Deriving ARX Models for Synchronous Generators

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***Abstract*—Parameter identification of a synchronous generator based on Phasor Measurement Unit (PMU) data requires autoregressive exogenous (ARX) models. This paper presents ARX model derivation and validation of two synchronous generator models. The first one is a third-order ARX model including electromechanical dynamics, turbine-governor dynamics and primary frequency control. Input of the model is power while output is frequency. The second one is a fifth order generator model extending from the simplified one, with additional rotor flux dynamics and excitation dynamics. Generator terminal voltage is treated as input while real power, reactive power and frequency are treated as outputs. Nonlinear model simulation data from Power System Toolbox (PST) are used as PMU data. The derived ARX models are validated by comparing the measurement outputs with the outputs generated from the ARX models. High matching degrees are observed. The coefficients of the ARX models are also compared based on two approaches: (i) calibration from the known generator parameters and (ii) system identification. High matching degrees are also observed.**

***Index Terms*—linear analysis, non-linear simulation, ARX model.**

I. Introduction

Accurate estimation of synchronous generator parameters and states plays an important role in power system operation. Various dynamic state and parameter estimation methods have been developed recently for Phasor Measurement Unit (PMU) data. There are two major system identification methods. The first approach is Kalman filter based estimation [1]–[8]. The estimation can be carried out at each time step. The second approach is least squares estimation (LSE) [9]–[11]. A time window of measurements and a discrete ARX model are required to carry out the estimation.

For phasor measurement unit (PMU) data based estimation, [9] shows a linear continuous model in Laplace domain is converted to a Z-domain model. A discrete ARX model is derived based on the z-domain model. Converting a Laplace continuous model to a z-domain requires specific treatment for each transfer function. Thus, this method (zero-order hold) documented in [9] requires time consuming derivation. This method will be difficult to be applied to a general model.

The objective of this paper is to provide a more general approach to form an ARX model for LSE for PMU data based generator parameter estimation. we will start from linear continuous state-space models, which will be further converted to discrete linear state-space models. Discrete ARX models are then obtained. With a discrete ARX model, a linear estimation problem can be formulated and the coefficients of the ARX model can be found. From these coefficients, we can further recover the generator parameters.

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In this paper, we present two ARX models with different level of complexity. The first one is a simplified generator model including swing dynamics, turbine-governor dynamics and primary frequency control droop. The second ones extends the first one by including rotor flux dynamics and exciter dynamics. For the first model, real power is treated as the input while the frequency is treated as the output. For the second model, the terminal voltage magnitude is treated as the input while the power, reactive power and frequency are treated as the outputs.

For validation, PMU data is generated by time-domain simulation, which is carried out for a two-area four-machine test system via PST [12]. The ARX models are validated by comparing output of real data with those generated from the ARX models. The ARX model coefficients obtained from two approaches (calibration based on known parameters and system identification) are also compared. The experiments show high matching degrees on both output data and coefficients, which demonstrates the feasibility and accuracy of the derived ARX models.

The rest of this paper is organized as follows. Section II describes the PMU data and the synchronous generator. Section III presents two ARX model. Section IV presents the numerical estimation results. Section VI concludes the paper.

II. Synchronous Generator and PMU Data

This paper uses data recorded by PST (generator terminal voltage V_a, machine speed ω, electrical power P_e and Q_e) at the terminal of the synchronous machine after a fault within few seconds. The synchronous generator is assumed to include electromechanical parameters, primary frequency control droop, turbine-governor dynamics and excitation dynamics. The following parameters: inertia constant H, damping coefficient D_1, regulation parameter R, governor time constant T_g, exciter time constant T_e, voltage regulator gain K_A, d-axis open circuit transient time constant T'_{do}, reactance X_q, X_d and transient reactance X'_d), are to be estimated through PMU data.

A synchronous machine connected to power grid is shown in the Figure. 1. The generator is modeled by a subtransient model in PST. All system data can be found in Appendix. The PMU data include the terminal voltage V_a, phase angle θ, frequency f, and real and reactive power (P and Q).

