Identification of Synchronous Generator Parameters Using Continuous ARX Model and Least Square Estimation

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***Abstract*—In this paper, least square estimation (LSE)-based synchronous generator model identification is carried out. The synchronous generator model including electromechanical dynamics and primary frequency control is first expressed by continuous-time AutoRegressive with eXogenous (ARX) models. The high-order derivatives in a continuous ARX model are approximated by expressions in terms of discrete-time data. The proposed approximation method (central difference approximation) is applied to second and third order derivatives. The continuous time model is now converted to a discrete time model. Generator parameters are then identified by LSE. We proposed two estimation structures to apply LSE. Benchmark model is simulated with and without input noise. The simulation data is used for estimation. It is found that one estimation structure is better than the other in terms of noise handling.**

***Index Terms*—Least square estimation(LSE), Discrete-time ARX model, Continuous-time ARX model, Parameter identification, Central difference, ARX structures, Natural condition.**

I. INTRODUCTION

Accurate estimation of synchronous generator parameters and states plays a very important role in the stability studies of the power systems. Accurate parameters (*e.g.* inertia) and states (*e.g.* synchronous speed) for synchronous generators are essential for a valuable analysis of stability and dynamic performance of the system.

During recent years, various methods have been developed. There are two major systematic methods of parameter estimation have been applied. The first approach is Kalman filter based estimation [1]–[9]. The estimation can be carried out at each time step. The second approach is least squares estimation (LSE) [10]–[12]. A time window of measurements and a discrete ARX model are required to carry out the estimation.

There are plenty of research on LSE-based offline generator parameter estimation based on either time-domain data and frequency response data. For phasor measurement unit (PMU) data based estimation, [10] shows a linear continuous model in Laplace domain is converted to a Z-domain model. A discrete ARX model is derived based on the Z-domain model. Converting a Laplace continuous model to a Z-domain requires specific treatment for each transfer function. Thus, this method (zero-order hold) documented in [10] requires time consuming derivation. This method will be difficult to be applied to a general model.

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The objective of this paper is to provide a more general approach to form an ARX model for LSE for PMU data based generator parameter estimation. we will start from linear continuous ARX models which can be directly obtained from transfer functions. The high-order derivatives will be approximated by discrete-time data. Discrete ARX models for LSE are then obtained. With an discrete ARX model, a linear estimation problem can be formulated and the parameters of the ARX model can be found. The input and output data are obtained from a synchronous generator simulation model consisting electromechanical dynamics and primary frequency control droop. We analyze LSE of two ARX structures to estimate the parameters of a generator with central difference approximation, which is used for derivative approximation. Numerical results are presented to illustrate the performance of the proposed structures in LSE estimation. Effect of noise on estimation is also discussed.

The rest of this paper is organized as follows. Section II describes the continuous-time model. Section III describes two structures of ARX model and central difference approximation method to convert a continuous ARX model into a discrete model. Section IV presents the numerical estimation results and discussion on noise effect. Section VI concludes the paper.

II. CONTINUOUS-TIME MODEL

For the system identification, the input and output of the system have to be known, it shows in Fig. 1. For parameter estimation, a system's structure should be known while the parameters need to be identified based on the input and output signals.

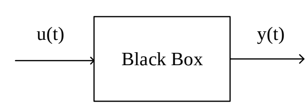

Fig. 1. Black box of the system.

In this paper, the model is a synchronous generator with primary frequency control. Its structure is shown in Fig. 2, where δ is the rotor angle in radius, ω is the rotor speed or frequency in rad/s, P_m is the mechanical power in pu and P_e is the electric power in pu. δ is obtained by integrating the synchronous speed ω. The input of the system is electrical power, ΔP_e, and the output is the synchronous frequency, $\Delta\omega$. The transfer function of this system can be expressed based

