaller oscillation.

Conventional PSS

Robust PSS

Cond 7: -0.8991 ± 5.7661i
-0.2979 ± 5.0637i

Cond 7: -6.5566 ± 2.6118i
-8.9842 ± 53.0798i
* Better robustness because of multiple operation conditions considered.

* The control design is based on LMI solving using convex programming tools.
Thank you!