In order to carry out LSE-based system identification using PMU data, ARX models are required. There are multiple ways to separate the PMU data into input and outputs. In this paper, two ways are examined:

1) P as input, frequency as output. We will show in Section III that this model is suitable to estimate parameters related to swing dynamics and turbine-governors: H, D_1, R, and T_g. However, this model does not reflect the other parameters such

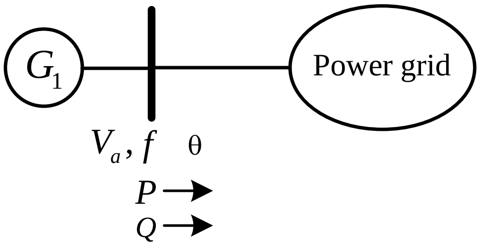

Fig. 1. Synchronous generator connected to power grid.

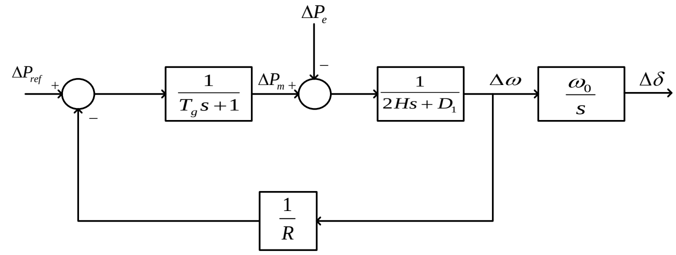

Fig. 2. Block diagram of a simplified synchronous generator.

as T'_{d0}, X_d, X'_d, etc.
2) V_a as input, frequency, P and Q as output. This model reflects all the aforementioned parameters.

III. ARX MODELS DERIVATION

A. ARX model

The ARX model structure can be expressed as:

$$A(z)y(k) = B(z)u(k-n_k) + e(k) \tag{1}$$

where $u(k)$ is the system inputs, $y(k)$ is the system outputs, n_k is the system delay and $e(t)$ is the white noise disturbance. $A(z)$ and $B(z)$ are polynomials and defined by the following equations:

$$A(z) = 1 + a_1 z^{-1} + \cdots + a_{n_a} z^{-n_a}$$
$$B(z) = b_0 + b_1 z^{-1} + \cdots + b_{nb-1} z^{-n_b+1}$$

$[n_a, n_b, n_k]$ represent the ARX model's orders and time delay. In Matlab software, we can use ARX command to find the coefficients of the polynomials for the given input/output data and $[n_a, n_b, n_k]$.

B. Model 1: Power (ΔP_e) to frequency $\Delta\omega$

The swing equations are expressed as follows.

$$\begin{cases} \frac{d\omega}{dt} &= \frac{1}{2H}(P_m - P_e - D_1(\omega - 1)) \\ \frac{d\delta}{dt} &= \omega_0(\omega - 1) \end{cases} \tag{2}$$

where δ is the rotor angle in radius, ω is the rotor speed (frequency) in p.u., P_m is the mechanical power (p.u.), and P_e is the electrical power (p.u.).

The small-signal model is as follows.

$$\begin{cases} \Delta\dot{\omega} &= \frac{1}{2H}(\Delta P_m - \Delta P_e - D_1\Delta\omega) \\ \Delta\dot{\delta} &= \omega_0\Delta\omega \end{cases} \tag{3}$$

The mechanical power P_m is produced after the turbine which is controlled by a governor. The dynamic of turbine governor with primary frequency control can be expressed as:

$$\Delta\dot{P_m} = \frac{1}{T_g}(\Delta P_{ref} - \frac{1}{R}\Delta\omega - \Delta P_m) \tag{4}$$

With (3) and (4), The block diagram of a simplified third order synchronous generator is designed as shown in Fig. 2, where T_g is the time constant of the turbine-governor (seconds), and R is the speed regulation constant (p.u.). ΔP_e is considered as input and $\Delta\omega$ is output. We named this generator model as Model 1.

Based on Fig. 2, the plant model will be built in the form of linearized state-space vector differential equations.

$$\begin{bmatrix} \Delta\dot{\delta} \\ \Delta\dot{\omega} \\ \Delta\dot{P_m} \end{bmatrix} = \begin{bmatrix} 0 & \omega_0 & 0 \\ 0 & -\frac{D_1}{2H} & \frac{1}{2H} \\ 0 & -\frac{1}{T_g R} & -\frac{1}{T_g} \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta\omega \\ \Delta P_m \end{bmatrix} + \begin{bmatrix} 0 \\ -\frac{1}{2H} \\ 0 \end{bmatrix} \Delta P_e$$
$$\Delta\omega = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta\omega \\ \Delta P_m \end{bmatrix} \tag{5}$$

Discretizing the continuous state-space model leads to the following discrete state-space model.