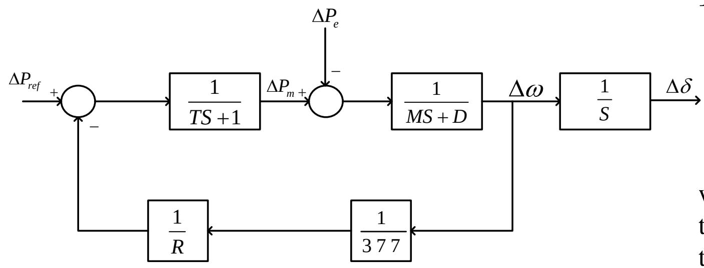

Fig. 2. Block diagram of a simplified synchronous generator.

on Fig. 2:

$$T_1(s) = \frac{\Delta\omega}{\Delta P_e} = -\frac{1+Ts}{TMs^2 + (M+TD)s + D + \frac{1}{\omega_0 R}} \quad (1)$$

where M is the generator inertia parameter, D is the damping factor, T is the time constant of turbine governor. R is the droop gain, and ω_0 is the fundamental frequency, 377rad/s. It is easy to find that $\frac{\Delta\omega}{\Delta P_e}$ is a second-order system. This model is named as Model 1 in this paper.

In Model 2, the rotor angle $\Delta\delta$ is used as the output signal of the system. It is the integrated result of $\Delta\omega$, so the transfer function of $\Delta\delta$ and ΔP_e is a third-order system as follows

$$T_2(s) = \frac{\Delta\delta}{\Delta P_e} = -\frac{1+Ts}{TMs^3 + (M+TD)s^2 + \left(D + \frac{1}{\omega_0 R}\right)s}. \quad (2)$$

III. Discrete ARX Model

With the transfer functions, continuous time models can be found. By approximating derivatives by discrete data, discrete ARX models will be found. There are several types of derivative numerical approximations used in the estimation. Euler forward, Euler backward and central difference are a few to approximate derivative. The central difference approximation is adopted in this paper. Furthermore, ARX model have various structures, the choice of ARX structure has a significant influence on the LSE.

A. ARX Structures

In this subsection, we will illustrate two different ARX structures, then compare the accuracy of the LSE with noise in the further.

1) Structure 1: Because the identification method is based on LSE which requires ARX model, the two transfer functions should be converted to continuous-time ARX models. For Model 1 $T_1(s)$, we have

$$\begin{aligned}\Delta P_e(t) = &-TMp^2\Delta\omega(t) + (-M-TD)p\Delta\omega(t)\\ &+\left(-D-\frac{1}{\omega_0 R}\right)\Delta\omega(t) + (-T)p\Delta P_e(t) + e(t).\end{aligned} \quad (3)$$

$T_2(s)$ is converted as

$$\begin{aligned}\Delta P_e(t) = &-TMp^3\Delta\delta(t) + (-M-TD)p^2\Delta\delta(t)\\ &+\left(-D-\frac{1}{\omega_0 R}\right)p\Delta\delta(t) + (-T)p\Delta P_e(t) + e(t)\end{aligned} \quad (4)$$

where $p^k f(t)$ is the kth-order derivative of $f(t)$ and $e(t)$ is the white noise. System identification is based on discrete time data. Let's assume the time step of data is h. The corresponding discrete-time ARX model for Model 1 can be formed as

$$\begin{bmatrix}\Delta P_e(k)\\ \vdots \\ i \\ \vdots \\ \Delta P_e(N)\end{bmatrix} = \begin{bmatrix} D^2\Delta\omega(k) & D\Delta\omega(k) & \Delta\omega(k) & D\Delta P_e(k)\\ \vdots & \vdots & \vdots & \vdots\\ i & i & i & i\\ \vdots & \vdots & \vdots & \vdots\\ D^2\Delta\omega(N) & D\Delta\omega(N) & \Delta\omega(N) & D\Delta P_e(N)\end{bmatrix} \cdot \begin{bmatrix}a_{12}\\ a_{11}\\ a_{10}\\ b_{11}\end{bmatrix} + e(t) \quad (5)$$

where $D^k\Delta\omega$ is the differential operator in discrete-time process. k is one sampling point of the data and N is the last sampling point which can be used. a_{ij} is the coefficient for the output signal and b_{ij} is for the input signal. The first subscript i of a and b is related to structure number and the second subscript j is corresponding to the order of differential operator. The row of the matrix has an increment by one means the sampling point increase by one, k+1. For Model 2, Discrete-Time ARX model is the same except that the order of $\Delta\delta$.