$$\begin{bmatrix} \Delta\delta_{k+1} \\ \Delta\omega_{k+1} \\ \Delta P_{m_{k+1}} \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & \omega_0 h & 0 \\ 0 & 1 - \frac{D_1 h}{2H} & \frac{h}{2H} \\ 0 & -\frac{h}{T_g R} & 1 - \frac{h}{T_g} \end{bmatrix}}_{A'} \begin{bmatrix} \Delta\delta_k \\ \Delta\omega_k \\ \Delta P_{m_k} \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ -\frac{h}{2H} \\ 0 \end{bmatrix}}_{B'} \Delta P_{e_k}$$
$$\Delta\omega_k = \underbrace{\begin{bmatrix} 0 & 1 & 0 \end{bmatrix}}_{C'} \begin{bmatrix} \Delta\delta_k \\ \Delta\omega_k \\ \Delta P_{m_k} \end{bmatrix} \tag{6}$$

where h is sampling time.

Convert the above model to z-domain:

$$\frac{y}{u} = C'(zI - A')^{-1}B'. \tag{7}$$

We can find the equivalent discrete time ARX model as:

$$A(z)y(k) = B(z)u(k-1) \tag{8}$$

where

$$A(z) = 1 + a_1 z^{-1} + a_2 z^{-2}$$
$$B(z) = b_0 + b_1 z^{-1}$$

The orders and delay of Model 1 is $[2, 2, 1]$.

Given time series measurements of input u and output y, an overdetermined problem $Y = \phi\theta$ can be built as follows:

$$\begin{bmatrix} y(k+2) \\ \vdots \\ y(i) \\ \vdots \\ y(N) \end{bmatrix} = \begin{bmatrix} y(k+1) & y(k) & u(k+1) & u(k) \\ \vdots & \vdots & \vdots & \vdots \\ y(i-1) & y(i-2) & u(i-1) & u(i-2) \\ \vdots & \vdots & \vdots & \vdots \\ y(N-1) & y(N-2) & u(N-1) & u(N-2) \end{bmatrix} \cdot \begin{bmatrix} -a_1 \\ -a_2 \\ b_0 \\ b_1 \end{bmatrix} \tag{9}$$

where $y(k+1), y(k), u(k+1), u(k)$ are delayed input and output data is refereed to as regressors. The coefficients can be solved by the normal equation of least square estimation $\theta = (\phi^T\phi)^{-1}\phi^T Y$ or Matlab ARX command.

C. Model 2: Terminal voltage (ΔV_a) to frequency $\Delta\omega$, electrical active power ΔP_e and reactive power ΔQ_e

Considering the swing equations and rotor flux dynamics, there are three state variables: E'_a, frequency, ω, and the rotor angle δ. The dynamic model of the generator is expressed as follows.

$$\begin{cases} \frac{d\delta}{dt} &= \omega_0(\omega - 1) \\ \frac{d\omega}{dt} &= \frac{1}{2H}(P_m - P_e - D_1(\omega - 1)) \\ \frac{dE'_a}{dt} &= \frac{1}{T'_{do}}\left(E_{fd} - E_a\right) \end{cases} \tag{10}$$

where E_{fd} can be viewed as field voltage v_F in equivalent stator terms, E'_a is an internal voltage whose magnitude is proportional to the rotor flux λ_F, E_a is the stator open circuit voltage. The voltage and current phasor diagram is shown in Fig. 3.

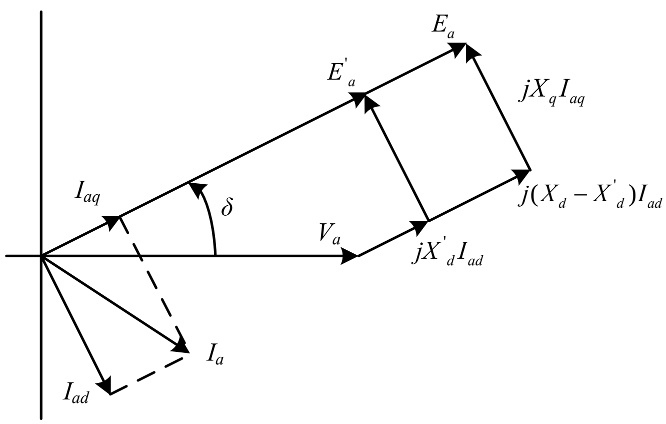

Fig. 3. Phasor diagram showing E_a and E'_a

E_a in terms of E'_a, δ and V_a can be expressed as:

$$E_a = \frac{X_d}{X'_d}E'_a + V_a\frac{X'_d - X_d}{X'_d}\cos(\delta) \tag{11}$$

Add a small perturbation to linearize the E_a equation:

$$\begin{aligned} \Delta E_a &= \frac{\partial E_a^0}{\partial E'_a}\Delta E'_a + \frac{\partial E_a^0}{\partial\delta}\Delta\delta + \frac{\partial E_a^0}{\partial V_a}\Delta V_a \\ &= \frac{X_d}{X'_d}\Delta E'_a + \left(\frac{X_d - X'_d}{X'_d}\right)V_a\sin(\delta^0)\Delta\delta \\ &+ \frac{X'_d - X_d}{X'_d}\cos(\delta^0)\Delta V_a \\ &= \frac{1}{k_3}\Delta E'_a + k_4\Delta\delta + e_1\Delta V_a \end{aligned} \tag{12}$$

Substituting (12) into the differential equation of $\dot{E'_a}$ in (10) leads to:

$$T'_{do}\frac{d\Delta E'_a}{dt} = \Delta E_{fd} - \frac{1}{k_3}\Delta E'_a - k_4\Delta\delta - e_1\Delta V_a$$

$$\Delta E'_a = \frac{k_3}{T'_{do}k_3 s + 1}(\Delta E_{fd} - k_4\Delta\delta - e_1\Delta V_a) \tag{13}$$

The electrical active power P_e in terms of E'_a, δ and V_a can be expressed as:

$$P_e = \frac{E'_aV_a}{X'_d}\sin(\delta) + \frac{V_a^2}{2}\left(\frac{1}{X_q} - \frac{1}{X'_d}\right)\sin(2\delta) \tag{14}$$

Using the same way to linearize (14),

$$\begin{aligned} \Delta P_e &= \frac{\partial P_e^0}{\partial E'_a}\Delta E'_a + \frac{\partial P_e^0}{\partial\delta}\Delta\delta + \frac{\partial P_e^0}{\partial V_a}\Delta V_a \\ &= \frac{V_a}{X'_d}\sin(\delta^0)\Delta E'_a \\ &+ \left[\frac{E'_aV_a}{X'_d}\cos(\delta^0) + V_a^2\left(\frac{1}{X_q} - \frac{1}{X'_d}\right)\cos(2\delta^0)\right]\Delta\delta \\ &+ \left[\frac{E'_a}{X'_d}\sin(\delta^0) + \left(\frac{1}{X_q} - \frac{1}{X'_d}\right)V_a\sin(2\delta^0)\right]\Delta V_a \\ &= k_2\Delta E'_a + T\Delta\delta + p_1\Delta V_a \end{aligned} \tag{15}$$

The electrical reactive power Q_e is given by:

$$Q_e = \frac{E'_aV_a}{X'_d}\cos(\delta) - V_a^2\left(\frac{\cos(\delta)^2}{X'_d} + \frac{\sin(\delta)^2}{X_q}\right) \tag{16}$$

Linearizing the Q_e equation:

$$\begin{aligned} \Delta Q_e &= \frac{\partial Q_e^0}{\partial E'_a}\Delta E'_a + \frac{\partial Q_e^0}{\partial\delta}\Delta\delta + \frac{\partial Q_e^0}{\partial V_a}\Delta V_a \\ &= \frac{V_a}{X'_d}\cos(\delta^0)\Delta E'_a \\ &+ \left[-\frac{E'_aV_a}{X'_d}\sin(\delta^0) + V_a^2\left(\frac{1}{X'_d} - \frac{1}{X_q}\right)\sin(2\delta^0)\right]\Delta\delta \\ &+ \left[\frac{E'_a}{X'_d}\cos(\delta^0) - \left(\frac{\cos(\delta^0)^2}{X'_d} + \frac{\sin(\delta^0)^2}{X_q}\right)2V_a\right]\Delta V_a \\ &= q_1\Delta E'_a + q_2\Delta\delta + q_3\Delta V_a \end{aligned} \tag{17}$$