Let us consider (5) can be presented as $y(k) = \phi(k)\theta + e(k)$. $\phi(k)$ and $y(k)$ matrix are created by the input and output data. θ is the vector of the coefficients. The role of LSE in the system identification method is to use $\phi(k)$ and $y(k)$ to identify θ. At time t, LSE estimate of x minimizes:

$$J_t = \sum_{i=1}^{t}[\Delta P_e(i) - \phi(i)\theta]^2 \quad (6)$$

Following $\frac{\partial J_t}{\partial\theta(t)} = 0$, the well-known direct solution can be written as

$$\hat{\theta} = (\phi^T\phi)^{-1}\phi^T y \quad (7)$$

The coefficients a_{ij} and b_{ij} can be obtained by (7). Once the coefficients are found, we will examine the accuracy by the comparison with true values from a bench mark model.

The coefficients are given by

$$\begin{cases} a_{12} &= -TM\\ a_{11} &= -M-TD\\ a_{10} &= -D-\frac{1}{377R}\\ b_{11} &= -T\end{cases} \quad (8)$$

Regarding the structure of this discrete-time ARX model, it is observed that y vecotr is related to ΔP_e and ϕ matrix contains the highest order variable of the system. It is named as Structure 1.

2) Structure 2: For Structure 2, the most important change is to use the highest order variable from as y. Therefore, the new continuous-time ARX models for Model 1 and 2 can be written as the follow:

$$p^2\Delta\omega(t) = -\frac{M+TD}{TM}p\Delta\omega(t) - \frac{D+\frac{1}{\omega_0 R}}{TM}\Delta\omega(t) - \frac{1}{M}p\Delta P_e(t) - \frac{1}{TM}\Delta P_e(t) + e(t) \tag{9}$$

$$p^3\Delta\delta(t) = -\frac{M+TD}{TM}p^2\Delta\delta(t) - \frac{D+\frac{1}{\omega_0 R}}{TM}p\Delta\delta(t) - \frac{1}{M}p\Delta P_e(t) - \frac{1}{TM}\Delta P_e(t) + e(t) \tag{10}$$

Based on the new continuous-time ARX model, let us consider Model 2 $T_2(s)$, the discrete-time ARX model for Model 2 can be expressed as:

$$\begin{bmatrix} D^3\Delta\delta(k) \\ \vdots \\ i \\ \vdots \\ D^3\Delta\delta(N) \end{bmatrix} = \begin{bmatrix} D^2\Delta\delta(k) & D\Delta\delta(k) & D\Delta P_e(k) & \Delta P_e(k) \\ \vdots & \vdots & \vdots & \vdots \\ i & i & i & i \\ \vdots & \vdots & \vdots & \vdots \\ D^2\Delta\delta(N) & D\Delta\delta(N) & D\Delta P_e(N) & \Delta P_e(N) \end{bmatrix} \cdot \begin{bmatrix} a_{22} \\ a_{21} \\ b_{21} \\ b_{20} \end{bmatrix} + e(t) \tag{11}$$

The corresponding coefficients are given by

$$\begin{cases} a_{22} &= -\frac{M+TD}{TM} \\ a_{21} &= -\frac{D+\frac{1}{377R}}{TM} \\ b_{21} &= -\frac{1}{M} \\ b_{20} &= -\frac{1}{TM} \end{cases} \tag{12}$$

The advantage of Structure 2 will be discussed in the case study by comparing the identified coefficients from these two structures.