Linearizing dynamics (10) by substituting (15) and (12) into them:

$$\begin{cases} \Delta\dot{\delta} &= \omega_0\Delta\omega \\ \Delta\dot{\omega} &= \frac{1}{2H}\left(\Delta P_m - (k_2\Delta E'_a + T\Delta\delta + p_1\Delta V_a) - D_1\Delta\omega\right) \\ \Delta\dot{E'_a} &= \frac{1}{T'_{do}}\left(\Delta E_{fd} - (\frac{1}{k_3}\Delta E'_a + k_4\Delta\delta + e_1\Delta V_a)\right) \end{cases} \tag{18}$$

A simplified exciter model is used in the single machine infinite bus system. In this, the transfer function is $K_A/(1 + T_e s)$. The dynamic of ΔE_{fd} can be expressed as:

$$\Delta\dot{E_{fd}} = \frac{1}{T_e}(K_A\Delta V_{ref} - K_A\Delta V_a - \Delta E_{fd}) \tag{19}$$

With (18), (4) and (19), the block diagram of fifth order generator model is designed as shown in Fig. 4. The input is the terminal voltage V_a, the outputs are frequency w, real power P_e and reactive power Q_e.

Finally, writing (18), (4) and (19) in compact form, we

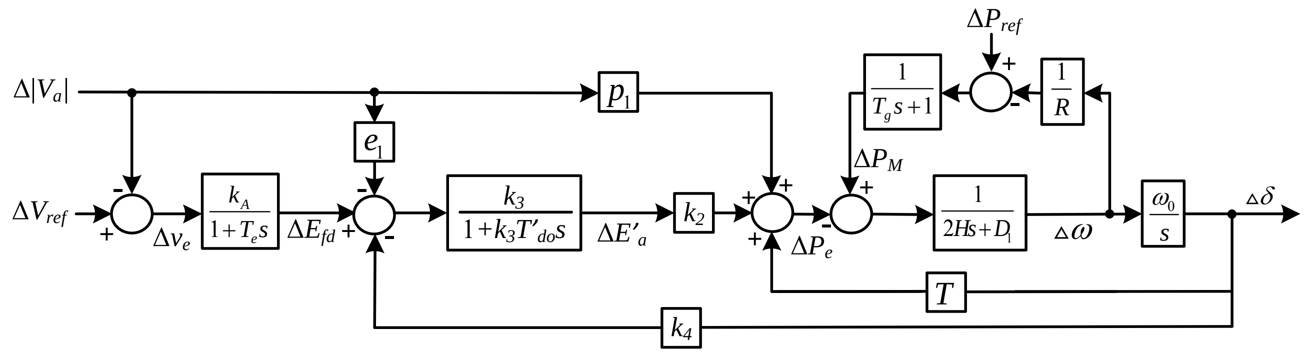

Fig. 4. Linearized fifth order generator model considering terminal voltage variation

obtain

$$\begin{bmatrix}\Delta\dot{\delta}\\ \Delta\dot{\omega}\\ \Delta\dot{E}'_a\\ \Delta\dot{E}_{fd}\\ \Delta\dot{P}_m\end{bmatrix}=\begin{bmatrix}0 & w_0 & 0 & 0 & 0\\ -\frac{T}{2H} & -\frac{D_1}{2H} & -\frac{K_2}{2H} & 0 & \frac{1}{2H}\\ -\frac{k_4}{T'_{do}} & 0 & -\frac{1}{k_3T'_{do}} & \frac{1}{T'_{do}} & 0\\ 0 & 0 & 0 & -\frac{1}{T_e} & 0\\ 0 & -\frac{1}{RT_g} & 0 & 0 & -\frac{1}{T_g}\end{bmatrix}\begin{bmatrix}\Delta\delta\\ \Delta\omega\\ \Delta E'_a\\ \Delta E_{fd}\\ \Delta P_m\end{bmatrix}+\begin{bmatrix}0\\ -\frac{p_1}{2H}\\ -\frac{e_1}{T'_{do}}\\ -\frac{K_A}{T_e}\\ 0\end{bmatrix}\Delta V_a$$

$$\begin{bmatrix}\Delta\omega\\ \Delta P_e\\ \Delta Q_e\end{bmatrix}=\begin{bmatrix}0 & 1 & 0 & 0 & 0\\ T & 0 & k_2 & 0 & 0\\ q_2 & 0 & q_1 & 0 & 0\end{bmatrix}\begin{bmatrix}\Delta\delta\\ \Delta\omega\\ \Delta E'_a\\ \Delta E_{fd}\\ \Delta P_m\end{bmatrix}+\begin{bmatrix}0\\ p_1\\ q_3\end{bmatrix}\Delta V_a \quad (20)$$