B. Approximation method

Central difference approximation is used to handle the derivative operators $D^k y(t)$ in discrete-time ARX models. Considering numerical approximation to time integrals, order one to third central difference approximation method can be written as

$$\begin{aligned} Dy(t) &= \frac{y(t+h) - y(t-h)}{2h} \\ D^2y(t) &= \frac{y(t+2h) - 2y(t) + y(t-2h)}{4h^2} \\ D^3y(t) &= \frac{y(t+3h) - 3y(t+h) + 3y(t-h) - y(t-3h)}{8h^3} \end{aligned} \tag{13}$$

Considering replacing $p^k f(t)$ with $D^k f(t)$, we will discuss a way to approximate $p^k f(t)$. The general linear approximation of the differentiation operator is expressed as

$$p^k f(t) \approx D^k f(t) \triangleq \frac{1}{h^k}\sum_j \beta_{k,j} f(t+jh) \tag{14}$$

where k denotes the order of $D^k f(t)$ and j denotes the number j sampling point of the data. There is a small bias $O(h)$ in the differential operator approximation

$$D^k f(t) = p^k f(t) + O(h), k = 0, \ldots, n \tag{15}$$

To gain an accurate approximation, let us introduce a condition to apply on the weights $\beta_{k,j}$. We refer to (15) as natural condition [11]. Consider a case of sampling intervals h, the series expansion of $D^k f(t)$ can be expressed as

$$\begin{aligned} D^k f(t) &= \frac{1}{h^k}\sum_j \beta_{k,j} f(t+jh) \\ &= \frac{1}{h^k}\sum_j \beta_{k,j}\left[\sum_{\nu=0}^{k}\frac{1}{\nu!}j^\nu h^\nu p^\nu f(t) + O(h^{k+1})\right] \\ &= \sum_{\nu=0}^{k}\frac{1}{\nu!}p^\nu f(t)\left[\sum_j \beta_{k,j} j^\nu\right] h^{\nu-k} + O(h) \end{aligned} \tag{16}$$

The natural condition can be written as

$$\sum_j \beta_{k,j} j^\nu = \begin{cases} 0, & \nu = 0, \ldots, k-1 \\ k!, & \nu = k \end{cases} \tag{17}$$

When the natural condition meets, the approximation has an accuracy with a bias of linear order of h [11]. We will check if central difference algorithm meet the natural condition. For verification of $\beta_{k,j}$, we obtain the estimated $\beta_{k,j}$ according to (17), then compare with $\beta_{k,j}$ in (13). Taking the natural conditions into account, the estimated $\beta_{k,j}$ can be evaluated as

$$\begin{bmatrix} j_1^0 & j_2^0 & \cdots & j_{k+1}^0 \\ j_1^1 & j_2^1 & \cdots & j_{k+1}^1 \\ \vdots & \vdots & \vdots & \vdots \\ j_1^k & j_2^k & \cdots & j_{k+1}^k \end{bmatrix} \begin{bmatrix} \beta_{k,1} \\ \beta_{k,2} \\ \vdots \\ \beta_{k,j} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ k! \end{bmatrix} \tag{18}$$

The estimated $\beta_{k,j}$ are listed in Table 1. They are the same as in (13). It is proved that the central difference approximation satisfies the natural condition. In other words, the central difference approximation method is reasonable.

TABLE I
ESTIMATED $\beta_{k,j}$ FOR THE ORDER K

k \ j	-3	-2	-1	0	1	2	3
1			$-\frac{1}{2}$		$\frac{1}{2}$		
2		$\frac{1}{4}$		$-\frac{1}{2}$		$\frac{1}{4}$	
3	$-\frac{1}{8}$		$\frac{3}{8}$		$-\frac{3}{8}$		$\frac{1}{8}$

IV. CASE STUDIES

To investigate the effectiveness of LSE estimation with the central difference approximation method, the benchmark model has been built in the MATLAB/Simulink according to Fig.2. A sine wave is considered as the input, ΔP_e. we select $\Delta\omega$ and $\Delta\delta$ as the output signals respectively. The data of input and output signals are collected and used for estimation. The parameter vector, θ, is solved with LSE direct solution. After that, we compared the solved θ with true values to examine the accuracy of LSE for different ARX structures. In addition, the white noise is added to the input to test the noise tolerance of two structures.

We defined four cases based on the structure and mode.