Discretizing (20), the complete discrete state-space model has the following form:

$$\begin{bmatrix}\Delta\delta_{k+1}\\ \Delta\omega_{k+1}\\ \Delta E'_{a_{k+1}}\\ \Delta E_{fd_{k+1}}\\ \Delta P_{m_{k+1}}\end{bmatrix}=\begin{bmatrix}1 & w_0h & 0 & 0 & 0\\ -\frac{Th}{2H} & \frac{2H-D_1h}{2H} & -\frac{k_2h}{2H} & 0 & \frac{h}{2H}\\ -\frac{k_4h}{T'_{do}} & 0 & \frac{k_3T'_{do}-h}{k_3T'_{do}} & \frac{h}{T'_{do}} & 0\\ 0 & 0 & 0 & \frac{T_e-h}{T_e} & 0\\ 0 & -\frac{h}{RT_g} & 0 & 0 & \frac{T_g-h}{T_g}\end{bmatrix}\begin{bmatrix}\Delta\delta_k\\ \Delta\omega_k\\ \Delta E'_{a_k}\\ \Delta E_{fd_k}\\ \Delta P_{m_k}\end{bmatrix}+\begin{bmatrix}0\\ -\frac{p_1h}{2H}\\ -\frac{e_1h}{T'_{do}}\\ -\frac{K_Ah}{T_e}\\ 0\end{bmatrix}\Delta V_{a_k}$$

$$\begin{bmatrix}\Delta\omega_k\\ \Delta P_{e_k}\\ \Delta Q_{e_k}\end{bmatrix}=\begin{bmatrix}0 & 1 & 0 & 0 & 0\\ T & 0 & k_2 & 0 & 0\\ q_2 & 0 & q_1 & 0 & 0\end{bmatrix}\begin{bmatrix}\Delta\delta_k\\ \Delta\omega_k\\ \Delta E'_{a_k}\\ \Delta E_{fd_k}\\ \Delta P_{m_k}\end{bmatrix}+\begin{bmatrix}0\\ p_1\\ q_3\end{bmatrix}\Delta V_{a_k} \quad (21)$$

Three ARX models can be formulated in terms of corresponding output. The orders and delay of them are presented in Table I.

The structure of ARX model for output w is given by:

$$A(z)y(k) = B(z)u(k-1) \quad (22)$$

where

$$A(z) = 1 + a_1z^{-1} + a_2z^{-2} + a_3z^{-3} + a_4z^{-4} + a_5z^{-5}$$
$$B(z) = b_0 + b_1z^{-1} + b_2z^{-2} + b_3z^{-3} + b_4z^{-4}$$

The orders and delay of Model 2 is [5 5 1].

The structure of ARX model for output P and Q are same, both can be expressed as:

$$A(z)y(k) = B(z)u(k) \quad (23)$$

where

$$A(z) = 1 + a_1z^{-1} + a_2z^{-2} + a_3z^{-3} + a_4z^{-4} + a_5z^{-5}$$
$$B(z) = b_0 + b_1z^{-1} + b_2z^{-2} + b_3z^{-3} + b_4z^{-4} + b_5z^{-5}$$

The orders and delay of Model 2 is [5 6 0].

TABLE I

input V_a			
Output	$A(z)$ order	$B(z)$ order	Delay
ω	5	5	1
P_e	5	6	0
Q_e	5	6	0

IV. Case studies

The two-area four-machine test system is shown in Fig. 5. The synchronous generators are modeled by subtransient models in PST. The generators are equipped with turbine-governors, primary frequency droop, excitation systems. A three-phase fault occurs at $t = 0.1$s on Bus 3 to bus 101. The fault is cleared at Bus 3 after 0.05s. The power system dynamics study is carried out by PST [12]. The simulation time is set to 5 sec and the sampling time is $h = 0.001$ sec. Machine speed ω, machine terminal voltage V_a, electrical active power P_e and reactive power Q_e of generator 1 are recorded as shown in Fig. 6. The recorded raw data are pre-processed. The data for ARX model estimation are taken starting from 0.5 sec. In addition, the DC offset will be removed from recorded data. The smooth data are used to identify the ARX models.

A. Case study-I: Model 1 Validation

In this case study, P_e is considered as input and ω is output. The ARX model is generated with the $[n_a = 2, n_b = 2, n_k = 1]$. We compares the output of the ARX model with the measurement data as shown in Fig. 7. The normalized root mean square measure of the goodness of fit is 95.63%.