$Case\ 1a$: Structure 1, Model 1

$Case\ 1b$: Structure 1, Model 2

$Case\ 2a$: Structure 2, Model 1

$Case\ 2b$: Structure 2, Model 2

The simulation time is set to 10 sec and the sampling period h is set to 0.01 sec. The benchmark model parameters used in the simulation studies as well as in the theoretical calculation are provided in Table II. The corresponding model is simulated in MATLAB/Simulink. The screen shot of the Simulink model is shown in Fig. 3.

TABLE II
THE BENCHMARK MODEL PARAMETERS

R	T	M	D	h
0.05	0.15	0.0265	0.1	0.01

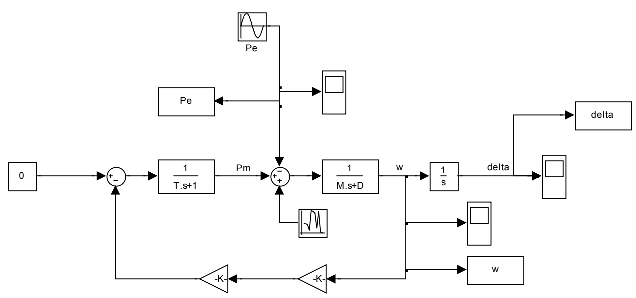

Fig. 3. Matlab/Simulink simulation blocks.

When h=0.01s, the generator states, ΔP_e, $\Delta\omega$ and $\Delta\delta$ with no noise are shown in Fig. 4. These states with the noise, σ^2=0.1, are plotted in Fig. 5. The corresponding derivative operators $D\Delta\delta$, $D^2\Delta\delta$ and $D^3\Delta\delta$ are shown in Fig. 6 and Fig. 7.

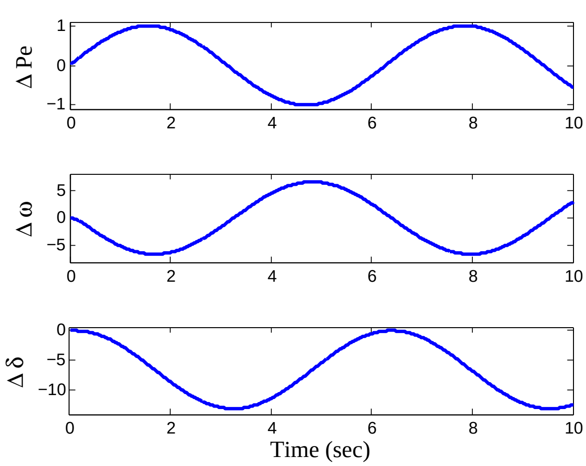

Fig. 4. ΔP_e, $\Delta\omega$ and $\Delta\delta$ with no noise.

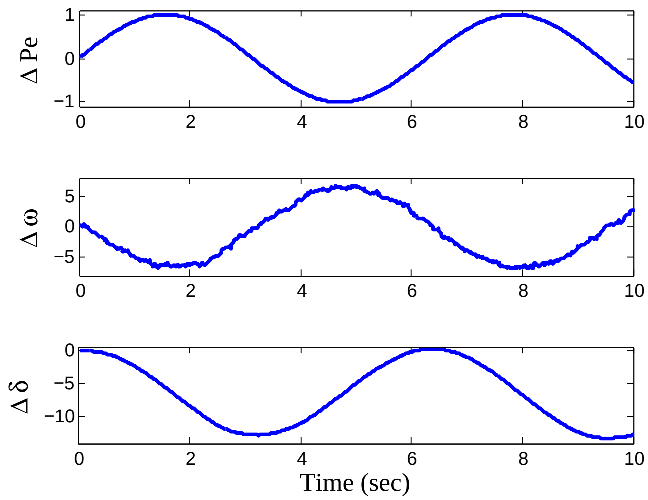

Fig. 5. ΔP_e, $\Delta\omega$ and $\Delta\delta$ with noise σ^2=0.1.

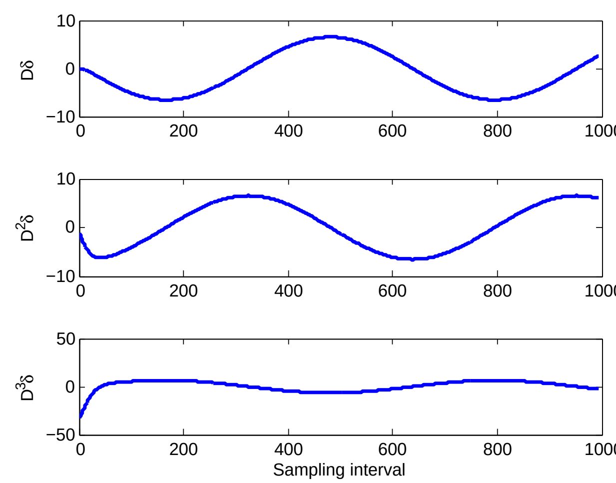

Fig. 6. $D\Delta\delta$, $D^2\Delta\delta$ and $D^3\Delta\delta$ with no noise.

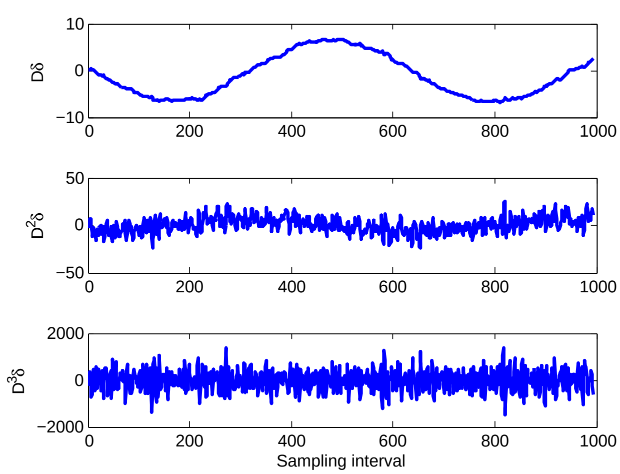

Fig. 7. $D\Delta\delta$, $D^2\Delta\delta$ and $D^3\Delta\delta$ with noise σ^2=0.1.

TABLE III

$Case\ 1a$							
Coefficients	True Value	$\sigma^2=0$	error(%)	$\sigma^2=10^{-8}$	error(%)	$\sigma^2=10^{-7}$	error(%)
a_2	-0.0040	-0.0040	0	-0.0036	10.0	-0.0019	52.5
a_1	-0.0415	-0.0415	0	-0.0389	6.27	-0.0275	33.7
a_0	-0.1527	-0.1531	0.26	-0.1530	0.20	-0.1527	0
b_0	-0.1500	-0.1498	0.13	-0.1325	11.7	-0.0575	61.7

TABLE IV

$Case\ 2a$							
Coefficients	True Value	$\sigma^2=0$	error(%)	$\sigma^2=10^{-5}$	error(%)	$\sigma^2=10^{-4}$	error(%)
a_2	-10.4403	-10.4390	0.01	-10.1145	3.12	-9.3961	10.0
a_1	-38.5032	-38.5236	0.05	-37.0937	3.66	-52.0069	35.1
a_0	-37.7358	-37.7111	0.07	-36.8109	2.45	-20.5194	45.6
b_0	-251.5723	-251.7038	0.04	-242.1864	3.73	-338.3568	34.5

TABLE V

$Case\ 1b$							
Coefficients	True Value	$\sigma^2=0$	error(%)	$\sigma^2=10^{-8}$	error(%)	$\sigma^2=10^{-7}$	error(%)
a_2	-0.0040	-0.0040	0	-0.0038	5.00	-0.0026	35.0
a_1	-0.0415	-0.0415	0	-0.0401	3.37	-0.0322	22.4
a_0	-0.1531	-0.1531	0	-0.1530	0.07	-0.1529	0.13
b_0	-0.1500	-0.1498	0.13	-0.1409	6.07	-0.0885	41.0