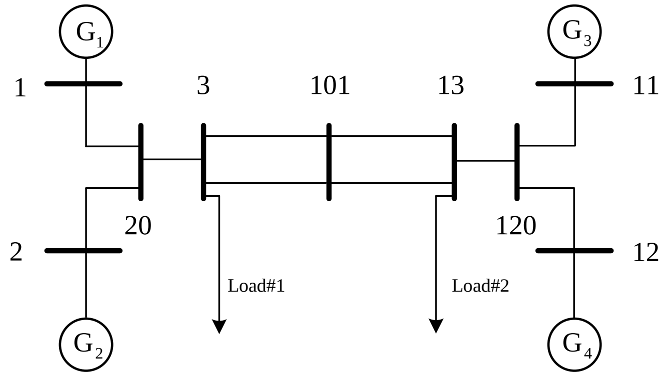

Fig. 5. Two-area four-machine system

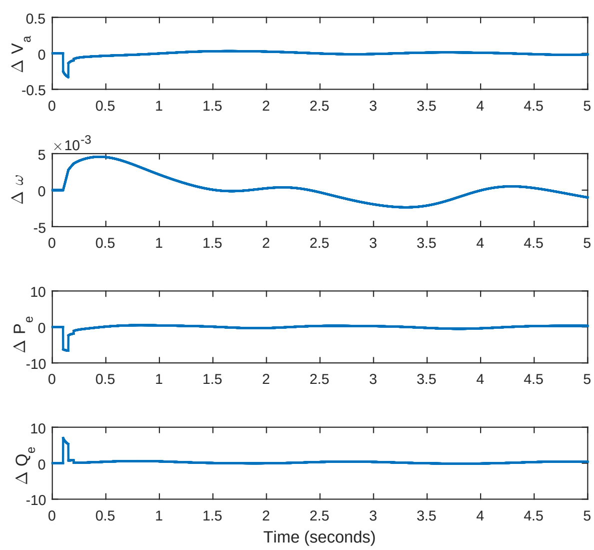

Fig. 6. Recorded data by PST: magnitude response to three-phase fault

The coefficients of predicted ARX model and the computed coefficients based on known paramters are presented in Table II.

TABLE II

Case 2		
Coefficients	Model 1	Data by PST
a_1	- 1.999	-1.995
a_2	0.9995	0.9951
b_0	-8.736 $\times$ 10^{-5}	-7.692 $\times$ 10^{-5}
b_1	8.731 $\times$ 10^{-5}	7.677 $\times$ 10^{-5}

It can be observed that the coefficients of $A(z)$ polynomial from measurement based identification match very well with the computed coefficients. The error of estimation of b_0 and b_1 is approximately 10%.

B. Case study-II: Model 2 Validation

In this case study, V_a is input. There are three outputs to be considered. For ω as output, the goodness of fit is 97.5%.

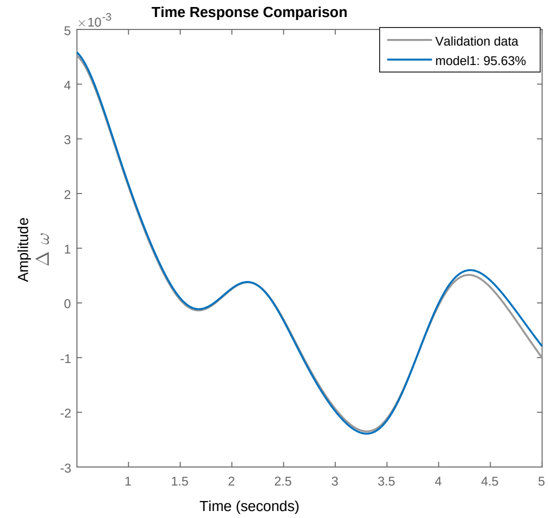

Fig. 7. Comparison of the validation data vs model 1. Output signal ($\Delta\omega$) is depicted

For P_e or Q_e is output, the goodness of fit are 85.9% and 83.1% respectively. The coefficients of predictive ARX model and the real ARX model are presented in Table III.

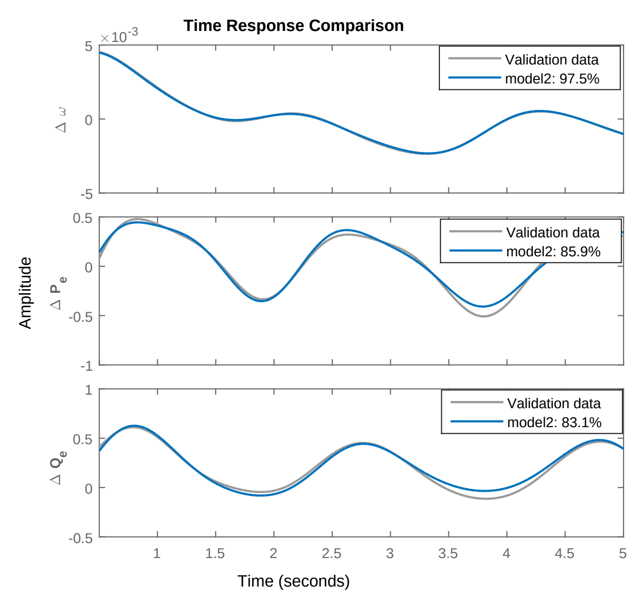

Fig. 8. Comparison of the validation data vs Model 2. Output signals ($\Delta\omega, \Delta P_e$ and ΔQ_e) are depicted.

TABLE III

Output $\Delta\omega$			Output ΔP_e		Output ΔQ_e	
Coefficients	Estimation	Calibration	Estimation	Calibration	Estimation	Calibration
a_1	-5	-4.992	-4.999	-4.992	-4.995	-4.992
a_2	10	9.97	9.99	9.97	9.98	9.97
a_3	-10	-9.95	-9.99	-9.95	-9.97	-9.95
a_4	5	4.97	4.99	4.97	4.98	4.97
a_5	-1	-0.99	-1	-0.99	-1	-0.99
b_0	-1.86 $\times$ 10^{-4}	-4.93 $\times$ 10^{-4}	-9.41	-6.39	-15.95	-1.03
b_1	7.41$\times$ 10^{-4}	19.74 $\times$ 10^{-4}	46.92	32.00	79.65	5.16
b_2	-11.1$\times$ 10^{-4}	-29.65 $\times$ 10^{-4}	-93.56	-64.10	-159.1	-10.31
b_3	7.4 $\times$ 10^{-4}	19.79 $\times$ 10^{-4}	93.29	64.20	158.9	10.29
b_4	-1.85 $\times$ 10^{-4}	-4.95 $\times$ 10^{-4}	-46.51	-32.15	-79.33	-5.14
b_5	0	0	9.276	6.44	15.85	1.03

The comparison shows that the ARX model output has a high matching degree with the measurement data. The predicted ARX model coefficients for $A(z)$ polynomial are very close to the values computed based on the known parameters. The coefficients for $B(z)$ polynomial are not close to the real values. This experiment indicates that if the coefficients of $A(z)$ from system identification can be used for parameter recovery. However, the coefficients of $B(z)$ from system identification need to be further examined.

V. Conclusion

This paper presents ARX model derivation and validation of two synchronous generator models. The first one is a third-order ARX model including electromechanical dynamics, turbine-governor dynamics and primary frequency control. Input of the model is power while output is frequency. The second one is a fifth order generator model extending from the simplified one, with additional rotor flux dynamics and excitation dynamics. Generator terminal voltage is treated as input while real power, reactive power and frequency are treated as outputs. Nonlinear model simulation data from Power System Toolbox (PST) are used as PMU data. The derived ARX models are validated by comparing the measurement outputs with the outputs generated from the ARX models. High matching degrees are observed. The coefficients of the ARX models are also compared based on two approaches: (i) calibration from the known generator parameters and (ii) system identification. High matching degrees are also observed.

The derived ARX models are suitable for generator parameter estimation.

Appendix

Generator model data for the swing model in per unit is as follows:

$$H = 6.5 \quad T_g = 0.1s \quad R = 0.04 \quad D_1 = 0$$

Generator model data for the 5th order model in per unit is as follows:

$$\begin{array}{ccccc} H = 6.5s & T_g = 0.5s & T'_{do} = 8s & R = 0.04 & D_1 = 37.7 \\ X_d = 1.8 & X'_d = 0.3 & X''_d = 0.25 & X_q = 1.7 & X'_q = 0.55 \\ & X''_q = 0.25 & X_f = 4 & r_f = 0.5 & \end{array}$$

Excitation system parameters in per unit are as follows:

$$k_A = 46 \quad T_e = 0.46s$$

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