TABLE VI

$Case\ 2b$							
Coefficients	True Value	$\sigma^2=0$	error(%)	$\sigma^2=10^{-5}$	error(%)	$\sigma^2=10^{-4}$	error(%)
a_2	-10.4403	-10.4390	0.01	-10.2935	1.41	-9.6007	8.04
a_1	-38.5032	-38.5236	0.05	-38.8664	0.94	-48.4404	25.8
a_0	-37.7358	-37.7105	0.07	-36.5393	3.17	-24.6070	34.8
b_0	-251.5723	-251.6996	0.05	-253.8207	0.89	-315.3912	25.4

A. Comparison 1: $Case\ 1a$ versus $Case\ 2a$

Let us investigate the possible estimation with various noise levels σ^2 in $Case\ 1a$ and $Case\ 2a$. We will increase the noise level until the LS estimation is not accurate from σ^2=0. The estimated coefficients of two structures are shown in Table III and Table IV.

Based on Table III and Table IV, the estimated coefficients are exactly same as the true value without any noise. Moreover, the noise tolerance is 10^{-8} in $Case\ 1a$ while $Case\ 2a$ has a higher noise tolerance, 10^{-5}.

B. Comparison 2: $Case\ 1b$ versus $Case\ 2b$

The estimated coefficients of two cases are shown in Table V and Table VI.

According to Table V and Table VI, the same result can be observed. The coefficients of $Case\ 1b$ can be estimated correctly when σ^2 from 0 to 10^{-8}, while that range for $Case\ 2b$ is from 0 to 10^{-5}.

In conclusion, the performance of LSE in Structure 2 is better than its performance in Structure 1 because the previous one has a better noise tolerance. The reason why Structure 2 is better at handling the noise than Structure 1 will be discussed in the following subsection.

C. Analysis of different noise structures

The different noise structure usually can cause the different noise tolerance. For the analysis of the noise structures, developing the mathematical model will be helpful.

First it is easy to rewrite (3) for $Case\ 1a$ as

$$(1+Ts)\Delta P_e(t) = \left(-TMs^2 - Ms - TDs - D - \frac{1}{\omega_0 R}\right) \cdot \Delta\omega(t) + e_1(t) \quad (19)$$

and hence

$$\Delta\omega(t) = -\frac{1+Ts}{TMs^2+Ms+TDs+D+\frac{1}{\omega_0 R}}\Delta P_e(t) + \frac{1}{TMs^2+Ms+TDs+D+\frac{1}{\omega_0 R}}e_1(t) \quad (20)$$

Next, use (9) for $Case\ 2a$ to conclude

$$\left(s^2 + \frac{M+TD}{TM}s + \frac{D+\frac{1}{\omega_0 R}}{TM}\right)\Delta\omega(t) = \left(-\frac{1}{M}s - \frac{1}{TM}\right) \cdot \Delta P_e(t) + e_2(t) \quad (21)$$

Hence

$$\Delta\omega(t) = -\frac{1+Ts}{TMs^2+Ms+TDs+D+\frac{1}{\omega_0 R}}\Delta P_e(t) + \frac{TM}{TMs^2+Ms+TDs+D+\frac{1}{\omega_0 R}}e_2(t) \quad (22)$$

The difference between Structure 1 and 2 is a extra part TM beyond $e_2(t)$. TM is 0.003975 in our case study. It means

$e_1(t)$ is 250 times lager value than $e_2(t)$. Therefore, Structure 2 has a better noise tolerance than Structure 1.

V. Conclusion

In this paper, central difference approximation of the derivatives is used to investigate the LSE estimation of continuous-time ARX model. The coefficients have been estimated based on two ARX structures with different levels of noise. It is found that the choice of ARX structures is crucial for the LS estimation when noise is considered. Based on the estimated results comparison, Structure 2 have better noise tolerance than Structure 1. The different noise structure was presented to explain the better performance of Structure 2. However, the noise tolerance is still not good enough, $\sigma^2 = 10^{-5}$, even if using Structure 2. In the future paper, this method will be improved or a new structure will be investigated to handle a higher level noise.

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